

Approximative determination of failure probabilities in probabilistic fracture mechanics

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1 INTRODUCTION

In statistical reliability theory, approximation methods play an important role for the calculation of failure probabilities. The first-order-second-moment method (FORM) (Hohenbichler & Rackwitz 1983, Madsen 1986 and references therein) has been successfully used in structural reliability theory to determine the failure probabilities of structural systems, and several computer codes have been developed for this purpose.

The main advantages of FORM in comparison with the "exact" Monte Carlo simulation is that it is much easier to handle and less consuming of computer time. On the other hand, there is no possibility of estimating the error of the approximation. This means, that FORM calculations have to be verified with simulation results unless experience indicates that the result will be in the correct order of magnitude. Thus, the applicability of FORM has to be demonstrated for each field separately.

In this paper, the possibility of using FORM in probabilistic fracture mechanics (PFM) is investigated. After a short review of the method and a description of some specific problems occurring in PFM applications, results obtained with FORM for the failure probabilities in a typical PFM problem are compared with those determined by a Monte Carlo simulation.

2 BASIC IDEAS OF FORM

Statistical reliability theory in structural mechanics deals with the determination of failure probabilities P_f of components from the scatter of the applied loads and of the resistance of the structure. The failure behaviour of a component is described by a failure function g which depends on the basic random variables X_i ($i = 1, \dots, n$) denoting the applied loads and the structural resistance. $g(x) < 0$ implies failure, whereas no failure occurs for $g(x) > 0$. $g(x) = 0$ defines the so-called failure surface, a hyper-surface in the n -dimensional space of basic variables which separates the failure domain F ($g(x) < 0$) and the safe domain ($g(x) > 0$). P_f is defined as the probability content of the failure domain F and is determined by

$$(1) P_f = \int_F f(x_1) \dots f(x_n) dx_1 \dots dx_n$$

where $f(x_i)$ denotes the probability density of the basic variables X_i which, for the sake of simplicity, are assumed to be independent.

For linear functions $g(x)$ and standard normally distributed basic variables X_i eq. (1) can be solved analytically and the failure probability is equal to $\Phi(-\mathbb{R})$, where $\Phi(\cdot)$ denotes the standard normal integral and the reliability index \mathbb{R} is the distance of the hyperplane $g(x) = 0$ from the origin.

For non-linear $g(x)$ and non-normal basic variables X_i , the first order reliability method (FORM) yields an approximate solution of the failure integral, eq. (1). The approximation consists in

1. a linearization of $g(x)$ at the so-called design point on the hyper-surface $g(x) = 0$, and
2. a substitution of the distribution of the basic variables by "equivalent" normal distributions which coincide with the original distributions in the design point.

The design point and the substitute normal distributions are determined with an iterative procedure (Ditlevsen 1981). The approximate value of P_f is then given by

$$(2) P_f \approx \Phi(-\mathbb{R})$$

with $\Phi(\cdot)$ and \mathbb{R} defined as above.

3 APPLICATION OF FORM TO PROBABILISTIC FRACTURE MECHANICS

In almost all problems considered in probabilistic fracture mechanics, the failure integrals, eq. (1), have some common features:

- Generally, the components of interest are very reliable, so that the failure probabilities are expected to be very small.
- The shape of the hyper-surface $g(x)=0$ is rather smooth and does not show a pronounced curvature or sudden turns and bendings. Therefore the error in the linearization should be small. However, there is no way of estimating this error.
- The failure function $g(x)$ depends on a rather limited number of independent basic variables (usually less than 10). This means that the problems arising in applications of FORM to multi-dimensional cases ($n = 20$ or larger) do not occur.
- In PFM, the stress parameter characterizing the loading of the structure is a function of the crack size and shape and of the applied load.
- If fatigue crack growth is taken in consideration, crack size and shape change with time. This implies that the corresponding probability densities become dependent on time and have to be updated if the failure integral is evaluated as a function of time. This problem is avoided by using the initial values for the crack size and shape as basic variables.

- Fatigue crack growth can lead to local failure by leakage or to global instability by breakage (Munz 1984), i.e. a bi-modal failure function is called for. It will be shown, however, that this difficulty can be avoided by determining two different failure integrals separately.

Let σ_{2D} be the critical stress for local instability, and σ_{1D} the critical stress for global instability. If the applied stress σ exceeds σ_{2D} for a specific surface crack in a component, the crack will become locally unstable and penetrate the wall leading to a through-wall crack (leak). In case of $\sigma \geq \sigma_{1D}$ global instability (break) leading to excessive damage will follow, whereas the through-wall crack will remain stable for $\sigma < \sigma_{1D}$.

These considerations lead to the following expressions for the leak and the break probabilities.

$$(3) \quad P_{\text{failure}} = P(\sigma \geq \sigma_{2D})$$

$$(4) \quad P_{\text{break}} = \begin{cases} P(\sigma \geq \sigma_{1D}) & \text{for } \sigma_{1D} > \sigma_{2D} \\ P(\sigma \geq \sigma_{2D}) & \text{for } \sigma_{1D} \leq \sigma_{2D} \end{cases}$$

$$(5) \quad P_{\text{leak}} = \begin{cases} P(\sigma_{2D} \leq \sigma < \sigma_{1D}) & \text{for } \sigma_{1D} > \sigma_{2D} \\ 0 & \text{for } \sigma_{1D} \leq \sigma_{2D} \end{cases}$$

For $\sigma_{2D} < \sigma_{1D}$, P_{failure} and P_{break} can be evaluated with the failure functions

$$(6) \quad P_{\text{failure}} = P(\sigma \geq \sigma_{2D}) \quad \text{with } g_{2D} = \sigma_{2D} - \sigma$$

$$(7) \quad P_{\text{break}} = P(\sigma \geq \sigma_{1D}) \quad \text{with } g_{1D} = \sigma_{1D} - \sigma$$

The leak probability can be derived from eqs. (6)-(7)

$$(8) \quad P_{\text{leak}} = P_{\text{failure}} - P_{\text{break}}$$

This difference becomes negative for $\sigma_{2D} \geq \sigma_{1D}$. In this case we have:

$$(9) \quad P_{\text{leak}} = 0 \quad \text{and}$$

$$(10) \quad P_{\text{break}} = P_{\text{failure}}$$

Equations (6)-(10) show that no bi-modal failure function is necessary to perform a probabilistic leak-before-break analysis and simple FORM can be applied without any modifications.

4 EXAMPLE

A probabilistic fracture mechanical reliability analysis of a pipe elbow in the German fast breeder reactor is presented in (Theodoropoulos & Brückner 1987) where the Monte Carlo method was used to deter-

mine the failure probabilities. With the fracture mechanical and statistical model given in (Theodoropoulos & Brückner 1987) one of the circumferential welds of the pipe elbow is used to compare FORM with Monte Carlo in a typical PFM example.

Figures 1-3 show the leak probability as a function of time for the point of maximum stress in a circumferential weld. The fatigue loads causing crack growth are the design load cycles of the fast breeder reactor. As there is no information available about the distribution of crack size and shape characterized by the depth a and the depth-to-length ratio a/c the results of an extensive parameter study are reported in (Theodoropoulos & Brückner 1987) to investigate the influence of the model selected on the results. For example, the following crack depth distributions were considered:

exponential distribution with parameter $\lambda = 1 \text{ mm}^{-1}$ (basic model)

exponential distribution with parameter $\lambda = 0.5 \text{ mm}^{-1}$

exponential distribution with parameter $\lambda = 0.161 \text{ mm}^{-1}$ (Marshall 1976)

lognormal distribution with parameters $\mu = 1 \text{ mm}$ and $\sigma = 1$.
(Becher & Hansen 1974)

The results shown in figures 1-3 indicate that

- the numerical agreement between FORM and Monte Carlo results is excellent in all cases considered;
- the error in the approximation does not depend significantly on the input distributions;
- the modelling error (2-3 orders of magnitude) dominates completely the uncertainty of the results compared to which the approximation error (less than 10 % in this case) is negligible.

5 CONCLUSIONS

The results presented indicate that a reliability analysis in probabilistic fracture mechanics can be performed with sufficient accuracy with the first order reliability method. The deviations of the approximate results from the exact ones obtained with Monte Carlo simulation are small compared to the modelling uncertainties.

The main advantage of the first order reliability method is that sensitivity studies can be performed in a very efficient way by merely changing the input variables, while in Monte Carlo computations, the variance reduction techniques used to improve the convergence properties of the simulation have to be adapted to each problem. If e.g. importance sampling is applied, a new set of importance sampling functions has to be determined when the input distributions are changed.

On the other hand, a few checks with Monte Carlo results are always required to establish confidence in the results obtained by first-order reliability methods and to give a rough estimate of the numerical error of the approximation.

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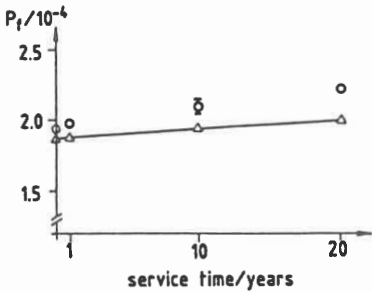


Fig. 1: Exponentially ($\lambda = 1$ mm) distributed crack depth

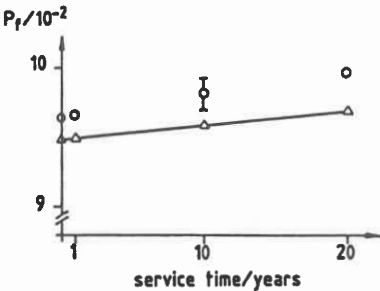


Fig. 2: Exponentially ($\lambda = 0.161$ mm) distributed crack depth

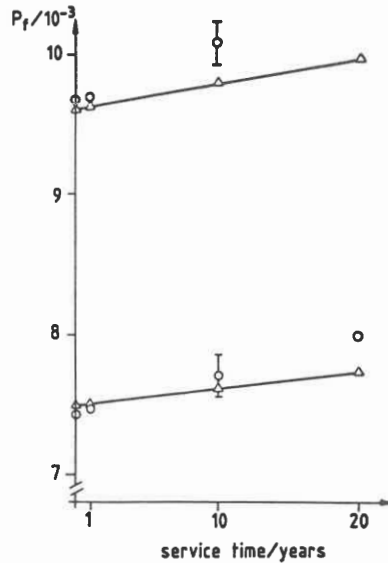


Fig. 3: Exponentially ($\lambda = 0.5$ mm⁻¹) and lognormally ($\mu = 1$ mm, $\sigma = 1$) distributed crack depth

Fig. 1-3: Leak probabilities computed with

△ FORM

○ Monte Carlo method (N = 5000 simulation runs);

bars indicate statistical error of Monte Carlo simulation