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Design Rules Against Elasto-Plastic Buckling

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Abstract

This paper presents a simplified method of analysis elastoplastic buckling. It is based on the determination of the perfect structure critical load, corrected by the use of diagrams in order to take into account the effects of geometrical imperfections. Particular attention is paid to plasticity and a possible unstable buckling behavior history. The analysis requires a knowledge of the Euler load (results of elastic calculations) and defect shapes so that diagrams can be used.

1. Introduction

The purpose of this paper is to present a simplified method of design to prevent buckling, the principles and validity of which are based on 10 years French experience evaluating the buckling risk of thin structures typical of those associated with pool type fast breeder reactors. Buckling can involve changes in shape large enough to reduce structural strength. Therefore, the designer must take steps to avoid this risk. Buckling loads related to these large and thin structures are very sensitive to unavoidable geometrical defects. A careful analysis taking account the shape of those defects and material non linearities into account can be very difficult and expensive. It is therefore necessary to dispose a simplified and reasonably conservative method to perform preliminary analysis. Such a method was presented by D. Moulin in June 1984 [1]. The present suggestion differs from the latter by the way in which possible unstable post buckling behavior is taken into account, and gives results, which are more realistic, but always conservative.

2. Diagram Representation

If we consider a perfectly plastic beam with a rectangular cross section, a width = 1, an initial deflection f_o, then the defect amplification due to an axial load can be written in a first approximation as: (1)

$$f - f_o = f\ P/P_E$$

where f is the final deflection

P the axial load

P_E the Euler's load.

The efforts sustained by the most stressed section is due to the axial load P and the moment Pf.

Also $M = Pf = \frac{P f_o}{1 - P/P_E}$ (2)

At the elastic yield point P, the whole section is plastic and we can write the classical equation:

$$\frac{M}{M_L} + \left(\frac{P}{P_L}\right)^2 = 1 \quad (3)$$

where P_L = yield load without defects

M_L = yield moment

From equation two, we obtain the following relation between P/P_L and P/P_E:

$$4 f_o/e\ P/P_L + (P/P_L)^2(1 - P/P_E) - (1 - P/P_E) = 0 \quad (4)$$

Also, in the plane defined by the axis P/P_L and P/P_E, the load reduction diagrams can be presented with parametric curves depending on f_o/e (fig. 1).

The same presentation can be used for elastoplastic buckling diagrams with the conventional elastic load limit instead of the limit load.

3. Stable or Unstable Post Buckling Behavior

With a second order approximation equation (1) could be write:

$$\frac{Pf}{e} = P_E \left(\frac{f-f_o}{e}\right) + \mu_1 \left(\frac{f-f_o}{e}\right)^2 \quad (5)$$

where the second term of the right member determines unstable post buckling behavior.

If $\mu_1 \geqslant 0$, the post buckling is stable and the load increases continuously with the deflection, but if $\mu_1 < 0$ the post buckling is unstable and the load increases to a maximum value called "buckling load" (fig.2). For the most stressed section of our beam the elastic moment (equation 2) can be written: $M = Pf = EI\chi$ (6)

where E is Young's modulus

I the inertia

χ the curvature variation.

From equation (5), we can calculate the curvature as follows:

$$\chi = \frac{P_E}{EI}(f-f_o) + \frac{\mu_1}{EIe}(f-f_o)^2 \quad (7)$$

Here, the unstable behavior is seen as an alteration in the relationship between curvature and deflection.

4. Hypotheses and Approximations

4.1 Plastic curvature. Equation seven is true only in the elastic regime. We make the hypothesis that it can also be used in plastic regime.

4.2 μ_1 value. An arbitrary conservative value of μ_1 can be chosen as $\mu_1 = -0.31\ P_E$. There corresponds to the results obtained by Koïter to a cylindrical shell with an axisymmetric imperfection. Indeed, the critical elastic load reduction of this structure is given by Koïter [2]: $P/P_E = A_i$; A_i is determined by the equation:

$$(1 - A_i)^2 = K \frac{f_o}{e} A_i \quad (8)$$

where e is the shell thickness and K a coefficient equal to $\frac{3}{2}$ $(3(1-\nu^2))^{1/2}$; K = 2.48 for ν=0.3.

In the first approximation, equation (8) can be written:

$$P/P_E = 1 - (1.24\ f_o/e)^{1/2} \qquad (9)$$

From our equation (5) we can determine the maximum deflection f_1 at which the buckling occurs. There is given by the equation $\frac{dP}{df} = 0$. Also, the reduction load factor due to an unstable behavior P/P_E can be calculated with to substitute of f by f_1 in the equation 5.

Finally, we obtain in the first approximation:

$$P/P_E = 1 - (4\mu_1/P_E\ f_o/e)^{1/2}$$

and μ_1 is determined from equations (9) and (10).

5. Diagram Equations

The real mechanical materials properties of the beam (stress-strain curve) allow the moment associated with the curvature χ and therefore with defects f and f_o and the load P to be sustained. Also M is a function of P, and f_o, $M = g(P,f,f_o)$. The same load gives a moment $M = Pf$, and $Pf = g(P,f,f_o)$: the buckling occurs when this equation has any solution. Graphically (figure 3) when the straight line M = Pf in the plan defined by the axis M and f is a tangent to the curve $M = g(P,f,f_o)$. P reaches its largest possible value when $(P - \frac{\partial g}{\partial f})df = 0$ for any deflection variation df.

Therefore $P = \frac{\partial g}{\partial f}(P,f,f_o)$.

This equation can be solve by an iteration method.

6. Results

Diagrams for stainless steel type 316L are given: figure 4 corresponds to stable buckling behavior ($\mu_1 = 0$) and figure 5 to unstable buckling behavior for a deflection $\delta = f_o/e$ from 0.01 to 3 times the shell thickness. In order to use these diagrams it is necessary to compute elastically a) the first bifurcation load P_E and b) the plastic load P_L } to the perfect structure where the first point on the shell reaches a stress equal to the conventional elastic limit and the ratio P_L/P_E. It is also necessary to know the ratio δ between the amplitude of the largest initial defect and the shell thickness. The predicted buckling load for an elasto-plastic structure with initial defects is determined by the coordinates (P/P_L or P/P_E) of the intersection between the straight corresponding to a slope equal to P_L/P_E and the curve associated with the normalized defect δ.

7. Conclusions

We propose a simplified method to conservatively estimate the critical buckling load of thin soft shells such as those encountered in type Liquid Metal Fast Breeder Reactors. The method takes into account load reductions due to plasticity, initial deflection and unstable post buckling behavior. It can be used to establish simplified analysis rules to prevent buckling. The method has been validated from 10 years of experimental tests in France and the results will be presented in Paris SMIRT post seminar.

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References

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[2] KOITER, W.T., "The Effect of Axisymmetric Imperfections on the Buckling of Cylindrical Shells under Axial Compression", Konink1.Ned.Akad - Weterschap Proc. B66, pp 265-273, (1963).

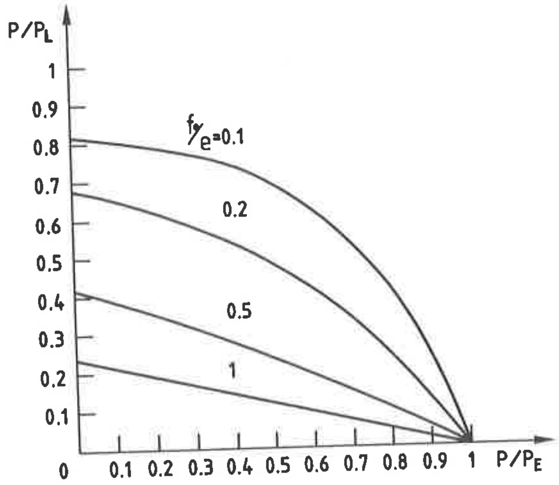

Fig. 1 LIMIT LOAD DIAGRAM

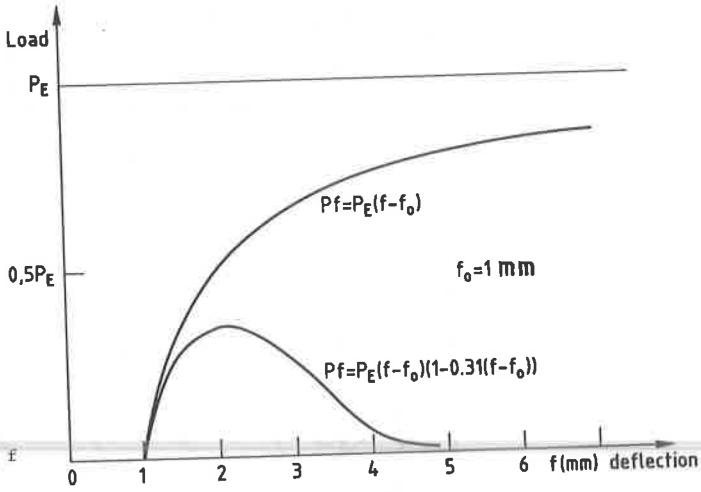

Fig. 2 LOAD P - DEFLECTION f

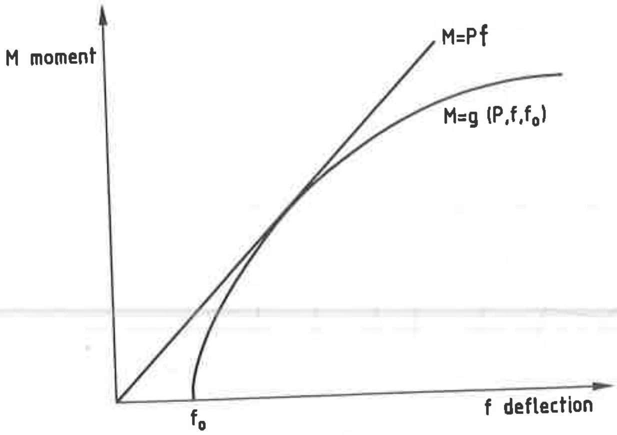

Fig. 3 MOMENT M - DEFLECTION f

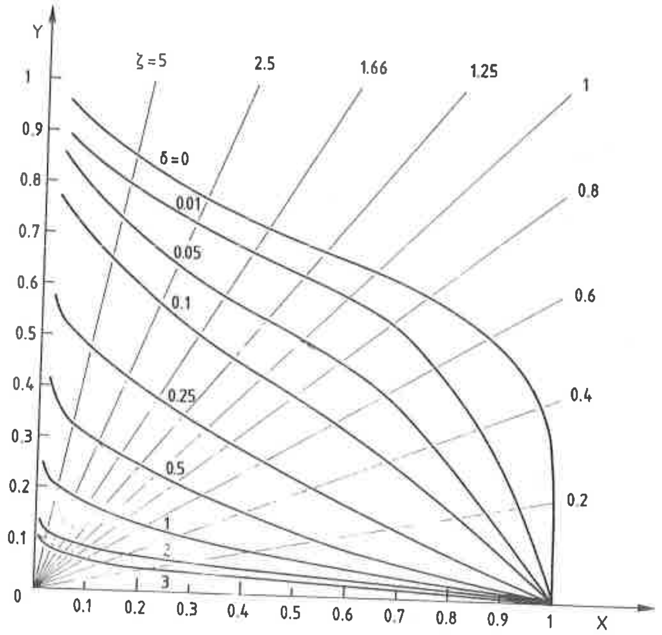

Fig. 4 BUCKLING DIAGRAMS - STABLE POST BUCKLING

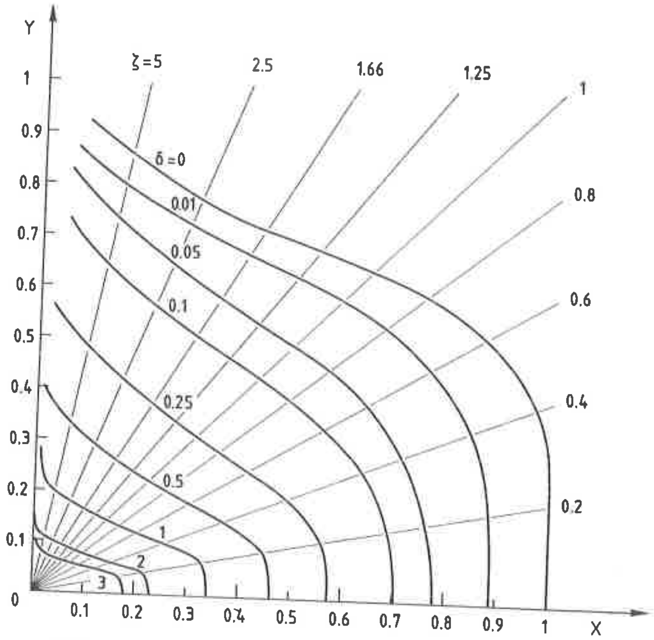

FIG. 5 BUCKLING DIAGRAMS - UNSTABLE POST BUCKLING