Nonelastic soil-structure interaction by BE- and FE-methods

L.Hadjikov & P.Dineva
Institute of Mechanics and Biomechanics, Sofia, Bulgaria
Ts.Rangelov
Institute of Mathematics, Sofia, Bulgaria

I. INTRODUCTION

The main purpose of this paper is to study the soil-structure interaction for multilayered,nonlineary physically soil system with arbitrary geometric form of boundaries between the layers,accounting the topographic specifics on the free surface.The behaviour of the structure is described by the finite element method(FEM).For the description of the propagation of the seismic waves in the soil the boundary element method (BEM) is used.

2. MECHANICAL TREATEMENT OF THE PROBLEM

To investigate the problem we shall study the propagation of the seismic waves in the soil using BEM.As a product we shall obtain the accelerogramm of the seismic signal wich entrance the structure.The dynamic response of the structure under such a seismic signal is described by FEM.

The assumptions for solving the problem for propagating the seismic waves in the soil are the next:

-the domain in which the seismic waves propagate is multi layered,nonelastic with nonparallel boundaries between the layers and topographic peculiarities on the free surface (see Fig.I).

-the nonelastic behaviour of the soil is described by the model of Maxwell-Gourevich giving an account of absorbing the seismic waves with small amplitude. This is a consequence of elasto-relaxational processes with a limited and continuous spectrum of relaxation times

-the incident two dimensional harmonic wave is SH- wave

$$\text{(I)} \quad u_p^i = \exp(ik^* (y\cos Q - x\sin Q)).\exp(-i\omega t)$$

where: k^* is a complex wave vector,the representation of which depends on the choosed physical model; ω is the frequency of the wave;t is the time;x,y are the coordinates and Q is the angle of the incident wave with respect to the y axis.

-since we deal with the two dimension problem at the plane (x,y) we get the stress

$$(2)\quad P_z=\bar{\mu}^*\frac{\partial u}{\partial n} \quad , \quad \bar{\mu}^*=\frac{\rho\,\omega^2}{(k^*)^2}$$

where ρ is a density; $\bar{\mu}^*$ is a complex shear moduli which depends on the physical parameters of the model of Gourevich

The assumptions for solving the problem for dynamic behaviour of the structure are the next:

- the structure is discretisized by FEM
- the lose of the energy under the motion of the structure is account on two ways:either-damping matrix for physical linear constructive material or with the physical model of Maxwell-Gourevich(Gourevich I974) for residual deformations of the structure with a plastic behaviour.

3. THE BASIS FOR THE CHOOSED PHYSICAL MODEL

The model of Gourevich(Gourevich I974) describe the dissipation of energy under the elasto-relaxational behaviour of the soil.This model is valid for seismic waves with a small amplitude(ε :10^{-4} -10^{-6}) for isothermic process,macroscopic homogeneous and isotropic medium.The overall deformation ε_{ij} is a sum of the elastic deformation e_{ij} and elasto-relaxational deformation e_{pij} with a limited and continuous spectrum of relaxation times

$$(3)\qquad \varepsilon_{pij}(t)=\int_{T_p}^{T_M}\varepsilon_{ij}(t,T_p^*)\cdot\frac{1}{T_p^*}\,dT_p^*$$

where T_p, T_M are the lower and upper times of relaxation and $T_p \ll T_M$, $\varepsilon_{ij}(t,T_p^*)$ is described by a constitutive equation of the Kelvin-Foight type.

$$(4)\qquad \sigma_{ij}=\mu e_{ij}=\mu_p^*\varepsilon_{ij}(t,T_p^*)+(\mu+\mu_p^*)T_p^*\dot{\varepsilon}(t,T_p^*)$$

here- μ , μ_p^* are the elastic and nonelastic shear moduli.

The equation for the SH wave on the Gourevich medium takes the form

$$(5)\qquad \frac{\mu}{\rho}\cdot\frac{\partial}{\partial t}\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right]=\frac{\partial^3 u}{\partial t^3}+\frac{\mu}{\mu_p^* R}\cdot\int_{S_m}^{S_M}e^{-St}\left[\int_0^t e^{S\tau}\frac{\partial^3 u}{\partial\tau^3}\,d\tau\right]ds$$

here

$$(6)\qquad R=\ln\left(\frac{T_M}{T_p}\right),\quad S_m=\left[\left(1+\frac{\mu}{\mu_p^*}\right)T_M\right]^{-1},\quad S_M=\left(\frac{T_M}{T_p}\right)S_m$$

We take the solution of (5) at the form

$$(7)\qquad u(x,y,t)=u(x,y)\cdot\exp(i\omega t)$$

and obtain the Helmholtz equation with a complex wave vector for u(x,y).The complex wave vector depends on the physical moduli in the model of Gourevich.The dispersal curve of the velocity $c(\omega)$,the coefficient of damping $\alpha(\omega)$ and logarithmic decrement of damping $\Theta(\omega)$ inquiried by the Gourevich model describe clearly the typical properties of the seismic wave.The linear dependance $\alpha(\omega)$,the inpedance $\Theta(\omega)$and the weak dispersion $c(\omega)$ are shown by the experimental results (Molotova 1966,Bokanenko 1967).

4. MATHEMATICAL DESCRIPTION OF THE PROBLEM

For obtaining the propagation of the seismic SH wave in the multilayered,nonelastic medium with nonparallel boundaries between the layers and topographic peculiarities on the free surface under the assumption from 2. and the basis of 3 for u(x,y) the next problem is posed:

$$(8)\quad \begin{aligned} &\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+(K^*)^2 u(x,y)=0;\\ &\left.\frac{\partial u}{\partial n}\right|_{\Gamma_t}=\left.\frac{\partial u}{\partial n}\right|_{\Gamma_s}=0;\ u^i(x,y)=u^{i+1}(x,y)\text{ and }\sigma^i(x,y)=\sigma^{i+1}(x+y)\text{ for }(x,y)\in\Gamma^i_{t,B}\\ &\qquad u^0_B=u_0+u^0=u^{(1)}_B,\ \sigma^0_B=\sigma^1_B\text{ for }(x,y)\in\Gamma^1_B\end{aligned}$$

Here: u_0 is the displacement,obtained by interference of the incident and the reflect waves of the free boundary Γ_s^0 :

$$(9)\qquad u_0(x,y)=exp(-ik^*\sin Q\ ).[exp(ik^*y\cos Q)+exp(-ik^*y\cos Q)]$$

u^0 is the displacement in Ω_0 obtained by the scattered by Γ_B^1 waves; u_B^1 is the displacement on Γ_B^1 in Ω_1 ; (u_0+u^1) satisfies the Sommerfield's radiation condition at the infinity.

The dynamic response of structure under seismic interaction satisfies the next equations:

$$(10)\quad \left|\begin{aligned} &[M]_K\{\ddot{X}\}_K+[K]_K\{X\}_K-\sum_{j=1}^{N}[K]_K\{\Delta_j^*\}X_{fj}-\sum_{j=1}^{N}[K]_K\{\Delta_j^\varphi\}\varphi_j=-[M]_K\{B\}\ddot{X}_0\\ &m_{f\tau}(\ddot{X}_0+\ddot{X}_{f\tau})+k_{f\tau}\cdot X_{f\tau}=Q_\tau\\ &I_{\varphi\tau}\ddot{\varphi}_\tau+K_{\varphi\tau}\varphi_\tau-M_\tau-Q_\tau\cdot h_{f\tau}=0\end{aligned}\right.$$

Here $[M]_K=diag[m_i]$is the mass matrix for the structure,i=1, 2,...n;n is the number of concentrated masses; $[K]_K$ is the stiffness matrix obtained by FEM; $\{B\}$ is the cosine-director's vector of external action, $\{x\}_K$ is the vector of the displacement for masses of the structure; $\{\Delta_j^x\}$ and $\{\Delta_j^\varphi\}$ are the vectors of displacements for the unit displacement and rotation of the j foundament;N is the number of foundaments; X_{fj} , φ_j are displacement and rotation of the j foundament; $K_{f\tau}$, $K_{\varphi\tau}$ are the stiffness of soil by the displacement and rotation of the j foundament; Q_τ , M_τ are the cross force and moment with wich the structure loads the foundaments during its motion; $h_{f\tau}$ is the height of the foundaments; $\ddot{x}_0(t)$is the external seismic action's acceleration and is received of the solution of the problem which is described in 4.

5. THE SOLUTION OF THE PROBLEM

Using (Costabel & Stephan 1985) we substantiate the possibility of applying the BEM for solving the problem(2).We obtain for i layer:

$$\text{(II)}\quad u^{(i)}(x,y)C_j=\int_{\Gamma^{(i)}}\left[u^*(x,y,x_0,y_0)\cdot\frac{\partial u^{(i)}(x_0,y_0)}{\partial n_0}-u^{(i)}(x_0,y_0)\cdot\frac{\partial u^*(x,y,x_0,y_0)}{\partial n_0}\right]d\Gamma$$

here $(x,y)\in\Omega_i$, u^* is the Green function for the Neuman problem in the half-space

$$\text{(I2)}\quad u^*(x,y,x_0,y_0)=\frac{i}{4}\left[H_0^{(2)}(\kappa^*\tau)+H_0^{(2)}(\kappa^*\tilde{\tau})\right]$$

$H_0^{(2)}$ is the Hankel function of 2^{nd} type,0^{th} order, τ , τ_0 are the radius vectors of the observation point, sours point and its simetric point from y=0,n is the unit normal vector, c is a constant which depends only on the geometry of the domain.Taking (12) for $(x,y)\in\Gamma_t^i$, $(x,y)\in\Gamma_B^i$, $(x,y)\in\Gamma_S^i$ resp.(see Fig.I)we obtain a system of integral equations. After discretizing this equations,using a constant boundary elements,we get a system of complex algebraic equations

$$\text{(I3)}\quad \begin{bmatrix} H_{tt}^i & H_{tB}^i & H_{ts}^i \\ H_{Bt}^i & H_{BB}^i & H_{Bs}^i \\ H_{St}^i & H_{SB}^i & H_{SS}^i \end{bmatrix} \begin{Bmatrix} u_t \\ u_B \\ u_S \end{Bmatrix}^i = \begin{bmatrix} G_{tt}^i & G_{tB}^i & G_{ts}^i \\ G_{Bt}^i & G_{BB}^i & G_{BS}^i \\ G_{St}^i & G_{SB}^i & G_{SS}^i \end{bmatrix} \begin{Bmatrix} u_{t,n} \\ u_{B,n} \\ u_{S,n} \end{Bmatrix}^i$$

here: $u_t^i=\{u_{t_j}^i\}$, $u_B^i=\{u_{B_j}^i\}$, $u_S^i=\{u_{S_j}^i\}$ i is the layer with the boundaries Γ_t^i, Γ_B^i, Γ_S^i ,j=I,2,...N,N is the number of the knots on the boundary.To satisfy the boundary conditions (8)we use the method of transfer matrix(Maier & Novati 1985)and finaly we obtain the formula

$$\text{(I4)}\quad \tilde{A}^*\cdot\{u_0(x,y)\}=I^1 I^2 I^3\ldots I^L\cdot\begin{Bmatrix} u_t \\ u_S \end{Bmatrix}^L$$

where L is the number of layers.We can find the displacement on the free boundary with the situated structure by formula(I4) if we know the incident wave on the boundary of the first domain Ω_1 ,the geometry and the physical characteristics of the medium represented by A^*,I,I...I^L.Moreover in this way we can obtain the displacement and the traction at the arbitrary point from the boundary of the multilayered medium.

6. FINAL NOTES

The numerical results for the behaviour of the soil-structure interaction for nuclear power plant on a real geological site are obtained.The influence of the nonelastic soil and structure properties and the influence of the topographic specifities of the geological region are investigated.

REFERENCES

Gourevich ,G.I.1974.Deformiruemost sred i rasprostranenie seismicheski voln.Moskow:Nauka,/in Russian/

Molotova,L.V. 1966.O dispersii scorostei obemnih voln v tverdih porodah,J.Physic of Earth, /in Russian/
Bokanenko,L.I. Dispersia i poglostenie naprechnih voln v viniplaste i plexiglase,J.Phys.of Earth, 8/in Russian/
Costabel,M.& Stephan E.1985.A direct boundary integral equation method for transmission problems.J.Math.Anal. Appl.106:367-413
Maier,G.& Novati G.1985.On boundary element transfer matrix analysis of layered elastic systems.7th Int.Conf. on BE in Eng.Como.Italy

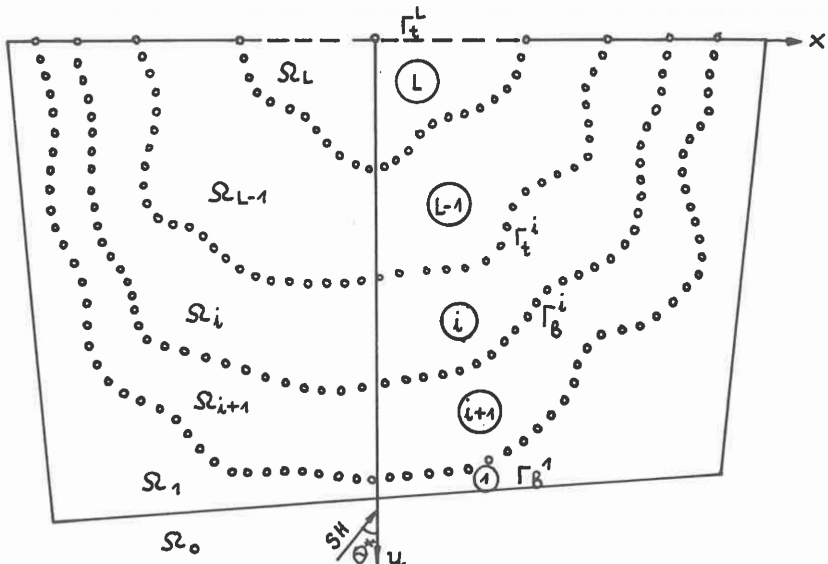

FIG. 1