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Wave propagation in layered ground considering the compressibilities of the constituents

Mitsuji K., Xue S., Sugimura Y.
Tohoku University, Faculty of Engineering, Japan

ABSTRACT

Bowen's theory of mixture is applied to wave propagation in stead of Biot's theory. In Bowen's theory, porosity is defined by the rate type equation. This allows porosity to change. In frequency domain, it is shown that porosity change means to introduce the compressibilities of the constituents. Bowen's theory is applied to porous layer which is included in the layered ground. Wave propagation in the several kinds of the layered ground is studied.

1 INTRODUCTION

It is useful to consider soil to be two-phased media in order to understand the dynamic behavior of the ground. Biot's theory[1] has been applied to earthquake engineering as the basic equations [2][3][4][5]. In Biot's theory, porosity which is one of the important parameters is defined as a constant. On the other hand, mixture theory has been developed since the 1950's in continuum mechanics[6]. Bowen proposed mixture theory[7] which is different from Biot's theory. According to mixture theory in continuum mechanics, volume fraction which can be extended to multi-phased media plays an important role. In Bowen's theory, volume fraction has been introduced to govern equations of two-phased media where volume fraction is equivalent to porosity in soil mechanics. Therefore, when soil is considered to be two-phased medium which consists of an elastic solid and one fluid, Bowen's theory allows porosity to change during dynamic loading. In this paper, Bowen's theory is applied to porous media, and basic characteristics of attenuation in the mixture and wave propagation in the layered ground including the porous layer are studied. Attenuation is discussed in the frequency domain through Q value. After Fourier transform, the equation of porosity defined by Bowen's theory is converted to the equations of motions of solid and fluid. It is shown that the porosity change means considering the compressibilities of the constituents in the frequency domain. Wave propagation in the layered ground including the porous layer has been studied using Biot's theory[2][3]. Porous layer modeled by Bowen's theory is applied to the layered ground, and basic characteristics of wave propagation are discussed considering the porosity change, namely, the compressibilities of the constituents. As basic examples, three types of layered ground are discussed which consist of three, four and five layers. The effect of the combination of layers including porous layers modeled by Bowen's theory is studied through amplitude of surface. Also, wave propagation in the time domain is shown using Ricker wavelet.

2 MIXTURE THEORY

2.1 Volume fraction

Volume fraction is considered to be an important variable in continuum mechanics. It is possible to measure a local structure of the mixture using this quantity. If ρ_a designates the mass of the ath constituent, the density of the mixture is defined by

$$\rho(\mathbf{x}, t) = \sum_{a=1}^{N} \rho_a(\mathbf{x}, t) \tag{1}$$

where ρ_a represents the mass density of the ath constituent per unit volume of the mixture. This quantity is bulk density. The true density of the ath constituent, which means the mass of the ath constituent per unit volume of the mixture, is defined by $\gamma_a(\mathbf{x}, t)$. The volume fraction of the ath constituent ϕ_a is defined by

$$\phi_a = \frac{\rho_a(\mathbf{x}, t)}{\gamma_a(\mathbf{x}, t)} \tag{2}$$

The volume fraction is the volume of the ath constituent per unit volume of the mixture. Therefore, it has the relation as follows,

$$\sum_{a=1}^{N} \phi_a(\mathbf{x}, t) = 1 \tag{3}$$

If the local structure of the mixture doesn't affect the response of the mixture, this kind of mixture is called miscible. On the other hand, immiscible mixture allows the volume of the mixture to be distinguished from that of each constituent. When the mixture consists of an elastic solid and one fluid, the volume fraction is equivalent to the porosity in soil mechanics. Since soil modeled as a two-phased mixture is dealt with in this paper, the volume fraction, solid and fluid are referred to as the porosity, soil particle and pore water, respectively. Also, estimation of the effect of the volume fraction means taking the porosity change into account during dynamic loading. Figure 1 shows the visible explanation of Bowen's mixture theory.

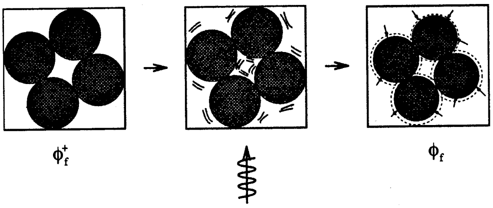


Fig. 1: Mixture with porosity change

2.2 Wave equation

The equations of motions proposed by Bowen[7] are described below. The third equation governs the behavior of the porosity.

$$\begin{aligned}
\rho_f \frac{\partial^2 \mathbf{u}_f}{\partial t^2} &= \lambda_f \mathrm{GRAD}(\mathrm{Div}\mathbf{u}_f) + \lambda_{sf}\mathrm{GRAD}(\mathrm{Div}\mathbf{u}_s) & (4)\\
&+\Gamma_f \mathrm{GRAD}\phi_f - \xi(\frac{\partial \mathbf{u}_f}{\partial t} - \frac{\partial \mathbf{u}_s}{\partial t}) \\
\rho_s \frac{\partial^2 \mathbf{u}_s}{\partial t^2} &= (\lambda_s + \mu_s)\mathrm{GRAD}(\mathrm{Div}\mathbf{u}_s) + \mu_s \mathrm{Div}(\mathrm{GRAD}\mathbf{u}_s) & (5)\\
&+\lambda_{sf}\mathrm{GRAD}(\mathrm{Div}\mathbf{u}_f) + \Gamma_s \mathrm{GRAD}\phi_f + \xi(\frac{\partial \mathbf{u}_f}{\partial t} - \frac{\partial \mathbf{u}_s}{\partial t}) \\
\frac{\partial \phi_f}{\partial t} &= -\Lambda_f\{(\phi_f - \phi_f^+) + \frac{\Gamma_s}{\Phi_f}\mathrm{Div}\mathbf{u}_s + \frac{\Gamma_f}{\Phi_f}\mathrm{Div}\mathbf{u}_f\} & (6)
\end{aligned}$$

where $\mathbf{u}_s$ and $\mathbf{u}_f$ are displacements of solid skeleton and fluid; ρ_s^+ and ρ_f^+ are the densities of solid and fluid, ρ_s and ρ_f are described with porosity $\rho_s = (1 - \phi_f^+)\rho_s^+$, $\rho_f = \phi_f^+ \rho_f^+$;ϕ_f^+ and ϕ_f are the initial volume fraction and the volume fraction (variable); g is gravity acceleration; λ_s, μ_s are Lamé's constants of the solid, λ_f is modulus of expansion of fluid, λ_{sf} is the coefficient of coupling damping between solid and fluid. The coefficient Λ_f has the physical dimension of the reciprocal time and determines the characteristic time of the relaxation process. Namely, Λ_f is considered to be a damping factor h. Γ_s, Γ_f and Φ_f are the coefficients related to the compressibilities of solid, fluid and pore of the mixture and can be expressed as $\Gamma_s = 1/C_s$, $\Gamma_f = 1/C_f$ and $\Phi_f = 1/C_p$ where C_s, C_f, C_p are the compressibility of solid, fluid and pore respectively. It means Γ_s, Γ_f and Φ_f are bulk modulus of solid, fluid and pore. Those coefficients can be decided according to Ishihara[5]. k_d is the permeability coefficient. ξ is the dissipation coefficient defined by

$$\xi = \phi_f^{+2} \rho_f^+ g / k_d \tag{7}$$

When the terms $\Gamma_f \mathrm{GRAD}\phi_f$ and $\Gamma_s \mathrm{GRAD}\phi_f$ are eliminated, Biot's equations can be obtained where the porosity is a constant. Based on Bowen[7], stresses are described with the effect of the porosity change.
Pore pressure is

$$p_f = \lambda_{sf}\mathrm{tr}\mathbf{E}_s + \lambda_f \mathrm{Div}\mathbf{u}_\mathrm{f} + \Gamma_f(\phi_f - \phi_f^+) \tag{8}$$

Effective stress is

$$\mathbf{T}^e = \lambda_s(\mathrm{tr}\mathbf{E}_s)\mathbf{I} + 2\mu_s \mathbf{E}_s + \lambda_{sf}\mathrm{Div}\mathbf{u}_f \mathbf{I} + \Gamma_s(\phi_f - \phi_f^+)\mathbf{I} \tag{9}$$

Therefore, total stress of mixture is

$$\mathbf{T} = (\lambda_s + \lambda_{sf})(\mathrm{tr}\mathbf{E}_s)\mathbf{I} + 2\mu_s \mathbf{E}_s + (\lambda_f + \lambda_{sf})\mathrm{Div}\mathbf{u}_f \mathbf{I} + (\Gamma_s + \Gamma_f)(\phi_f - \phi_f^+)\mathbf{I} \tag{10}$$

3 WAVE ATTENUATION

It is well known that the dispersion relation due to the interaction between solid and fluid appears in the porous media. Two kinds of P wave are obtained, namely P1 and P2 wave. P1 wave can be regarded as P wave in elastic media and P2 wave is very slow and highly attenuated [1][2] [3]. In order to study one dimensional attenuation of mixture, applying Fourier transform to (4) ~ (6), and substituting (6) into (4) and (5) in the frequency domain, the characteristic equation of P wave is derived. Solving the characteristic equation (11) and (13), phase velocities and Q which indicates attenuation are obtained.

$$
\begin{aligned}
&\{(\lambda_f - X_f)(\lambda_s + 2\mu_s - X_s) - (\lambda_{sf} - X_{sf})^2\}(\frac{k}{\omega})^4 \\
&-[\rho_f(\lambda_s + 2\mu_s - X_s) + \rho_s(\lambda_f - X_f) - \frac{i\xi}{\omega}\{(\lambda_f - X_f) + (\lambda_s + 2\mu_s - X_s) \\
&+2(\lambda_{sf} - X_{sf})\}](\frac{k}{\omega})^2 + \rho_s\rho_f - \frac{i\xi}{\omega}(\rho_s - \rho_f) = 0 \qquad (11)
\end{aligned}
$$

where X_f, X_s, X_{sf} are the terms which express the effect of the volume fraction and defined as follows.

$$X_f = \frac{\Lambda_f\Gamma_f^2}{\Phi_f(\Lambda_f + i\omega)} \quad X_s = \frac{\Lambda_f\Gamma_s^2}{\Phi_f(\Lambda_f + i\omega)} \quad X_{sf} = \frac{\Lambda_f\Gamma_s\Gamma_f}{\Phi_f(\Lambda_f + i\omega)} \qquad (12)$$

These relations show that the porosity defined by (6) affects the elastic coefficients related to the compressibilities of the constituents in the frequency domain.
The characteristic equation of S wave is not affected by volume fraction, because the terms about volume fraction are expressed by GRADϕ_f in the equations (4) and (5). Therefore, the characteristic equation of S wave is written as follows [1][2].

$$(-\rho_f\mu_s + \frac{i\xi}{\omega}\mu_s)(\frac{k}{\omega})^2 + \{\rho_s\rho_f - \frac{i\xi}{\omega}(\rho_s + \rho_f)\} = 0 \qquad (13)$$

Figures 2 and 3 show Q of P1 wave in the 2nd. and 4th.layer of Table 1, where $\Gamma_f = 2000 (kg/m^2)$, $\Gamma_s = 20000(kg/m^2)$, $\Lambda_f = 0.02$ in both cases. Attenuation is expressed the inverse of Q. In the low frequency domain, Q decreases when the compressibilities are considered. As the value of $\Phi_f(kg/m^2)$ is less, Q decreases more. When bulk modulus of pore is smaller, namely, pore is more likely to be collapsed, waves are more attenuated. Q changes only in the low frequency domain. It is because the form of (12) has the effect only in the low frequency domain.

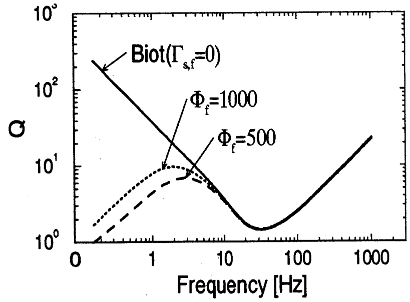


Fig. 2: Q of P1 wave in the 2nd. layer (Table 1)

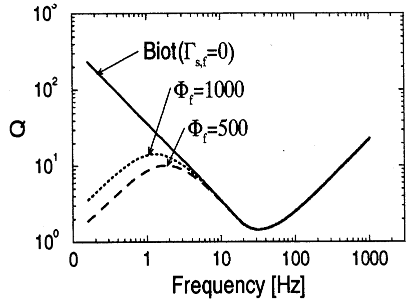
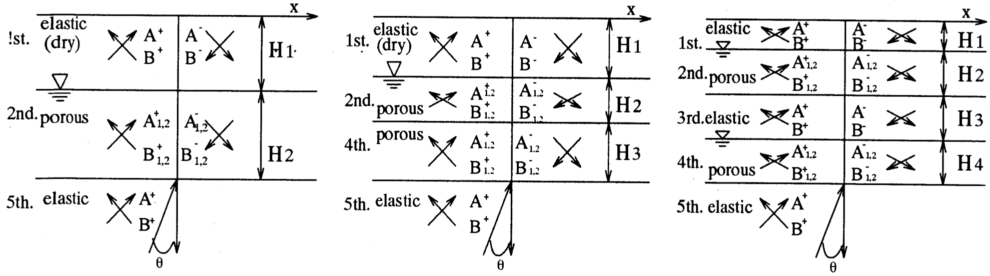


Fig. 3: Q of P1 wave in the 4th. layer (Table 1)

4 WAVE PROPAGATION

In this section, Bowen's mixture theory is applied to wave propagation in the layered ground. As basic examples, three kinds of the layered ground(Fig.4) are examined using Haskell method[8]. To carry out calculations, boundary conditions described below should be satisfied [2][3].



(a)3-layered ground (b)4-layered ground (c)5-layered ground

Fig. 4: Configurations of the layered ground

At the top of the surface layer, tractions are considered to be zero.

$$T_{xz} = 0, \quad T_{zz} = 0 \tag{14}$$

At the top of the porous layer, tractions and displacements of the elastic layer are equal to those of the porous layer and pore pressure vanishes.

$$T^e_{xz} = T^b_{xz}, \quad T^e_{zz} = T^b_{zz}, \quad u^e_{x,z} = u^b_{x,z}, \quad p_f = 0 \tag{15}$$

At the bottom of the porous layer, tractions and displacements of the porous layer are equal to those of the elastic half space and water flow doesn't occur.

$$T^b_{xz} = T^e_{xz}, \quad T^b_{zz} = T^e_{zz}, \quad u^b_{x,z} = u^e_{x,z}, \quad \phi^+_f(\dot{u}^b_z - \dot{u}^f_z) = 0 \tag{16}$$

where T, u and p_f mean traction, displacement and pore pressure respectively; and subscripts x and z mean x and z directions respectively; superscripts b and e represent bulk and elastic media respectively.

The parameters are shown in Table 1. In this table, (e) means elastic layer, (p) means porous layer modeled by Bowen's theory. In Fig.4, Ground(a) consists of the 1st., 2nd. and 5th. layer in Table 1. Ground(b) consists of the 1st., 2nd., 4th. and 5th. layer in Table 1. Ground(c) consists of all the layers in Table 1. The thicknesses of each layer are shown in Table 2. In order to study the effect of the porosity change, incident P wave problem is examined ($\theta = 0$).
Amplitude of the surface in each case in the frequency domain due to the vertical incident P wave are shown in Fig.5 to Fig8. Figures 5 and 6 are the cases of Ground(a) (3-layers) and Ground (b) (4-layers). Figures 7 and 8 have different patterns of layers in Ground(c) (5-layers) which are shown in Table 2. The parameters of the compressibilities are described in Table 3. The effect of the compressibilities are compared with Biot's

Table 1: Parameters of each layer of the ground

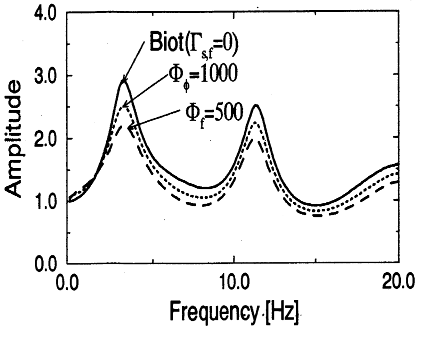
	1st.(e)	2nd.(p)	3rd.(e)	4th.(p)	5th.(e)
$V_s(m/s)$	150	250	200	300	450
$V_p(m/s)$	450	1000	700	1200	1500
$\rho(t/m^3)$	1.8	1.8	1.6	1.9	2.2
$\rho_s(t/m^3)$		2.23		2.2	
$\rho_f(t/m^3)$		1.0		1.0	
$k_d(cm/s)$		1.0		10.0	
$\phi_f^+(\%)$		35	20		
h	0.03	0.02	0.02	0.005	0.005

Table 2: Thickness of each layer of ground

(a)3-Layers Ground : $H_1 = 30(m)$, $H_2 = 50(m)$
(b)4-Layers Ground : $H_1 = 20(m)$, $H_2 = 30(m)$, $H_3 = 30(m)$
(c)5-Layers Ground : (I) $H_1 = 20(m)$, $H_2 = 25(m)$, $H_3 = 15(m)$, $H_4 = 20(m)$
(II) $H_1 = 20(m)$, $H_2 = 15(m)$, $H_3 = 25(m)$, $H_4 = 20(m)$

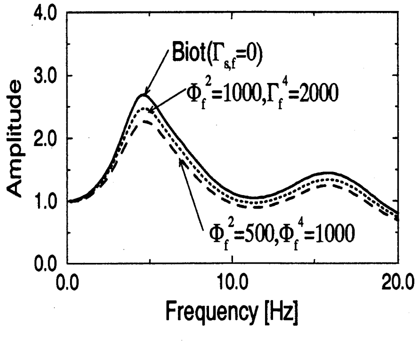
Table 3: Parameters of compressiblities

	the 2nd. layer	the 4th. layer
Γ_f	$20000(kg/m^2)$	$20000(kg/m^2)$
Γ_s	$200000(kg/m^2)$	$300000(kg/m^2)$



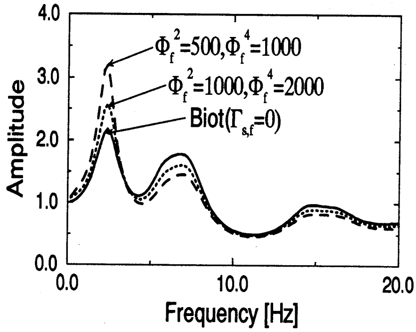
3-layers ground

Fig. 5: Amplitude of surface



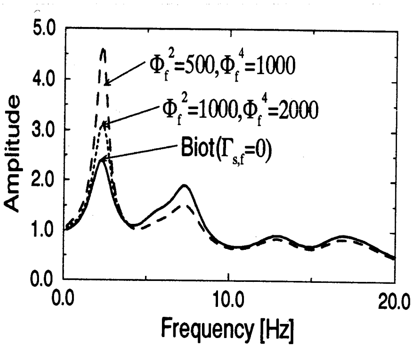
4-layers ground

Fig. 6: Amplitude of surface



5-layers ground (I)

Fig. 7: Amplitude of surface



5-layers ground (II)

Fig. 8: Amplitude of surface

theory. Especially, in the low frequency domain, amplitude is lower, considering the compressibilities in Fig.5 and Fig.6. The results obtained from amplitude of surface reflect those from Q in the previous section. However, In Fig.7 and Fig.8, amplitude is greater, considering the compressibilities. In Fig.8, the layered ground has the middle elastic layer (the 3rd. layer) which is thicker than in Fig.7. Amplitude depends on the thickness of the middle layer. In order to study wave propagation in the time domain, Ricker wavelet is examined. Ricker wavelet is defined by[2]

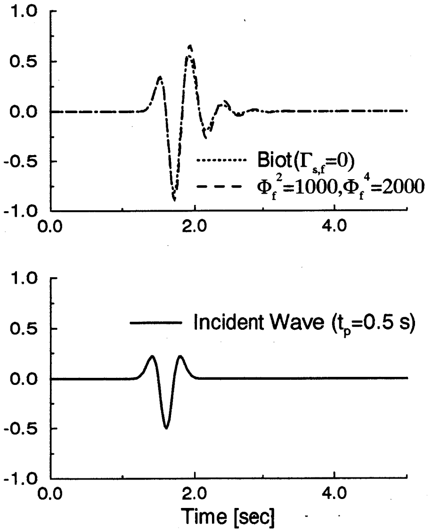
$$W(t) = (a - 0.5)e^{-a}, \quad where \quad a = \{\pi(t - t_0)t_p\} \tag{17}$$

Fig.9 shows the time domain wave propagation of amplitude of Fig.7, and Fig.10 shows that of Fig.8. Dispersion phenomenon which is one of the characteristics in the porous media, appears. When the middle elastic layer is thick (Fig.10), the wave is amplified more greatly.

5 CONCLUSIONS

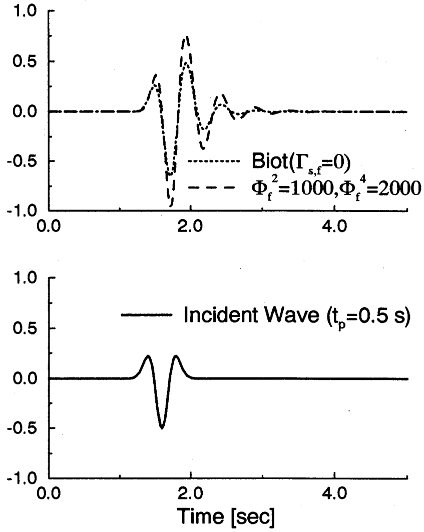
In this paper, Bown's theory is expanded and applied to the model of porous media. The basic characteristics in wave propagation are studied. In Bowen's theory, the porosity is defined as a variable and described by the rate type differential equation.

Applying Fourier transform, it is clear that in the frequency domain, the variable porosity affects the elastic coefficients in the low frequency domain. The results of Q in the porous layers reflect this fact. Compared with the model by Biot's theory, Q decreases in the low frequency. Amplitude of the surface layer becomes low. It can be said that Q decreases, that is, waves are more attenuated, considering the compressibilities of the constituents. According to the results of 5-layers ground, the middle elastic layer which is put between the porous layers plays an important role in wave propagation in the multi layered ground. Bowen's theory can express the dynamic behaviors of the porous media more precisely. In this paper, the basic characteristics of the theory was examined. In the future, the parameters of the compressibilities will be studied more exactly.



5-layers ground (I)

Fig. 9: Ricker wavelet of the surface



5-layers ground (II)

Fig. 10: Ricker wavelet of the surface

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