The Stress Analysis of the Secondary Piping System, Uncoupled from a Main Piping System

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ABSTRACT

Whenever the stress analysis for some secondary pipes is not necessary, it is desirable to uncouple the main system from these secondary pipes. The uncoupling, however, is only allowable on the basis of some acceptability criteria defined in terms of mass and frequency ratios of the main system to the secondary system.

At the request of the authorities, it often becomes necessary, at a later date, to also investigate the secondary pipes. The important reduction of costs, the elimination of the effect of ill-conditioning that are more serious in large systems with large differences in the stiffnesses of the elements, the exceeding of the program or computer capacity can be arguments for a separate stress analysis of the secondary uncoupled system. For the stress analysis of the secondary uncoupled system, it is generally not possible to neglect the influence of the main system.

This paper presents a method to separately perform the stress analysis of the secondary uncoupled piping system from the main system already calculated.

NOTATION

δ_i	Displacement of a point i of the pipe system 1, in the analyzed direction by the action of response spectra $S_1(\omega)$
δ_m	Maximum displacement in the piping system 1, in the analyzed direction, by the action of response spectra $S_1(\omega)$
$S_1(\omega)$	Input floor-response spectrum used for the dynamic stress analysis of the main system 1
$S_1(\omega_j)$	Acceleration according to response spectrum $S_1(\omega)$, at the natural frequency ω_j
ω	Natural frequency
ω_M	Principal natural frequency of the system
ω_j	Natural frequency of the one mass uncoupled system
φ_{mq}	Eigenvector value for mode m, at the uncoupling point q of the main system 1
$\emptyset_{mq}$	Maximum deflection coordinate for the m-th mode at point q of system 2
ψ_m	Particiation factor for mode m

N	Number of natural frequencies
$(AF_{m})_{j}$	Amplification factor of the acceleration of mass m_j, vibrating with the natural frequency ω_j, excited from the main system vibrating with the natural frequency ω_m
A	Maximum amplification factor at resonance
ζ_i	Damping ratio for the system i

1 THE STRESS ANALYSIS OF THE SECONDARY UNCOUPLED SYSTEM - GENERAL CONSIDERATION

To take into account the influence of the main piping system, it is proposed to consider the uncoupling point as a fixed point of the secondary system (point 204, Fig. 1).

The displacement of the uncoupling point (computed by the stress analysis of the main system), is to be reconsidered as relative displacement to the first fix support to the uncoupling point of the secondary system. The sign of the displacement is to be taken into account. For example, for the earthquake dynamic stress analysis, the most unfavorable of the four possible combinations will be considered for every direction. The calculations in most cases are simplified, because the displacements of the fixed point under consideration of the secondary piping system are very small compared with the displacement of the uncoupling point and can be ignored.

For the dynamic stress analysis of the secondary piping system, it is moreover necessary to calculate the response spectra at the uncoupling point. For the calculation of response spectra we assume that the dynamic stress analysis of the main system was performed. The input response spectra used by the stress analysis of the main system are known as well.

2 THE GENERAL METHOD OF CALCULATION OF RESPONSE SPECTRA AT THE UNCOUPLING POINT

For the understanding of the complex problem, a two-degree-of-freedom system is initially analyzed (Figure 2). The problem was studied by Kapur and Shao.

The acceleration of system 2 can be written as a function of the acceleration of the main system 1,

$$\ddot{X}_{2m} = \ddot{X}_{1m} (AF_1)_2. \tag{1}$$

The variation of the amplification factor with the variation of the natural frequency ratio ω_2/ω_1 is given in Fig. 3.

The techniques used for the two-degree-of-freedom system can be extended for a multiple-degree-of-freedom system. Any assumed maximum deflection of mass r in point q can be expressed as [4]:

$$\phi_{rq} = \sum_{m=1}^{N} \psi_m \phi_{rqm} \tag{2}$$

It is known that the vibration in a normal mode must always be harmonic. The equation (2) can be rewritten for the time t as:

$$\phi_{rqt} = \sum_{m=1}^{N} \psi_m \phi_{rqm} \sin \omega_m t \tag{3}$$

The response spectrum gives the response of any linear single-degree-of-freedom system, having the selected damping to the particular earthquake ground motion. In our situation, the ground motion is given by the point q of the main system (see Fig. 4). Starting from the response-spectrum definition, we can write the value of the acceleration of the mass M_j as a function of the acceleration at the point q for a given frequency ω_j as:

$$S_q(\omega_j) = \sum_{m=1}^{N} \psi_m (AF)_{mj} \emptyset_{rqm} \tag{4}$$

where

$$(AF_m)_j = \frac{\omega_j^2}{|(\omega_j^2 - \omega_m^2)|} \tag{5}$$

or in the particular case when $\omega_j \simeq \omega_m$: $(AF_j) = A$, see equation (6).

The above-presented method for the determination of response spectra is not convenient because it consumes a large amount of time for calculations and in addition, the results obtained by the computer stress analysis of the main system 1, generally do not furnish all the element data for the calculation according to equation (4).

3 THE SIMPLIFIED METHOD OF CALCULATION OF RESPONSE SPECTRA AT THE UNCOUPLING POINT

At the beginning, "the principal frequency" of the system has to be defined as the frequency that provides the largest contribution to the response spectra value at the point q (see equation (3)). The principal mode M is then the normal mode with the largest value of the product $\omega_m^2 \psi_{mq} \emptyset_{mq}$. Initially, it will be assumed that the point q, is the point with the largest displacement of the system in the analyzed direction.

There are to be defined also the significant frequencies of the system, as the normal modes with the value of the participation factor of the same order as the principal frequency ψ_M of the system. ($\psi_{ig} \geq 0.1\psi_{Mg}$)

The maximum value of the factor $(AF)_{mj}$, j = 1 ...N, is given for the frequency m nearest to the frequency j by using the equation (5). In the most unfavorable case $\omega_m \simeq \omega_j$, this is the resonance value of the amplification factor and can be calculated, for example, as a function of the damping ratios of the systems 1 and 2. The amplification factor according to Newmark is:

$$A = \frac{1}{\zeta_1 + \zeta_2} \tag{6}$$

To establish a simplified formula for calculation of response spectra, it is necessary to make the following assumptions:

I The main piping system 1 can be uncoupled from the secondary piping system 2, according to the uncoupling criteria, see [1]. This means that the vibration of the main piping system 1 is practically not influenced by the vibration of the secondary system 2.

II The Eigenvector value at the uncoupling point equals 1.

III For the calculation of the response spectra at point q, corresponding to a frequency ω_j near the principal frequency, the contribution of this frequency in the response

spectra according to, equation 6, is much bigger than the terms due to the other frequencies, so that the last terms can be neglected.

With these assumptions, the acceleration of mass system m_j coupled to the main system at point q (see Fig. 4) can be written for the known frequency (ω_j) of this system as:

$$S_{2q}(\omega_j) = S_{1q}(\omega_j) + \psi_{Mq}(A-1)\, S_1(\omega_j) \tag{7}$$

For the calculation of the acceleration of the system 2 with the frequency ω_j, coupled to the main system at a point i different from q, an influence coefficient will be considered:

$$\alpha_i = \frac{\delta_i}{\delta_m} \tag{8}$$

Considering the equation (8), we can write the equation (7) as:

$$S_{2i}(\omega_j) = S_{1i}(\omega_j)\,[1 + \psi_{Mq}\,\alpha_i\,(A-1)] \tag{9}$$

For the calculation of the acceleration $S_{2i}(\omega)$, at a frequency ω different from ω_j, we conservatively assume that ω is also a principal frequency of the main system and the acceleration is to be calculated with the same values ψ_{Mq}, α_i, and A as for the principal frequency:

$$S_{2i}(\omega) = S_{1i}(\omega)\,[1 + \psi_{Mq}\,\alpha_i(A-1)] \tag{10}$$

The equation (10) is the general equation for the calculation of response spectra. With the exception of the value of A, that can be calculated according to equation (6) for example, the other values are already known as input data or results of the stress analysis of the main system 1.

Let us analyze the assumptions I - III used to formulate the equation (10):

I The first hypothesis implies a limitation of the validity of equation (10) to the situation when the secondary system 2 can be uncoupled from main system 1

II The value 1 in the equation (10) for the Eigenvector value φ_{Mq} at the uncoupling point q, gives conservative results. At the point q, φ_{Mq} generally has subunit values near to 1 or 1

III The third assumption will be analyzed for 3 different cases as shown in Fig. 5.
Fig. 5.1 - The system 1 has many significant frequencies. Any of them are close to each other in the interval between (0.7 .. 1.4) ω_M. Neglecting other frequencies brings unacceptable errors and eventually unconservative results. In this situation, according to the uncoupling conditions, none of the frequencies of the secondary system is in this interval and the error by the calculation of response spectra does not influence the results of the calculation of the secondary system 2.

Fig. 5.2 - The frequencies of the main system 1 are spread out outside of the interval between (0.7 - 1.4) ω_m. The amplification factor can be calculated at the left and right limit of the hatched interval

$$A_r = A_1 = \frac{\omega_M^2}{\omega_{r,1}^2 - \omega_j^2} = 1.7$$

For $\zeta_1 = \zeta_2 = 0.01$, the amplification factor A is 50. For the calculation of the amplification of the ground motion, the contribution of the important frequencies out of the hatched interval can be neglected because of the abrupt decrease of the amplification factor. The assumption that no more than 3 significant frequencies with the same value of the product $\psi_{Mq} \emptyset_{Mq}$ are in the boundary line give a possible error of 9 %. By judging this error, it should not be overlooked that the analysis of such a peculiar case is only made for one frequency. By the construction of response spectra and the usage of modal analysis, the eventual errors for one frequency are usually compensated because of the conservative results for other frequencies. The probability to meet such peculiar cases increase with the increase of a number of important frequencies and this is the case with big, many-degree-of-freedom system. As a supplementary condition to eliminate such cases, it is recommended to lower the limit of the value of ψ_{Mq} to 0.1.

Then for $\psi_{Mq} \leq 0.1$ is to choose $\psi_{Mq} = 0.1$

The case of a frequency-ω_i, which is not a significant frequency must also be analyzed. Similar reasonings, as above, bring a supplementary condition to be taken into account by using equation (10). The mean slope given from the variation of acceleration with the frequency for the analyzed response spectra in the interval (ω_i-ω_j), must be smaller than the slope of the variation of the amplification factor with frequency (see Fig. 3) for the same frequency interval. This condition is generally satisfied by using smoothened response spectra with broadened peaks, according to Regulatory Guide 1.222.

Fig. 5.3 - None of the significant frequencies appear in the interval of large accelerations ($a_m \leq 0.1$, a_{max}) neither does a grouped repartition of the frequencies of the main system. In these peculiar cases, the value of the amplification factor A in the equation (10) will be for every frequency except resonance replaced with $(AF_n)_j$ calculated with the equation (5), whereas M is the first signifant frequency of the group (see Fig. 5.3). For $\omega_j \simeq \omega_M$ we will choose the minimum value calculated with equations (5) and (6).

4 NUMERICAL RESULTS

Dynamic stress analysis was carried out for the piping system, Fig. 1, with the input response spectra, Fig. 6.1 and 6.2. The system was initially calculated as a whole with the computer program KWU-ROHR. The secondary piping system (between the points 204 and 228) was later uncoupled and separately calculated, using the computer program PIPESTRESS and the multiple support excitation method. The response spectra at the point 204 was calculated with the simplified method. The calculated stresses are between 15 % and 35 % higher (20 % mean value) than the stresses initially calculated for the whole system.

For the simple system, Fig. 7, an input acceleration time history was used to calculate the response spectra corresponding to every time history. The same response spectra

were calculated using the simplified method. The results confirm the conservativity of the simplified method. The results of the calculation were also used, to compare the value of α_i calculated using the approximative equation (8) and the response spectra calculated for different points. The results are presented in Table 1.

It is to be mentioned that the numerical verification is continued at the ETH, Zurich, using the programs "SAP IV", "SPECEQ" and "SPAC". The results will be given at the oral presentation.

REFERENCES

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/2/ Kapur and Shao, "Structural Design of Nuclear Plant Facilities", Published by American Society of Civil Engineers, 1973

/3/ U.S. Nuclear Regulatory Commission, "Regulatory Guides" 1.92, Rev. 1, February 1976 and 1.122, September 1976

/4/ Biggs, J.M., "Introduction to Structural Dynamics", McGraw-Hill Book Company, Inc., New York, 1964

TABLE 1

Point	$S^*_{2i}(\omega)_{max}$	$\alpha^*_i = \frac{S^*_{2i}(\omega)-S_1(\omega)}{S^*_{2m}(\omega)-S_1(\omega)}$	$\alpha_i = \frac{\delta_i}{\delta_m}$
6	10.08	1.00	1.00
5	7.36	0.71	0.72
4	4.64	0.42	0.46
3	2.30	0.17	0.23
2	0.77	0.02	0.06
1	0.75	0.00	0.00

$S^*_{2i}(\omega)_{max.}$ - The maximum acceleration according to response spectrum $S^*_{2i}(\omega)$ calculated from time history, by using the computer programs SAPIV, SPECEQ and SPAC

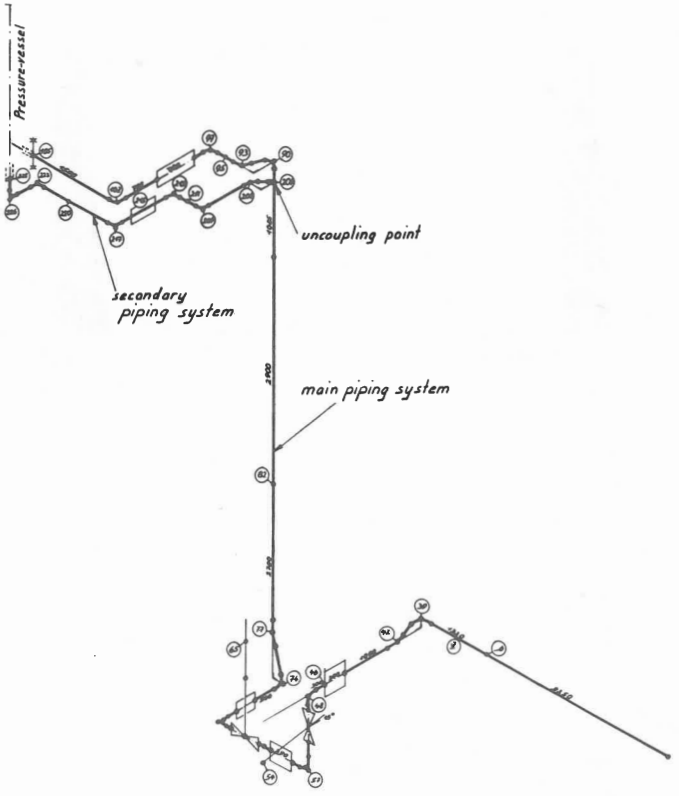

Figure 1

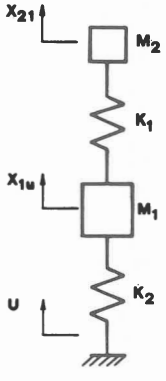

Figure 2

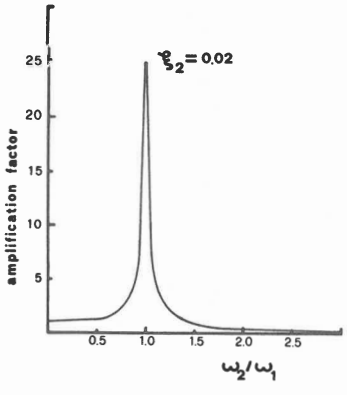

Figure 3

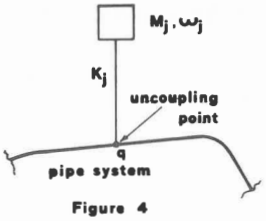

Figure 4

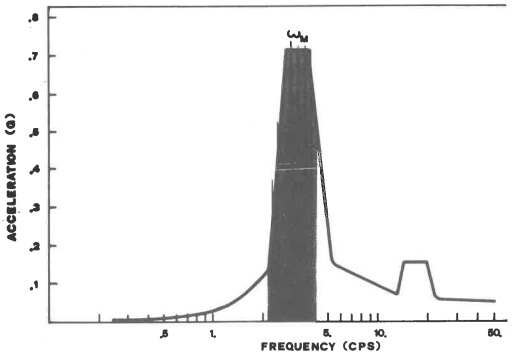

Figure 5.1

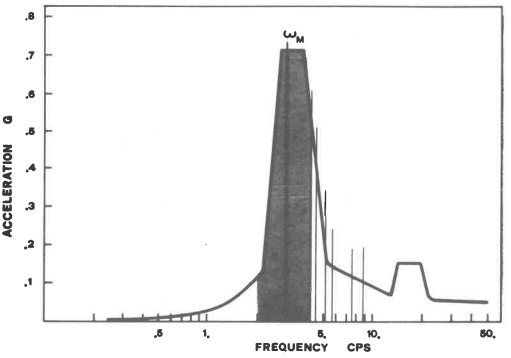

Figure 5.2

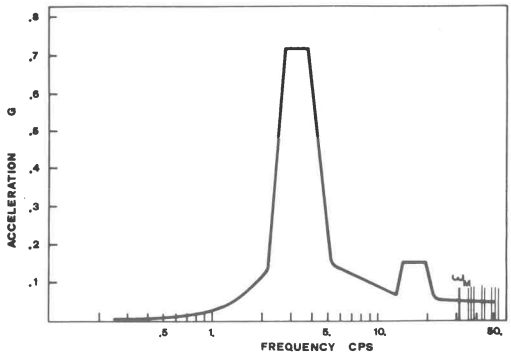

Figure 5.3

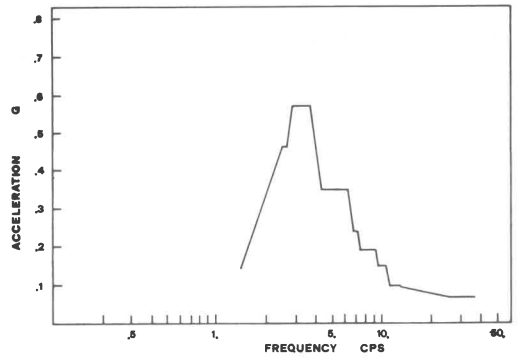

Figure 6.1

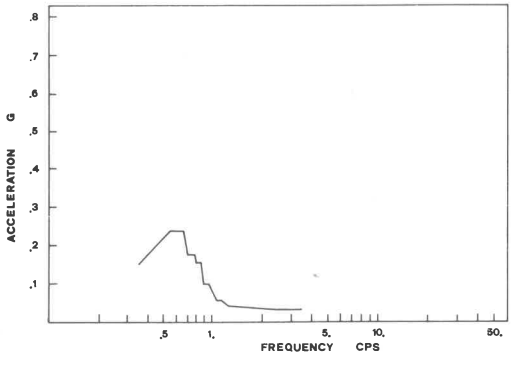

Figure 6.2

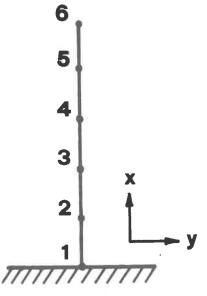

Figure 7