PHYSICS-INFORMED DEEP LEARNING MODEL FOR PREDICTION OF COMPLEX HYSTERESIS

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ABSTRACT

Accurate modeling of the hysteretic behavior of structural elements is essential for evaluating the seismic performance of nuclear power plant structures. While conventional mathematical models calibrated to hysteresis loops from quasi-static cyclic tests can capture a range of hysteretic behaviors, they are not universally applicable and may not effectively characterize the complex hysteretic responses observed from physical tests. In this study, we propose a deep learning-based hysteresis model which can predict far more diverse hysteretic behaviors than conventional hysteresis models. The network structure of the deep learning-based model is inspired by the numerical solution process of the Bouc-Wen-Baber-Noori (BWBN) model. The proposed model adopts a Long Short-Term Memory (LSTM) network to model the implicit features of hysteretic behaviors based on the displacement and velocity time histories. Additionally, the model incorporates a physics-informed loss function, which ensures adherence to the underlying physical principles of hysteresis. The performance of our proposed model is demonstrated by predicting the earthquake-induced hysteresis of various single-degree-of-freedom systems, such as the Ramberg-Osgood and BWBN models. It is expected that the proposed approach can be further developed to predict various force-displacement relationships obtained from experiments of structural elements.

INTRODUCTION

A hysteresis or a hysteretic behavior refers to a structural system's or element's force-deformation relationship which depends on the loading history. Accurate modeling of the hysteretic behavior of a structural system or elements is critical for response predictions of such systems subjected to random excitations. The predicted response then can be used to assess the seismic demands to the structure and equipment items housed in the structure. The accurate prediction of the structural response is especially important for nuclear power plant structures considering the high risk associated with potential failure.

Over the years, extensive research has been undertaken to predict the force-displacement hysteresis of structural systems. Significant advancement in this domain focused on the development of mathematical modeling for structural hysteresis. The foundational efforts by Bouc (1967) and Wen (1976) are presently known as the Bouc-Wen hysteresis model. However, the initial Bouc-Wen model exhibited limitations in adequately representing the strength and stiffness degradations, as well as the pinching behavior. This led to the development of the Bouc-Wen-Baber-Noori model, which added terms related to the amount of hysteretic energy associated with strength and stiffness degradations (Baber and Wen, 1981; Baber and Noori, 1985; Noori *et al.*, 1986).

Recently, the efforts to accurately predict various hysteretic behaviors led to an expansion from mathematical models to deep learning approaches. This evolution is largely attributed to the inherent complexities of structural hysteresis, which are often considered difficult to fully capture through a mathematical modeling. Ghaboussi *et al.* (1998) pioneered the use of neural networks for predicting the stress-strain behaviors of structural elements. More recently, advancements in computational technology have enabled the development of advanced deep learning hysteresis models such as the Long-Short Term Memory (LSTM) model (Ni *et al.*, 2022) or the Unrolled Attention (UA) model (Wang *et al.*, 2020).

Parallel to this progression, there have been attempts to incorporate physical insights into deep learning architectures. For example, Joghataie and Farrokh (2008) developed Prandtl Neural Network (PNN) in which the activation functions of the neurons were modified to the Prandtl-Ishlinskii operator. The model was based on the elasto-plastic behavior of materials and later was developed as a Generalized Prandtl Neural Network (GPNN) (Farrokh *et al.*, 2015) and Extended Preisach Neural Network (Farrokh *et al.*, 2022). Also, Borkowski *et al.* (2022) employed regularization techniques to ensure monotonically increasing plastic energy. Furthermore, there have been some studies to predict the parameters of mathematical models by deep learning models: Horton *et al.* (2021) developed a neural network that predicts the parameters of the modified-Ibarra-Krawinkler model (Lignos and Krawinkler, 2007). Gu *et al.* (2023) proposed an LSTM model which predicts the parameters of three mathematical models and simulated the hysteresis by the weighted sum of the outputs of the mathematical models, in which the weights were also estimated by the deep learning model. Lastly, adding a physics-informed loss to the data-driven loss to train a deep learning model was proposed to augment physics into the deep learning model: Zhang *et al.* (2020) developed physics-informed loss function related to the Bouc-Wen model, and Delgado-Trujillo *et al.* (2023) introduced a loss function which can impose general governing physics of hysteretic structures such as the Drucker's postulate or Il'iushin's postulate.

With the evolution of deep learning models in predicting structural hysteresis, many studies have been relying on hysteresis data obtained from the prescribed displacement histories, typically defined as a saw tooth-type cyclic loading protocol. Using experimental data from such loading protocol often leads to overfitting issue due to the sheer volume of trainable parameters that these models contain. Since real-world data typically exhibit a greater level of complexity than that observed from experiments using a prescribed displacement history, there is a pressing need to train deep-learning models based on realistic hysteresis behaviors.

This paper introduces an LSTM-based deep learning model, designed for robust performance in predicting earthquake-induced hysteretic behavior. Specifically, the deep learning model architecture, inspired by the Bouc-Wen class of hysteresis models, uses a physics-informed loss for training. This enables its applications across a broad range of hysteresis behavior including strength and stiffness degradation as well as pinching effects.

The structure of this paper is as follows: After reviewing the formulation of Bouc-Wen class models and deep learning, we present our LSTM-based model, inspired by Bouc-Wen class models, along with the method for physics-informed training. Our model is then tested using different earthquake-related hysteresis datasets, featuring not only Bouc-Wen class models including Bouc-Wen model and Bouc-Wen-Baber-Noori model, but also other models such as the bilinear and Ramberg-Osgood steel models. The paper is concluded by summarizing our results and discussing areas for future research.

FORMULATION OF BOUC-WEN CLASS MODELS

The resisting force $F(t)$ at time t of the Bouc-Wen class models is expressed as a linear function of the system displacement $u(t)$ and the auxiliary variable $z(t)$ representing the inelastic behavior. This relationship can be written as

$$F(t) = ak_i u(t) + (1 - a)k_i z(t) \tag{1}$$

where k_i represents the initial stiffness, and a denotes the post- to pre-yield stiffness ratio.

The auxiliary variable $z(t)$ is obtained by solving the first-order nonlinear ordinary differential equation. For example, the Bouc-Wen model's differential equation is

$$\dot{z}(t) = \frac{dz(t)}{dt} = \dot{u}(t)[A - \beta \mathrm{sgn}(\dot{u})|z(t)|^{n-1}z(t) - \gamma|z(t)|^n \tag{2}$$

where $\dot{u} = du/dt$ denotes the velocity, A and n are respectively the parameters determine the scale and sharpness of the hysteresis loop, and β and γ define the shape of hysteresis loops.

In the Bouc-Wen-Baber-Noori model, the dissipated energy $\varepsilon(t)$ is added to Eq. (2) to account for the energy-dependence of stiffness and strength degradation, and pinching effects through the pinching function $h(z(t), \varepsilon(t))$, strength function $\nu(\varepsilon(t))$, and stiffness function $\eta(\varepsilon(t))$. Additionally, A in Eq. (2) is modified to a linear function of $\varepsilon(t)$. As a result, the governing equation of the Bouc-Wen-Baber-Noori model is

$$\dot{z}(t) = \dot{u}(t)\frac{h(z,\varepsilon)}{\eta(\varepsilon)}\{A(\varepsilon) - \nu(\varepsilon)[\beta \mathrm{sgn}(\dot{u})|z|^{n-1}z - \gamma|z|^n]\} \tag{3}$$

Given an initial condition for the auxiliary variable at time t_0, i.e., $z(t_0)$, Eqs. (2) and (3) can be integrated to:

$$z(t) = z(t_0) + \int_{u(t_0)}^{u(t)} \psi(\boldsymbol{\alpha}(u); \boldsymbol{\theta})\, du \tag{4}$$

where ψ is a nonlinear function which takes $\boldsymbol{\alpha}(u)$ as an input vector, and $\boldsymbol{\theta}$ as a function parameter. For the Bouc-Wen model, $\boldsymbol{\alpha}(u) = [u,\ \ \dot{u}]^{\mathrm{T}}$, $\boldsymbol{\theta} = [A, \beta, n, \gamma]^{\mathrm{T}}$, and $\psi(\boldsymbol{\alpha}(u); \boldsymbol{\theta}) = [A - \beta\, sgn(\dot{u})|z(u)|^{n-1}z(u) - \gamma|z(u)|^n]$. For the Bouc-Wen-Baber-Noori model, $\boldsymbol{\alpha}(u) = [u\ \ \ \dot{u}]^{\mathrm{T}}$, $\boldsymbol{\theta} = [\varepsilon, A, \beta, n, \gamma]^{\mathrm{T}}$, and $\psi(\boldsymbol{\alpha}(u); \boldsymbol{\theta}) = \frac{h(z(u),\varepsilon)}{\eta(\varepsilon)}\{A(\varepsilon) - \nu(\varepsilon)[\beta\, sgn(\dot{u})|z(u)|^{n-1}z(u) - \gamma|z(u)|^n]\}$.

DEEP LEARNING MODEL FOR TIME SERIES PREDICTION

Long Short-Term Memory (LSTM) Model

Long Short-Term Memory (LSTM) model (Hochreiter and Schmidhuber, 1997) is an advanced type of Recurrent Neural Networks (RNNs) which refer to a class of artificial neural networks designed to handle sequential data. Unlike traditional feedforward neural networks, these models allow data to loop back on itself. The output $\boldsymbol{h}(t_n)$ at time step n of an RNN layer can be expressed as:

$$\boldsymbol{h}(t_n) = NN\big(\boldsymbol{x}(t_n), \boldsymbol{h}(t_{n-1})\big) \tag{5}$$

where NN denotes the typical fully connected neural network layer, which includes a linear transformation followed by an activation function, and $\boldsymbol{x}(t_n)$ is an input vector for the n-th time step. Figure 1 shows a basic graphical representation of RNNs.

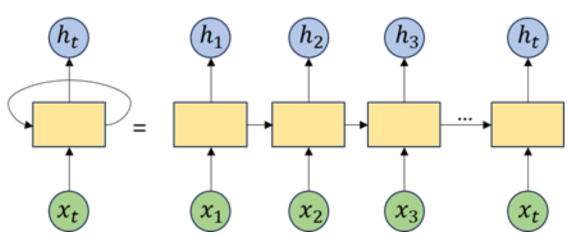
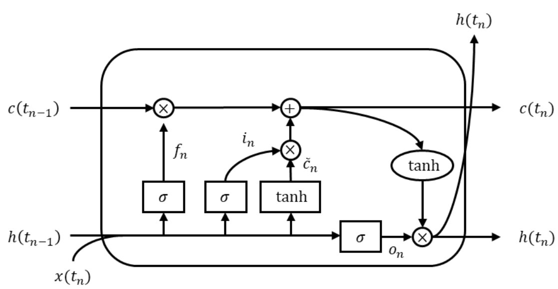


Figure 1. Graphical representation of RNNs

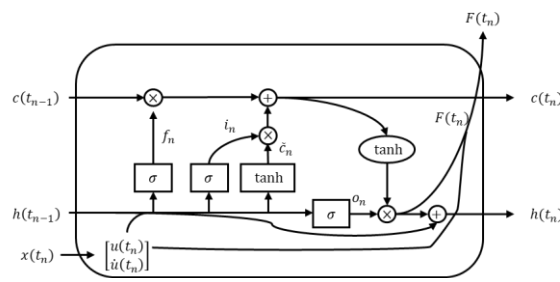
In LSTMs, a typical fully-connected neuron in the RNN model is replaced by LSTM cells. These cells take an additional long-term memory component called cell state ($\boldsymbol{c}(t_{n-1})$) as inputs along with $\boldsymbol{h}(t_{n-1})$ and $\boldsymbol{x}(t_n)$. The calculations for $\boldsymbol{h}(t_n)$ and $\boldsymbol{c}(t_n)$ are as follows:

$$
\begin{gathered}
\boldsymbol{f}_n = \sigma\left(\boldsymbol{W}_f \cdot [\boldsymbol{h}(t_{n-1}), \boldsymbol{x}(t_n)] + \boldsymbol{b}_f\right) \\
\boldsymbol{i}_n = \sigma(\boldsymbol{W}_i \cdot [\boldsymbol{h}(t_{n-1}), \boldsymbol{x}(t_n)] + \boldsymbol{b}_i) \\
\tilde{\boldsymbol{c}}_n = \tanh(\boldsymbol{W}_c \cdot [\boldsymbol{h}(t_{n-1}), \boldsymbol{x}(t_n)] + \boldsymbol{b}_c \\
\boldsymbol{c}(t_n) = \boldsymbol{f}_n \otimes \boldsymbol{c}(t_{n-1}) + \boldsymbol{i}_n \otimes \tilde{\boldsymbol{c}}_n \\
\boldsymbol{o}_n = \sigma(\boldsymbol{W}_o \cdot [\boldsymbol{h}(t_{n-1}), \boldsymbol{x}(t_n)] + \boldsymbol{b}_o) \\
\boldsymbol{h}(t_n) = \boldsymbol{o}_n \otimes \tanh \boldsymbol{c}(t_n)
\end{gathered}
\tag{6}
$$

where $\boldsymbol{f}_n$, $\boldsymbol{i}_n$, $\tilde{\boldsymbol{c}}_n$, and $\boldsymbol{o}_n$ respectively denote the forget gate, input gate, candidate cell state, and output gate, and $(\boldsymbol{W}_f, \boldsymbol{b}_f)$, $(\boldsymbol{W}_i, \boldsymbol{b}_i)$, $(\boldsymbol{W}_c, \boldsymbol{b}_c)$, and $(\boldsymbol{W}_o, \boldsymbol{b}_o)$ are respectively weights and biases corresponding to the forget gate, input gate, cell state and output gate. $\otimes$ and σ denote element-wise multiplication and sigmoid activation function, respectively. Figure 2(a) illustrates the LSTM cell's structure. Because the LSTM cell state gets updated based on the candidate cell state, the LSTM model can discern and retain critical information over extended sequences.



(a)



(b)

Figure 2. Graphical representation of RNNs: (a) Conventional LSTM cell, and (b) proposed LSTM cell for hysteresis prediction

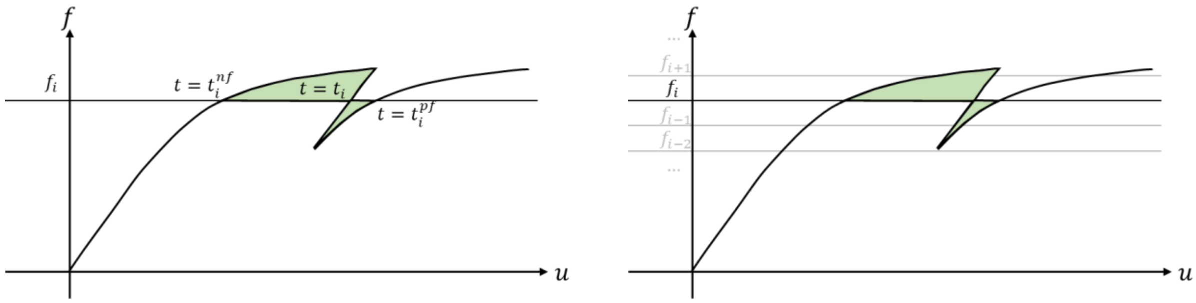
Physics-informed Deep Learning

Physics-informed deep learning is a deep learning method that incorporates physical principles. In many cases, the approach employs a loss function constructed to constrain the model to physical principles. Defining the data-driven loss as $\mathcal{L}_D$ and the physics-informed loss as $\mathcal{L}_P$, the total loss function that is used to train the model can be constructed as

$$
\mathcal{L} = (1-\alpha)\mathcal{L}_D + \alpha\mathcal{L}_P \tag{7}
$$

Delgado-Trujillo et al. (2023) proposed a loss function which is related to the Drucker's postulate in material engineering: for an arbitrary force f_i, the stress loop can be obtained as in Figure 3(a), and defining the previous time step where the force is equal to f_i as t_i^{nf} and the next time step as t_i^{pf}, the degree that the hysteresis violates the Drucker's postulate can be computed as

$$
\mathcal{L}_P = \max\left(0, \varepsilon(t_i) - \varepsilon\left(t_i^{pf}\right)\right) + \max\left(0, \varepsilon\left(t_i^{nf}\right) - \varepsilon(t_i)\right) \tag{8}
$$



(a) (b)

Figure 3. Graphical representation for obtaining (a) the specific stress loop at a certain force level, and (b) simultaneously obtaining stress loops from the equally spaced force levels

PROPOSED DEEP LEARNING MODEL FOR HYSTERESIS PREDICTION

Proposed LSTM Cell for Hysteresis Prediction

Our proposed LSTM cell starts from the idea that the auxiliary variable $z(t)$ for Bouc-Wen class models can be regarded as the hidden state $h(t)$ in the LSTM cell. Letting the LSTM cell model $z(t)$, it can increase flexibility of the Bouc-Wen class models. Given that force is computed as the linear combination of $z(t)$ and $u(t)$ in Eq. (1), our proposed LSTM cell emulates the mechanism by applying a fully connected operation of $u(t)$ and $h(t)$ to obtain $F(t)$. Also, the inputs of the model are considered to be the displacement $u(t_n)$ and the velocity $\dot{u}(t_n)$ at time t_n, to emulate operations that are detailed in Eq. (4). The architecture of the proposed LSTM cell is shown in Figure 2(b).

Proposed Physics-informed Drucker's Loss

Our proposed method also uses a physics-informed loss to sustain the underlying physical phenomena of hysteresis curves. The proposed method enhances the computational efficiency of the Drucker's loss proposed by Delgado-Trujillo *et al.* (2023) by obtaining stress loops by equally spacing the force level and then obtaining the time steps that match the force level as shown in Figure 3(b). Then, the Drucker's loss is computed from Eq. (8).

NUMERICAL EXAMPLES

We subjected it to a variety of hysteresis models, including Bouc-Wen class models (Bouc-Wen model and Bouc-Wen-Baber-Noori model) and the non-Bouc-Wen class models (bilinear model and the Ramberg-Osgood model) to demonstrate its capability to predict a wide range of hysteresis types. The proposed model was trained using earthquake-induced hysteresis data.

A total of 80 earthquake ground motion records were obtained from the NGA-West 2 Database (Power *et al.*, 2008). The records were scaled to ensure that they have the same Peak Ground Accelerations (PGAs). Utilizing the OpenSeesPy package in Python, we performed a nonlinear time history analysis on a single-degree-of-freedom oscillator with a normalized mass, producing hysteresis data for each earthquake. The initial stiffness of the oscillator was set to have the frequency of 1 Hz. The specific parameters adopted for hysteresis models of the Bouc-Wen and non-Bouc-Wen classes within OpenSeesPy are detailed in Tables 1 and 2, respectively. The force and the displacement of the hysteresis sample data were normalized to bound between $[-1, 1]$. The resulting samples of the hysteresis curve and the accumulated energy are shown as shadowed lines in Figures 4 and 5. From the 80 generated datasets for each hysteresis models, 40 hysteresis samples were allocated for training and validation, while the remaining 40 were reserved for testing prediction accuracy. Among 40 samples that were used for training and validation, 66 percent of the sample data were used for training, and the other 33 percent of the data were used only to compute the validation loss. The proposed model architecture was constructed with one LSTM layer which contains 64 LSTM cells. A weighted sum of the mean squared error (MSE) and the Drucker's loss were used to define a loss function. The loss from each data set was normalized by the loss computed from the model with random initialization. The physics informed weight of α of the Drucker's loss was set to 0.2. The model was trained with the Nadam optimizer (Dozat, 2016) with the learning rate of 10^{-3} and was trained 1,000 epochs. The optimal model for each hysteresis models was obtained by selecting the model parameters that result in the minimum validation loss. The model was constructed and trained using Pytorch.

Table 1: Parameters of the reference Bouc-Wen class hysteresis model

Type of hysteresis	α	k_0	n	γ	β	A_0	q	ζ	p	ξ	$\Delta\xi$	λ
Bouc-Wen model	0.1	30.5	1	-0.5	1.5	1	-	-	-	-	-	-
Bouc-Wen-Baber-Noori model	0.1	30.5	1	-0.5	1.5	1	0.1	0.97	1	0.2	0.002	0.1

Table 2: Parameters of the reference non-Bouc-Wen class hysteresis model

Type of hysteresis	F_y	E_0	b	a	n
Bilinear model	7	30.5	0.7	-	-
Ramberg-Osgood model	7.5	30.5	-	0.002	5

The dashed lines in Figures 4 and 5 represent the force prediction results for the testing data samples corresponding to the Bouc-Wen class hysteresis and non-Bouc-Wen class hysteresis, respectively. It is seen that the forces predicted by the proposed deep learning model match well with the reference data, regardless of the hysteresis model, and the displacement magnitude.

Figure 6 compares the predicted maximum force using test data set against the reference data. The data points on solid lines indicate that the maximum forces predicted by the proposed model is the same to that of the target data. It is seen that the maximum forces predicted by the proposed model are similar to those in the reference data. However, for the BWBN model and the Ramberg-Osgood model, the proposed deep learning surrogates tend to have some errors as the force magnitudes increase. This may be due to the lack of data points at large amplitudes. Further research is on its way for more robust prediction of the hysteresis curve with large amplitudes. Nevertheless, the model demonstrates successful predictions even though it was trained by earthquake-induced hysteresis data.

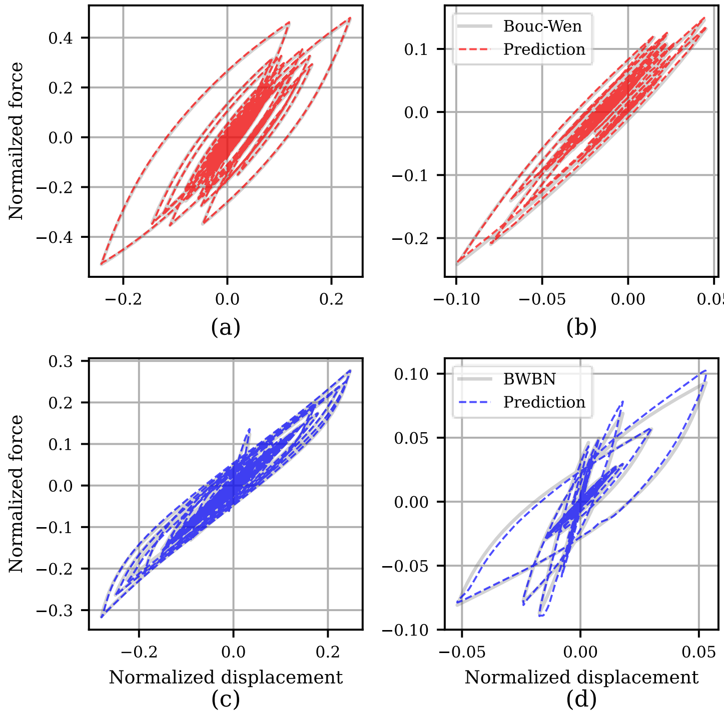


Figure 4. Sample hysteresis curves and the proposed model prediction result of Bouc-Wen class models: (a) Bouc-Wen model with large displacement, (b) Bouc-Wen model with small displacement, (c) BWBN model with large displacement, and (d) BWBN model with small displacement

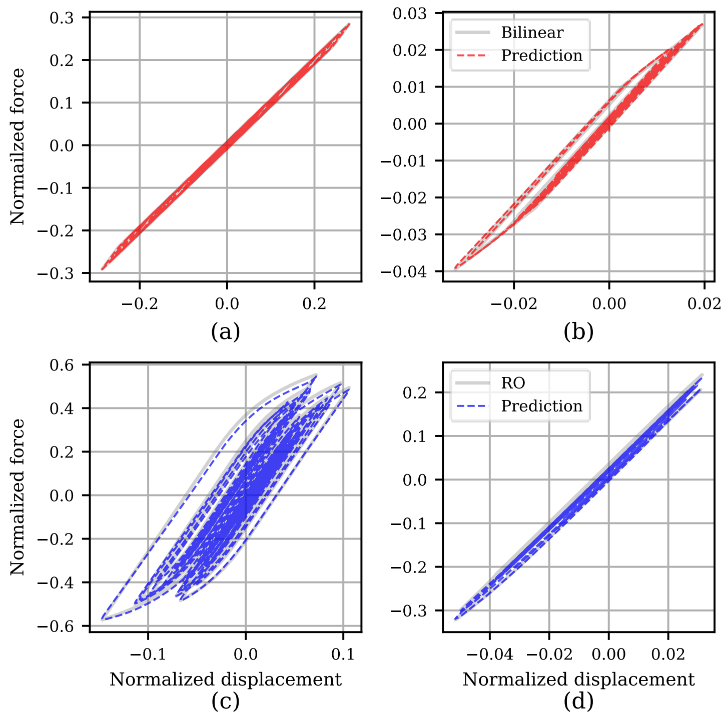


Figure 5. Sample hysteresis curves and the proposed model prediction result of non-Bouc-Wen class models: (a) Bilinear model with large displacement, (b) Bilinear model with small displacement, (c) Ramberg-Osgood model with large displacement, and (d) Ramberg-Osgood model with small displacement

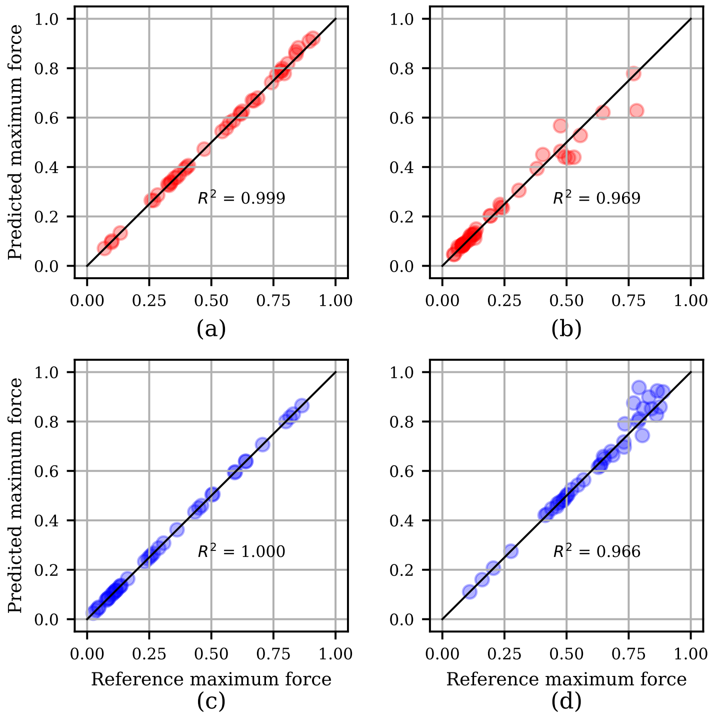


Figure 6. Prediction result of the peak responses: (a) Bouc-Wen model, (b) BWBN model, (c) Bilinear model, and (d) Ramberg-Osgood model

CONCLUSIONS

This paper introduces an LSTM-based deep learning model that can capture a wide range of hysteretic behaviors. The network architecture of the LSTM-based model is inspired by the formulation of Bouc-Wen class models and incorporates a physics-informed loss function grounded in Drucker's postulate, enhancing its robustness and accuracy in hysteresis prediction. We tested the model against four distinct types of hysteresis models using earthquake-induced hysteresis data. While the proposed model was able to predict the force sequence and the maximum forces accurately, further research is needed for accurate prediction of the maximum forces that are not usually observed in the training dataset. It is also noteworthy that the proposed model has not yet been tested with actual experimental data. Future research will focus on applying the model to nuclear power plant structures, by adding the additional physics-informed loss regarding the nuclear power plant-specific physical constraints.

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