# Ultimate Flexural Behaviour of Reinforced Concrete Shells Under Static and Dynamic Loading

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Abstract

A study was undertaken, in order to characterize the behaviour of concrete slabs subjected to low energy impacts by rigid missiles. Static and dynamic tests were performed on 1,8 m x 1,8 m slabs. Analysis with the SAMSON, BILBO and PLEXUS modules of CEASEMT Finite Element System was carried out in order to validate these codes. A definition of a safety coefficient was proposed, and validated against experimental results. This definition is fully compatible with finite element computations using the global plasticity method.

## 1. Introduction

Safety analysis of nuclear power plants implies situations where reinforced or pre-stressed concrete structures are subjected to extreme flexural loadings : impact of soft missiles, or low energy rigid missiles, on containments. The evaluation of safety margins requires a good definition of the ultimate state of such structures. On the other hand, a computational methodology has to be validated on simple tests before it can be applied to real structures. An experimental and theoretical study was undertaken in order to fullfill these requirements. The first part of this program was concerned with static loading ; a simple test was performed on square slabs, and computations were carried out in this case. In a second step, dynamic tests and analysis allowed to investigate the behaviour of the structure, and to qualify the computational tools, in case of an impact by a low energy rigid missile. The corresponding work is described in the following chapters.

## 2. Static tests

### 2.1 - Characteristics of the slab

Two static tests were performed on a square slab made of reinforced concrete. Its overall dimensions were 1,8 m x 1,8 m. Its current thickness was 0,125 m. Along its periphery, it was surrounded by walls of 0,250 m, as shown on Figure 1. In the upper part of the slab, the reinforcement was made of 4 mm diameter steel bars in both directions ; the spacing between bars was 50 mm. In the lower part, 6 mm diameter bars were used in both directions, with a spacing of 50 mm. The number of stirups was very low : one every four nodes of the longitudinal reinforcement.

The maximum size of the agregates in the concrete was 12,5 mm. Tests on specimens were made in order to determine the mechanical characteristics of the different materials.

For concrete, the following average values were found :

Compressive strength : 28 MPa

Tensile strength : 4,7 MPa

Young's modulus : 39 500 MPa

The mechanical characteristics of steels were different, depending upon the diameter of the bars :

| Diameter of bar (mm) | 0,2 % Elastic Limit (MPa) | Tensile strength (MPa) | Young's Modulus (MPa) |
|---|---|---|---|
| 4 | 594 | 651 | 203 000 |
| 6 | 370 | 519 | 169 000 |

TABLE 1 : MECHANICAL CHARACTERISTICS OF CONCRETE AND REINFORCED STEELS

2.2 - Loading of the slabs

The slabs were simply supported along the bottom ends of the outer walls. They were loaded at the center of their top face, by means of a steel cylinder of 28 cm diameter. A servo controlled activator allowed to run the test at a constant displacement rate of 0,10 mm/s. The force was continuously monitored, as well as the displacements, at various locations.

2.3 - Experimental results

Figure 2 shows the force-displacement curve, for the center of the slab. During the tests, the following observations were made :

- before the maximum load is reached, no visible shear damage can be observed within the structure. In particular, no circumferential crack develops at the bottom face of the slab, while radial cracks, corresponding to a typical flexural behaviour, progressively appear ;
- circumferential cracks are formed in a second step, while the force is practically constant, and the central displacement increases ;
- final punching of the slab occurs suddenly, with a drastic decrease of the applied force ;
- during the whole test, the corners slab tend to lift up.

From a practical view point, the main conclusion of the test is that the behaviour of the structure seems to be entirely determined by its flexural resistance, until the maximum load has been reached.

3. Static Analysis

3.1 - Global methods

In order to perform the analysis of the slab, the global method was used [1], [2], [3]. Its principle is presented by Figure 3 : starting from the mechanical characteristics of the components of the slab (steel and concrete), the pre-processor

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SAMSON is used in order to derive the stress-strain curve of an quivalent homogeneous material. The main hypothesis in this process is that the cross section remains plane. On the other hand, all the material non linearities due to plasticity of steel, as well as compressive and tensile failure of concrete are taken into account.

The equivalent stress-strain curve is then used as an input into a classical finite element code, where computations can be performed with shell elements.

The BILBO code of the CEASEMT System [4] was used for the analysis of the slab subjected to quasi static loading. Two different equivalent homogeneous materials had to be defined, in order to represent the current part of the slab, and the outer walls. Unilateral constraints were necessary, in order to modelize the lift up of the corners, which results into a progressive loss of contact between the slab and its outer support. Figures 4 and 5 show the deformed shape of the slab at maximum load, and the comparison between numerical and experimental force-displacment curves at the center of the slab, before final punching. These results validate the global method for this configuration. On the other hand, the good agreement between numerical and experimental results confirms that punching, or shear processes, do not play any significant role before the maximum load is reached, since they are not taken into account in the analysis.

## 4. Dynamic Tests

### 4.1 - Experimental set-up

Five dynamic tests were performed on the same slabs as the one used for quasi-static loading. For these tests, the slabs were only supported by four load cells, at the middle of the bottom part of each of the outer walls. A circular steel plate of 28 cm diameter and 4 cm thickness was fixed at the center of their top face. For the tests themselves rigid masses were droped from various heigths, in order to impact the circular steel plate with various kinetic energies. The following quantities were measured :

- Impacting velocity of the projectile by photo-electric cells.
- Displacements at various points of the slab, using LVDTs.
- Acceleration of various points using accelerometers.
- Reaction forces.

### 4.2 - Experimental results

Figures 6 and 7 show typical experimental curves (test n° 2). For reaction forces, the scatter between the 4 loads cells was ± 20 %, and ± 7 % for measurements corresponding to symetric points with respect to the center of the slab. The following table presents the maximum defections at the center extrapolated from displacements measured outside the impact area, for each test.

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| SLAB NUMBER (Test number) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Projectile mass (kg) | 135 | 201 | 250 | 250 | 250 |
| Impacting Speed (m/s) | 5,2 | 6,5 | 6,1 | 8,6 | 9,7 |
| Energy (J) | 1825 | 4245 | 4650 | 9245 | 11760 |
| Maximum deflection (mm) | 6 | 16 | 22 | 25 | 27 |

TABLE 2 : MAXIMUM DEFLECTION AT THE CENTER OF THE SLABS

Comparison between slabs 3 and 4 shows that the maximum deflection at the center, only undergoes a slight increase, while the impacting energy is doubled.

This effect is due to an increase in the mechanical properties of the reinforcing steels of the slabs used for tests 3 and 4. This effect can easily be accounted for in the computations.

The following damages could be observed after testing :

- when the kinetic energy is low, radial cracks typical of a flexion mechanism only appear at the bottom face of the slabs (Tests N° 1 and 2).

- when the kinetic energy increases, circumferential cracks appear on the impacted face (Test n° 3).

- When the energy is high, a punching mechanism is superimposed to the previous ones (Tests n° 4 and 5). In case of test n° 4, punching is incipient while for test n° 5 ; a very clear shear plug could be observed after impact. From this view point, slab n° 4 can be considered as being in a limit state, with respect to shear failure.

5. Dynamic analysis

Dynamic computations were performed with the PLEXUS code [5] of the CEASEMT System. The global plasticity method was again used. As a first approximation the projectile was modelized by use of additional masses with initial velocity, at the points of the mesh under going impact. On the other hand, a uniform vertical displacement constraint was imposed for these same points, in order to take into account the rigidity of the projectile. The deformed shapes of the slabs were similar to the one obtained in the quasi static cases (Figure 4). Figure 8 shows the displacements of various points of slab n° 3.

The comparison between numerical and experimental results shows that the maximum displacements are well predicted by the analysis. However, some discrepancies remain concerning the time at which maximum displacements are reached, and the shape of the reaction forces versus time curves.

Further studies will be carried out to solve this problem. In a first step, the projectile and the loading steel cylinder will be represented as such, in order to get a better representation of the impact conditions. 3D studies will also be performed, with a local model for concrete.

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## 6. Determination of a safety coefficient

In order to determine a safety coefficient, for the impact configurations considered in the present study, the following hypothesis were made :

- Under static loading, the ultimate behaviour of the structure is reached when the load just becomes constant under increasing displacement. Considering the results previously described, it is then supposed that the ultimate state of the structure only depends upon its bending resistance.

- Under dynamic loading, and when the displacement at the center reaches its maximum value, the deformed shape of the structure is identical to the one corresponding to the same displacement under static loading. Therefore, the strain energy for the maximum dynamic displacement is equal to the strain energy for the same displacement under static loading.

- The safety coefficient of the structure can be measured by the ratio between the strain energy at ultimate state, and the strain energy at maximum dynamic displacement.

The following methodology can then be applied to determine safety coefficients : the transient response of the slab subjected to impact is first computed, and the maximum displacement under the projectile is determined. By use of the static force displacement curve, it is then possible to determine the strain energy, for maximum dynamic displacement, and ultimate state. Therefore, the safety coefficient can be evaluated. This methodology is illustrated by Figure 9, it is fully compatible with computations using the global plasticity method.

This methodology was applied to the impacted slabs. A coefficient of 1 was found for slab n° 4. This suggests that the proposed definition for a safety coefficient is conservative enough, since it garanties that the overall behaviour of the structure will be governed by its bending resistance, under dynamic loading.

## 7. Conclusion

The study undertaken on reinforced concrete slabs subjected to quasi static loading and impact by low energy rigid missiles has led to the following conclusions :

- Under static loading, punching only occurs after the maximum bending resistance has been reached.

- Under dynamic loading, a certain level of energy is necessary before shear mechanisms play a significant role.

- The global method and the SAMSON and BILBO codes, of the CEASEMT System, were fully validated for static cases.

- Under dynamic loading, the maximum displacements can be accuraterly computed by the PLEXUS code.

- A methodology has been proposed to validate a safety coefficient, the values obtained are consistent with the experimental results.

More studies will be carried out in the future, in order to generalize these results. In particular, the case of a reduced surface of impact will be investigated, in order to establish wether flexural resistance is still reached, before punching.

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REFERENCES


[1] HOFFMANN A., LIVOLANT M., ROCHE R. "Plastic Analysis of Shells by Finite Element Methods. Global Plasticity model for any Shapes of Shells" Paper L 6/2 SMIRT 2, 1973.

[2] ROCHE R., HOFFMANN A. "Global Plasticity Models for Computerized Analysis", SMIRT 4, 1977.

[3] HOFFMANN A., MILLARD A., NAHAS G. "Application of a Global Plasticity Model to Determine the Ultimate Strength of a Reinforced Concrete Slab" Paper J 3/7, SMIRT 7, 1983.

[4] JEANPIERRE F. et al "CEASEMT - System of Finite Element Computer Programs" I.A.E.A./I.W.G.F.R. Specialists' meeting on High Temperature Structural Design Technology of LMFBRs - Champion, Pa, 1976.

[5] HOFFMANN A. et al "PLEXUS - A General Computer Program for Fast Dynamic Analysis" - Proceedings of the Conference on Structural Analysis and Design of Nuclear Power Plants, Porto Alegre, Brasil, 1984.


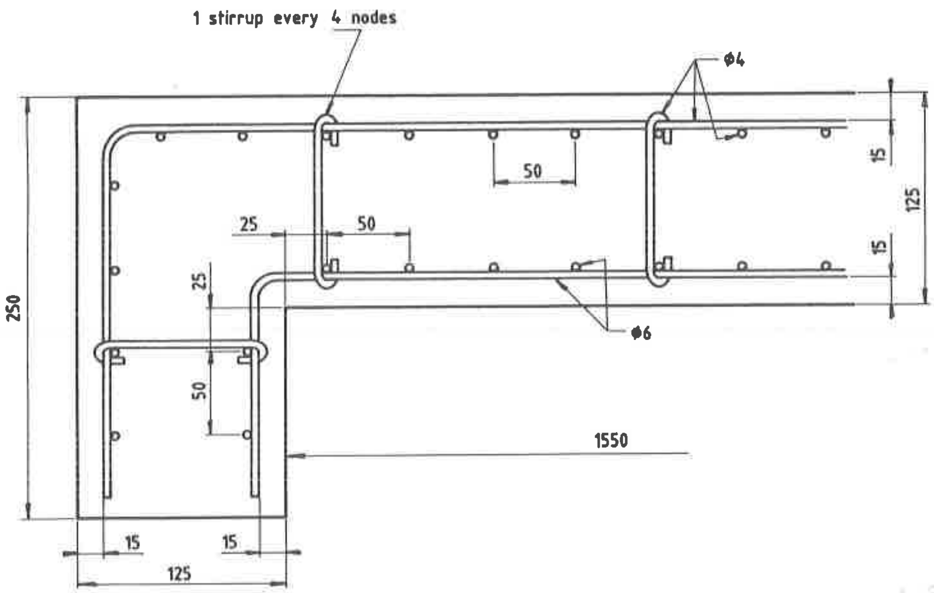


Figure 1 - GEOMETRY AND REINFORCEMENT OF THE EXPERIMENTAL SLABS

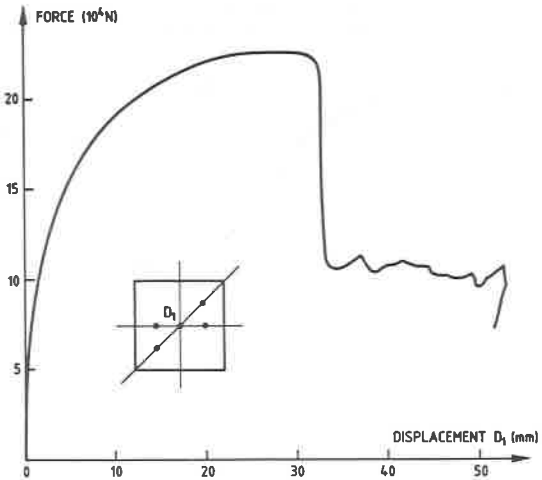


Figure 2 - EXPERIMENTAL FORCE-DISPLACEMENT CURVE FOR THE CENTER OF THE SLAB

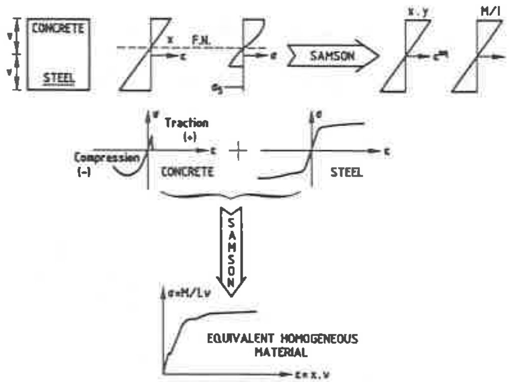


Figure 3 - DETERMINATION OF THE STRESS-STRAIN CURVE OF AN EQUIVALENT HOMOGENEOUS MATERIAL

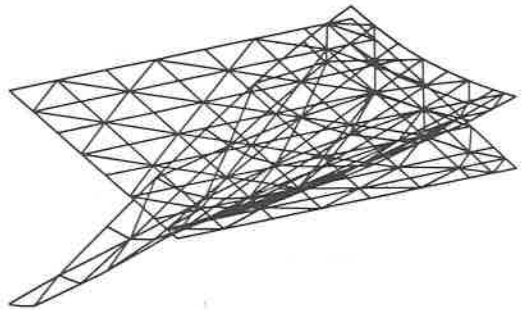

Figure 4 - MAGNIFIED DEFORMED SHAPE OF THE SLAB AT MAXIMUM LOAD

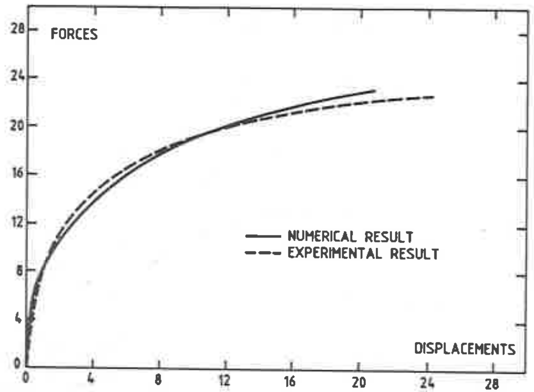


Figure 5 - NUMERICAL AND EXPERIMENTAL FORCE-DISPLACEMENT CURVES AT THE CENTER OF THE SLAB

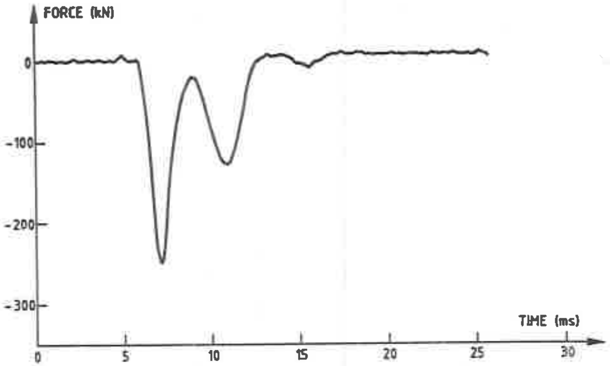


Figure 6 - REACTION FORCE VERSUS TIME, ON ONE LOAD CELL

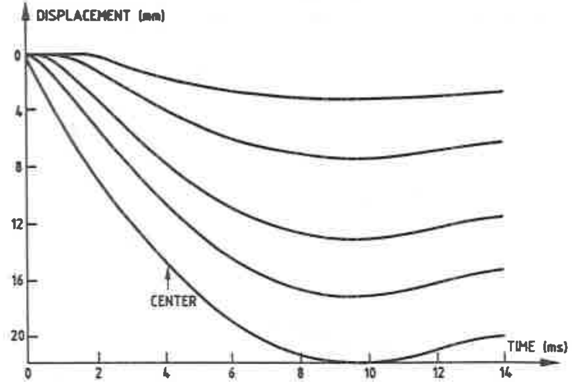


Figure 8 - COMPUTED DYNAMIC DISPLACEMENTS OF VARIOUS POINTS OF SLAB N°3

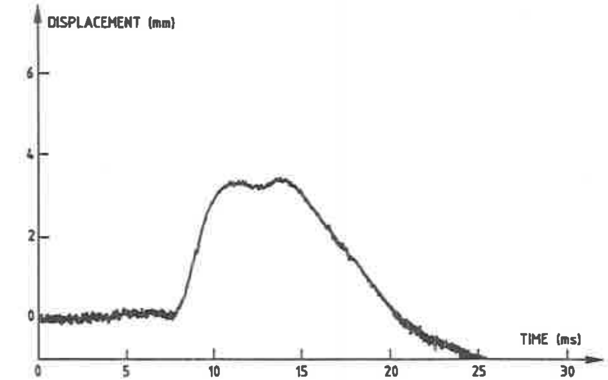


Figure 7 - DISPLACEMENT VERSUS TIME, AT THE MID-POINT BETWEEN THE CENTER OF THE SLAB AND ITS CORNER

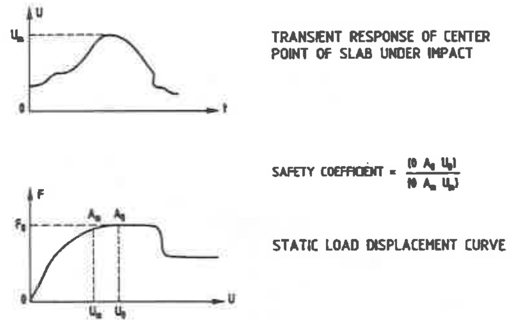


Figure 9 - DEFINITION OF A SAFETY COEFFICIENT