A SIMPLIFIED PROCEDURE FOR EVALUATING MODAL DAMPING FACTORS IN STRUCTURES WITH WIDELY VARYING DAMPING CAPACITIES

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SUMMARY

The dynamic response of composite structures by the modal superposition method can be conveniently performed only if the motion equations are decoupled by proper coordinate transformations. The decoupling of motion equations can be exactly obtained by using the damped complex eigenmodes of the structure. This rigorous procedure results into an expensive computational effort for structures discretized with a large number of degrees of freedom. For this reason some approximate solutions of the problem were proposed requiring only the computation of the undamped modal characteristics of the system.

Such techniques are based on the assumption that the damping matrix is diagonalized by the undamped modes so that the following relation holds:

$$X^T CX = 2\Omega N \qquad (1)$$

where C is the damping matrix, X is the matrix of the undamped modes, Ω and N are diagonal matrices, whose entries are the undamped frequencies and the equivalent modal damping ratios, respectively.

If the material damping properties are defined by a non-proportional damping matrix, Eq. (1) is used to define the equivalent modal damping ratios, assuming that the off-diagonal terms of the matrix X^TCX are negligible. Generally these terms are by no means negligible even if in some cases this approximation may produce acceptable results.

In this paper a simplified procedure for the evaluation of the equivalent modal damping ratios is proposed taking into account to some extent the contribution of the off-diagonal terms of matrix X^TCX.

This technique is based on the assumption that the complex eigenmodes can be expressed as a linear combination of the first few undamped modes. The coefficient of the linear combination and the approximate frequencies and damping modal factors are computed by solving a reduced generalized eigenvalue problem whose order is equal to the number of undamped modes considered.

To evaluate the efficiency of this procedure, a numerical experimentation was performed for the reinforced concrete containment and the internals of a typical pressure water nuclear reactor founded on different soil conditions. For this system the exact complex modes, frequencies, and damping factors were computed, and the results were compared with those obtained by the proposed technique. Additional comparisons were also made with the results obtained neglecting the off-diagonal terms of matrix X^TCX.

The numerical experimentation showed that a very small number of undamped modes ($5 \div 7$) were necessary to obtain modal damping factors nearly coincident with the exact ones. On the contrary, only in some cases, the results obtained using Eq. (1) were reasonably accurate.

1. Introduction

This paper presents a method to approximate the dynamic behaviour of structures with varying damping capacities using modal analysis. It is well-known that the modal superposition method is strictly applicable only in the case of viscous damping but in spite of this limitation, it is widely used for its practical advantages.

The application of this technique requires the definition of proper coordinate transformations to decouple the motion equations.

In general the decoupling cannot be obtained using the undamped modes of the structure but it is possible to retain modal superposition by working with the damped complex eigenmodes [1]. This rigorous procedure results into an expensive computational effort for structure discretized with a large number of degrees of freedom. For this reason approximate techniques are used requiring only the computation of the undamped modal characteristics of the system and the definition of equivalent modal damping ratios.

Among these techniques the most simple and straightforward procedure is the generalized form of Biggs' rule [2] which is based on the assumption that the damping matrix C is diagonalized by the matrix of undamped modes X. Even if this method provides reasonably accurate results in many cases, the assumption on which it is based is not generally verified. For this reason, the inspection of the off-diagonal terms of matrix $X^T CX$ is not sufficient to determine the actual range of applicability of this approximate technique.

In this paper a simplified procedure for the evaluation of the equivalent modal damping ratios is proposed taking into account to some extent the contribution of the off-diagonal terms of matrix $X^T CX$.

To evaluate the efficiency of this procedure, a numerical experimentation was performed for the reinforced concrete containment and internals of a typical pressure water nuclear reactor founded on different soil conditions.

2. Numerical method

The application of modal superposition in time domain for the dynamic analysis of structures with damping requires the solution of the generalized eigenvalue problem:

$$K\,\delta_i + \lambda_i\, C\,\delta_i + \lambda_i^2\, M\,\delta_i = 0 \qquad (2)$$

where K is the stiffness matrix, C is the viscous damping matrix, M is the mass matrix, λ_i is the ith eigenvalue and δ_i is the corresponding eigenmode. If matrix C satisfies the orthogonality condition:

$$X^T\, C\, X = D \qquad (3)$$

where X is the matrix of the undamped modes and D is a diagonal matrix, problem (2) is ultimately reduced to the solution of the undamped equations:

$$K\,X_i = \omega_i^2\, M\,X_i \quad ; \quad X_j^T\, M\,X_i = \begin{cases} 0 & j \neq i \\ 1 & j = i \end{cases} \qquad (4)$$

and one has $\delta_i \equiv X_i$.

If relation (3) is not verified, the eigenvectors δ_i are complex and each point of the structure may vibrate with a different phase angle.

An approximate solution of eq. (2) can be obtained by assuming that the complex eigenmodes are expressed as a linear combination of the first few undamped modes:

$$\delta_i = \bar{X}\;\bar{\delta}_i \qquad (i = 1, n) \qquad (5)$$

where $\tilde{X}$ is a matrix of order N x n whose columns are the first n undamped modes, $\tilde{\delta}_i$ is a vector of order n, N is the order of matrices K, C, M, and it is assumed that $n \ll N$.

Taking into account relation (5), eq. (2) premultiplied by $\tilde{X}^T$ becomes:

$$\tilde{\Omega}^2 \tilde{\delta}_i + \tilde{\lambda}_i \tilde{C} \tilde{\delta}_i + \tilde{\lambda}_i^2 \tilde{\delta}_i = 0 \qquad (6)$$

where $\tilde{\Omega}$ is a diagonal matrix of order n whose entries are the first n undamped frequencies, and $\tilde{C}$ is a matrix of order n x n whose elements are:

$$\tilde{C}_{ij} = X_i^T C X_j \qquad (i, j = 1, n) \qquad (7)$$

In this way, approximate values of the first complex eigenvectors and eigenvalues can be obtained by solving the reduced problem (6) which requires a computational effort much smaller than problem (2).

It must be noticed that if n = N, the solution of eq. (6) provides the exact solution of problem (2). The approximate complex eigenvectors δ_i and eigenvalues λ_i obtained by eqs. (5,6) can be used to perform modal superposition by working in complex arithmetics.

This consistent approach may involve some difficulties especially in the case of seismic analysis with response spectrum method. For this reason it may be convenient to apply a more simplified procedure based on the assumption that the complex modes δ_i are coincident with the undamped modes X_i, while the complex eigenvalues λ_i are those computed by eq.(6). In this case the equivalent damping modal ratios are:

$$\beta_{i\ eq} = \frac{-\mathrm{Re}\ (\tilde{\lambda}_i)}{|\tilde{\lambda}_i|} \qquad (8)$$

where Re $(\tilde{\lambda}_i)$ is the real part of $\tilde{\lambda}_i$, and $|\tilde{\lambda}_i|$ is its modulus.

It must be noticed that if the off-diagonal terms of matrix $\tilde{C}$ are neglected, the equivalent modal damping factors obtained by eq. (8) are coincident with those computed by the generalized form of Biggs' rule.

3. Numerical example

As a first test to evaluate the efficiency of the suggested procedure, we just computed the equivalent modal damping ratios for the reinforced concrete containment and internals of a typical pressure water reactor founded on different soil conditions [3] .

For this type of system, different damping properties are assumed for the fixed base structure and the soil. The damping of the fixed base structure is generally described by assigning equivalent damping ratios for each mode and assuming that the damping matrix C is defined by the relationship

$$\phi^T C \phi = 2\ \Omega\ N \qquad (9)$$

where ϕ is the matrix of the undamped modes of the fixed base structure and Ω and N are diagonal matrices whose entries are the undamped frequencies and equivalent modal damping ratios, respectively.

The damping properties of the soil are described by dashpot constants so that the assembled damping matrix is not diagonalized by the undamped modes of the soil-structure system. Therefore the application of the modal superposition method for soil-structure system can be carried out using the approximate procedure described in Sect. 2.

For the particular case considered, the containment structure and the internals were idealized as a 18 lumped mass model, and the soil-structure interaction was represented by two springs and related dampers: one set corresponding to horizontal motion and the other to rocking. Fig. 1 represents the lumped mass model of structure-foundation system and Table I summarizes the properties of the fixed base structure model. The damping of the fixed structure was neglected, and 4 soil conditions were considered corresponding to different shear wave velocities [4] . Table II quotes the equivalent foundation spring-damper constants relevant to the cases examined. The spring-damper constants K_x, C_x for swaying and K_ϕ, C_ϕ for rocking were chosen so that for each case the damping ratios for the foundation were constant and equal to:

$$\beta_x = \frac{C_x}{2\sqrt{K_x \cdot M}} = 41.6\%; \qquad \beta_\phi = \frac{C_\phi}{2\sqrt{K_\phi \cdot I}} = 7.2\%$$

where I and M are the total mass moment of inertia and the total mass of the system, respectively.

For the cases examined, the first four complex eigenmodes and eigenvalues were computed exactly by solving eq. (2) and the first four equivalent damping modal ratios were computed according to eq. (8) using 5 and 7 undamped modes, respectively. The results obtained are shown in Tables III, IV, V, VI where, for comparison's sake, also the equivalent modal damping ratios computed by the generalized form of Biggs' rule are quoted.

Moreover, only for case No. 1, the first four undamped modes of the structure-foundation system are shown in fig. 2.

The results quoted in the tables indicate that the procedure proposed provides reasonably accurate results for all the cases considered; the generalized Biggs' method supplies acceptable values of the equivalent modal damping ratios only for case No. 1 while it produces increasing errors when the ratios $\frac{K_x}{C_x}$, $\frac{K_\phi}{C_\phi}$ increase.

The equivalent damping ratios for the 5^{th} mode, computed with the proposed procedure, were not quoted as their accuracy was considered to be inadequate. Indeed a comparison between the values obtained working with 5 and 7 undamped modes showed differences in the 10÷40% range. This indicates that 5 undamped modes are not sufficient to compute 5 damping modal ratios with reasonable accuracy. It must be noticed that acceptable results for the first four modal damping ratios were obtained by solving two standard eigenvalue problems of order 10 and 14. Therefore the computational effort was much lesser than required for the solution of eq. (2) which is equivalent to a standard eigenvalue problem of order 76.

4. Conclusions

This paper presents an approximate procedure for computing the equivalent modal ratios to apply the normal mode superposition to structures characterized by a damping matrix which is not diagonalized by the normal modes. This technique is based on the assumption that the complex eigenmodes of the structure can be expressed as a linear combination of the first few undamped modes.

Obviously it is not possible to define a priori the number n of undamped modes to be used in view of results of acceptable accuracy. However in practice a small number of numerical experiments with different values of n is sufficient to decide if the results are adequate. In general it is reasonable to assume coincident results obtained with different values of n as exact. The validity of this criterium is confirmed by the numerical results presented in the paper. Indeed, the first four modal damping ratios obtained using 5 and 7 undamped modes were nearly coincident and in good agreement with the exact ones.

It must be noticed that the suggested procedure allows to obtain not only approximations of the complex eigenvalues but also of the complex eigenmodes. For this reason such procedure may be used to perform modal superposition - working in complex arithmetics - with a reduced computational effort.

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References

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Table I

Properties of the structural models of the containment building and internals

(Concrete: $E = 3.30 \cdot 10^{10}\ N \cdot m^{-2}$; $\nu = 0.27$)
(Internals: $E = 2.06 \cdot 10^{11}\ N \cdot m^{-2}$; $\nu = 0.3$)

Joint Properties				Member Properties			
Mass No	$m_i \times 10^{-6}$ (kg)	$I_i \times 10^{-6}$ ($kg \cdot m^2$)		Location between Joint No	Area (m^2)	Shear Area (m^2)	Moment of Inertia $\times 10^{-3}$ (m^4)
base	9.07	889.15	CONTAINMENT				
1	2.08	396.15		base to 1	130.06	65.03	24.16
2	1.90	358.19		1 to 2	"	"	"
3	"	"		2 to 3	"	"	"
4	"	"		3 to 4	"	"	"
5	"	"		4 to 5	"	"	"
6	"	"		5 to 6	"	"	"
7	2.09	396.11		6 to 7	"	"	"
8	1.37	248.63		7 to 8	91.97	46.45	16.40
9	1.12	155.92		8 to 9	"	"	12.94
10	0.96	71.64		9 to 10	"	"	6.90
11	0.08	4.21		10 to 11	"	"	1.72
12	1.27	101.14	INTERNALS	base to 12	185.81	122.63	9.49
13	1.14	80.06		12 to 13	273.83	144.93	10.35
14	2.85	210.70		13 to 14	205.32	135.64	10.35
15	1.70	278.12		14 to 15	182.09	67.82	11.22
16	3.87	530.97		15 to 16	161.65	55.74	7.76
17	0.55	33.71		16 to 17	72.46	33.44	1.72
18	0.37	4.21		17 to 18	17.65	6.50	0.01

Table II

Equivalent foundation spring-damper constants

Case number	Translational spring, K_x $(N \cdot m^{-1})$	Translational damper, C_x $(N \cdot s \cdot m^{-1})$	Rotational spring, K_ϕ $(N \cdot m)$	Rotational damper, C_ϕ $(N \cdot s \cdot m)$
1	$7.54 \cdot 10^{10}$	$1.41 \cdot 10^{9}$	$2.40 \cdot 10^{13}$	$1.10 \cdot 10^{11}$
2	$1.51 \cdot 10^{11}$	$1.99 \cdot 10^{9}$	$4.80 \cdot 10^{13}$	$1.56 \cdot 10^{11}$
3	$3.02 \cdot 10^{11}$	$2.84 \cdot 10^{9}$	$9.60 \cdot 10^{13}$	$2.21 \cdot 10^{11}$
4	$6.04 \cdot 10^{11}$	$4.02 \cdot 10^{9}$	$1.92 \cdot 10^{14}$	$3.13 \cdot 10^{11}$

Table III

Results for case No 1

		Equivalent modal damping ratios as a percentage			
Mode number	Undamped frequencies (Hz)	Exact method (eq.(2))	Generalized Biggs' method	Present method (5 modes)	Present method (7 modes)
1	3.3	4.4	4.6	4.4	4.4
2	9.0	45.0	40.7	44.6	44.8
3	14.6	3.7	4.2	3.8	3.7
4	17.4	11.6	11.1	11.0	11.4

Table IV

Results for case No 2

		Equivalent modal damping ratios as a percentage			
Mode number	Undamped frequencies (Hz)	Exact method (eq.(2))	Generalized Bigg's method	Present method (5 modes)	Present method (7 modes)
1	4.0	2.5	2.6	2.5	2.5
2	11.5	36.9	28.7	36.9	36.8
3	15.6	5.8	5.2	5.4	5.5
4	18.4	11.4	12.2	9.9	11.0

Table V

Results for case No 3

		Equivalent modal damping ratios as a percentage			
Mode number	Undamped frequencies (Hz)	Exact method (eq.(2))	Generalized Biggs' method	Present method (5 modes)	Present method (7 modes)
1	4.5	1.2	1.2	1.2	1.2
2	13.4	4.7	10.8	5.6	4.9
3	16.7	2.7	5.2	2.8	2.8
4	20.0	39.0	20.9	33.6	37.2

Table VI

Results for case No 4

		Equivalent modal damping ratios as a percentage			
Mode number	Undamped frequencies (Hz)	Exact method (eq.(2))	Generalized Biggs' method	Present method (5 modes)	Present method (7 modes)
1	4.8	0.5	0.5	0.5	0.5
2	14.2	1.6	2.5	2.0	1.8
3	17.4	0.8	1.3	0.9	0.9
4	23.7	4.1	20.4	2.9	3.6

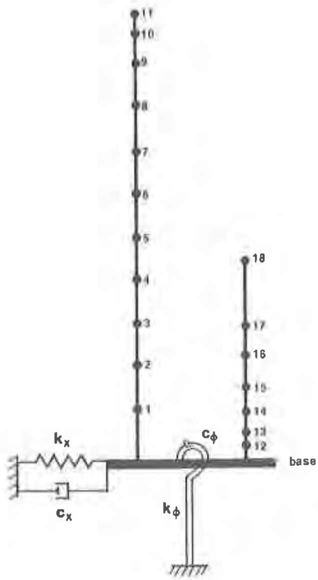

Fig. 1 Lumped mass model of structure-foundation system.

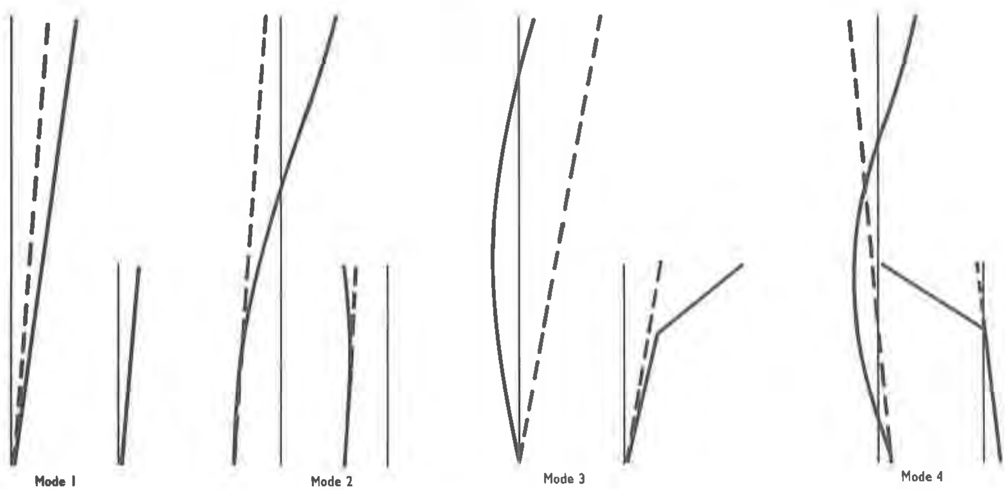

Fig. 2 First four modal shapes of flexible base structure (case No 1)