AXISYMMETRIC REACTOR AND INTERNALS MODEL FOR SEISMIC ANALYSES

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SUMMARY

The major internal components of most nuclear reactors are beam-type structural elements, i.e., fuel rods, CRD tubes and housings, etc.; while the reactor pressure vessels, shrouds, internal supporting plates, etc., are shell elements. To conduct a 2-dimensional finite element dynamic analysis for the combined reactor and internal lumped mass system due to seismic excitations, it is a common practice to consider the cross-section of each element is rigid. One then can treat the shell elements as beams and the plates as springs, and the beam theory alone can be utilized to establish a mathematical model for the entire system. The advantage of this method of approach is that the element stiffness matrices for various types of beams are readily available and the system equations of motion can be formulated and solved in a straight-forward manner. The drawback of this procedure, however, is that for vessels of large radius to thickness ratios, the resulting dynamic properties are exclusively stiffer than those yielded by the shell theory, and hence the solution so obtained may not be appropriate.

The purpose of the present paper is to introduce another method of attack, namely, establishing the mathematical models essentially by the shell theory alone. This can be accomplished by transforming all the internal tubular beam elements into equivalent shell elements, using the orthotropic theory of elasticity with suitable modifications and the basic law of similarity between the actual beam and the transformed shell elements. Through the present technique, a mathematical model is established with which a dynamic analysis has also been conducted. It is found that this new method not only yielded satisfactory solutions for both $n=0$ and $n=1$ shell modes but also required minimum computer input preparations. The applicability of the present method of analysis is further significantly increased by the incorporation of the hydrodynamic mass matrices due to the fluid or liquid sodium contained within the reactor vessel. These effects can be efficiently taken care of by the introduction of the axisymmetric fluid finite elements. As a consequence of the appearance of the hydrodynamic mass matrices which have off-diagonal terms, however, it is desirable to solve the following eigen-value system equations:

$$Ax = \omega^2 Bx \quad (1)$$

where
A = positive definite system stiffness matrix,
ω = circular frequencies,
B = non-negative definite system mass matrix,
x = eigen-vector.

To ensure the dynamic behavior of the transformed shell elements is essentially identical to that of the beam elements, eigen-values and vectors of the transformed shell elements have been derived by the Rayleigh-Ritz variational principle and checked with those for uniform beams having the classical boundary conditions.

1. Introduction

As a counter part to the conventional lumped-mass models which are usually established by the beam theory, an axisymmetric mathematical model for a reactor and internals is presented. This model is intended for use on the digital program ASHSD2 which accepts four types of finite elements, viz., solid, thin-shell, fluid and effective cylindrical elements [1]. Since the theoretical treatments for the development of the first three types of finite elements have been well established, in this presentation only the fundamental aspects of these elements are discribed briefly, and special emphasis will be given to the generation and verification of the effective cylindrical element.

The solid element may take the triangular section or have the combination of four triangles to form the quadrilateral shape [2]. The assumptions used for the formulation of the stiffness matrix are linear displacement field, orthotropic material properties and the degenerated 3-D elasticity. The displacement vector is decomposed into Fourier terms such that the asymmetric harmonics can be handled properly. This type of element is also suitable for thick shells, provided that an adequate number of layers are used through the thickness.

The thin-shell element has the form of a conical frustum. It can be used to fit the shape of any axisymmetric configuration approximately by rotating the semi-apex angle, making a cylindrical element on the one extreme and a flat plate on the other. A linear displacement field is used for both axial and tangential components, and polynominals up to the third degree are assumed for the transverse component. Orthotropic materials are also accepted.

The assumptions used for the establishment of the mass matrix of the fluid element are (a) that the fluid is homogeneous, incompassible and invicid, (b) that the motion of the entrapped fluid between the concentric flexible cylinders have no vertical component, (c) that the two-dimensional velocity field is linear, (d) and that leakage through the cylinder is permissible. Since the velocity vector can be expressed in Fourier harmonics, the modal hydrodynamic effects of the system can be obtained by the Lagrange equation. Due mainly to the coupling between the radial and tangential velocities, however, the resulting element mass matrix has off-diagonal terms, and a special algorithm for the solution of the undamped dynamic characteristics of the system under consideration has been incorporated in the present program [3].

2. Effective Cylindrical Element

The basic assumptions used for the establishment of the effective cylindrical shell elements are, as follows:

a. All tubular beam-column elements are identical in geometry and material properties.

b. All staggered elements except those located at the center of the vessel, if any, can be rearranged such that they are standing side by side along the circumference of a circle and each of which assumes a solid square transformed cross-section. (See Figure 1).

c. The state of stress for the effective shell elements is governed by the orthotropic theory of elasticity except that the normal stress, the shear strains and the Poisson effects (coupling terms) vanish identically.

d. For an arbitrary boundary condition and applied load, the transformed shell elements behave essentially both statically and dynamically the same as the actual individual elements.

Accordingly, transformation of the real elements to the effective shell elements shown in Figure 1 can be made by the application of the basic law of dynamic similarity, as follows: For any mode number m:

$$a_m \, (K_o/M_o)^{1/2} = a_m \, (K_e/M_e)^{1/2} \qquad (2)$$

where a_m = Constant

$K_o(K_e)$ = Actual (effective) stiffness of an arbitrary structural element

$M_o(M_e)$ = Actual (effective) mass of an arbitrary structural element

For elements of constant lengths, Eq. (2) may be expressed as

$$E_oI_o/M_o = E_eI_e/M_e \tag{3}$$

in which $E_o(E_e)$ = Modulus of elasticity of an actual (effective) element

$I_o(I_e)$ = Moment of inertia of an actual (effective) element

Referring to Figure 1, let n_o, a_o and R_e represent the number of identical actual elements, area of an idividual element and the radius of the effective shell elements, respectively. Then, the general expression for the effective modulus of elasticity can be derived from Eq. (3), namely:

$$E_e = \rho E_o I_o h_e^2 / I_e a_o \tag{4}$$

where the effective thickness $h_e = a_o\, n_o/2\pi R_e$ (5)

and ρ denotes the ratio of the actual and effective mass densities ρ_o/ρ_e.

For the purpose of verifying the validity of Assumption (d) mentioned above, the general expression for the undamped flexural vibrations of the effective cylindrical element with freely supported ends has been derived using the Rayleigh-Ritz procedure and the thin-shell theory. A brief description of the derivation follows.

The displacement components at any arbitrary location of a shell can be expressed as

$$\begin{Bmatrix} U \\ V \\ W \end{Bmatrix} = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} + \begin{Bmatrix} -w_{,s} \\ v/R_e - w_{,} \quad /R_e \\ 0 \end{Bmatrix} \tag{6}$$

where commas present partial derivatives, R_e is the effective radius of the cylinder and u,v,w are middle surface displacement components (See Figure 2c). [4]

Based on Assumption (c), the number of elastic constants for materials having orthotropic properties reduces to two and the pertinent stress-strain relations in the axial and circumferential directions are

$$\begin{Bmatrix} \sigma_s \\ \sigma_\theta \end{Bmatrix} = \begin{bmatrix} c_{11} & 0 \\ 0 & c_{22} \end{bmatrix} \begin{Bmatrix} \epsilon_s \\ \epsilon_\theta \end{Bmatrix} \tag{7}$$

The strains may be expressed in two parts, namely

$$\begin{Bmatrix} \epsilon_s \\ \epsilon_\theta \end{Bmatrix} = \begin{Bmatrix} \epsilon_s^o \\ \epsilon_\theta^o \end{Bmatrix} + z \begin{Bmatrix} \chi_s \\ \chi_\theta \end{Bmatrix} \tag{8}$$

where the middle-surface strains in turn can be expressed in terms of displacement components as

$$\begin{Bmatrix} \epsilon_s^o \\ \epsilon_\theta^o \end{Bmatrix} = \begin{Bmatrix} u_{,s} \\ (v_{,\theta} + w)/R_e \end{Bmatrix} \tag{9}$$

and the kinematic relations for the changes of curvature are

$$\begin{Bmatrix} \chi_s \\ \chi_\theta \end{Bmatrix} = \begin{Bmatrix} -w_{,ss} \\ -(w + w_{,\theta\theta})/R_e \end{Bmatrix} \tag{10}$$

The total strain energy for the deformed effective cylindrical shell is

$$S = \frac{R_e}{2} \int_0^L \int_0^{2\pi} \int_{-\frac{h_e}{2}}^{+\frac{h_e}{2}} (\sigma_s \epsilon_s + \sigma_\theta \epsilon_\theta)\, dz\, d\theta\, ds \tag{11}$$

where L denotes the length of the effective cylinder.

If the in-plane and rotatory inertia terms are neglected, the kinetic energy of the shell at any instant is

$$T = \frac{\rho_e}{2} R_e \int_0^L \int_0^{2\pi} \int_{-\frac{h_e}{2}}^{+\frac{h_e}{2}} \dot{w}^2\, dz\, d\theta\, ds \tag{12}$$

For simply supported classical boundary conditions, i.e., $v = w = M_s = N_s = 0$ at both ends, the following eigenfunctions are admissible

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = e^{i\omega_e t} \begin{Bmatrix} U_{mn} \cos \frac{m\pi s}{L} \cos n\theta \\ V_{mn} \sin \frac{m\pi s}{L} \sin n\theta \\ W_{mn} \sin \frac{m\pi s}{L} \cos n\theta \end{Bmatrix} \tag{13}$$

in which U_{mn}, V_{mn}, W_{mn} are functions of time, m, n are mode numbers and W_e is the circular frequency. The nodal patterns for the vibrations of the cylindrical shells are depicted in Figure 2ab.

The minimization of the Lagrangian formed by the energy expressions with respect to the generalized coordinates yields

$$\frac{\partial}{D_j}(S - \lambda T) = 0,\ j = 1, 2, 3 \tag{14}$$

where

$$D_j^T = [\ U_{mn} \quad V_{mn} \quad W_{mn}\ ] \tag{15}$$

and

$$\lambda = \rho_e \omega_e^2 / C_{11} \tag{16}$$

Suitable substitution of Eqs. (6) through (14) and performing the necessary mathematical manipulations lead to the following frequency equation.

$$\lambda = \frac{h_e^2}{12} \left[\frac{m^4 \pi^4}{L^4} + \frac{K(1+3n^2)}{R_e^4} \right]^{\frac{1}{2}} \tag{17}$$

where

$$K = C_{22} / C_{11} \tag{18}$$

It should be pointed out that, within the framework of the present assumptions, the axial vibration is decoupled with the tangential vibrations.

According to Equation (2), it is apparent that

$$a_o n_o \rho_o = 2\pi \rho_e h_e R_e \tag{19}$$

and the general expression for the flexural vibration of a simply supported beam is

$$\omega_o^2 = a_m^2 E_o I_o / a_o \rho_o L^4 \tag{20}$$

Simultaneous solution of Eqs. (17) through (20) provides the frequency ratio between the effective cylinder and the uniform beam, as follows

$$\tilde{\omega} = \omega_e / \omega_o = \left[\frac{\pi^4 m^4}{a_m^4} + \frac{L^4 K}{a^4 R_e^4} (1 + 3n^2) \right]^{\frac{1}{2}} \tag{21}$$

For axisymmetric vibrations, n=0 and C_{22}=0 by virtue of v=0, Eq. 21 reduced to

$$\tilde{\omega} = m^2 \pi^2 / a_m^2 \tag{22}$$

Equation (22) implies the frequencies for the breathing modes of the effective cylinder are identical to those for the bending modes of the actual beams. [5], [6]

For asymmetric vibrations, n=1, and to insure the cross-section of the shell remain circular, K=1 such that the tranverse displacement component (w) at θ=0 is exactly equal to the tangential component (v) at θ=π/2. Equation (21) then becomes

$$\tilde{\omega} = \left[\frac{\pi^4 m^4}{a_m^4} + \frac{4 L^4}{a_m^4 R_e^4} \right]^{\frac{1}{2}} \tag{23}$$

It is apparent that for small L/R_e, Eq. (23) approaches the exact solution as a limit. For some tubular elements of the present model, $\frac{L}{R_e} = 1.73$, Eq. (23) furnishes the results for the lowest three modes as shown in Table I. It appears that maximum discrepancy of the frequency ratio of 17 percent occurs in the fundamental mode. As the mode number increases, however, the error decreases quite rapidly.

3. Mathematical Model

The mathematical model is shown in Figure 3. This model is composed of 124 nodal points and 169 finite elements of which 119 are shells, 42 are fluids and 8 are solids. It is noted that there are two types of shell elements in this model, viz., regular and effective elements. The latter (shown in dash lines in Figure 3) represent the transformed tubular beamtype structural members. The cylindrical elements located at the axix of symmetry are actual tubular elements. Due to the small radius to thickness ratios, these central elements have relatively shorter lengths.

The present method of transformation seems to yield satisfactory solutions for the elasticity supported effective shell elements, i.e., elements with displacement boundary conditions such as that shown in Figure 4a. Considering other hinged boundary conditions such as shown in Figure 4b, c, however, it is desirable to modify the element stiffness by the method of static condensation. This can be achieved in the following way: The force-displacement relations of the elastically supported shell (Case 1) as shown in

Figure 4a may be defined as

$$[F_i] = [K_{ij}][u_j], \quad i, j = 1, 2, \cdots 8 \tag{24}$$

in which K_{ij} is the stiffness matrix of the element.

The corresponding stiffness matrix for the Case 2 can be obtained from Equation (24) by the elimination of the displacement component u_4 since $F_4 = 0$. Appropiate matrix manipulations yield the following results

$$[\tilde{F}_i] = [\tilde{K}_{ij}][\tilde{u}_j] \tag{25}$$

where

$$[\tilde{K}_{ij}] = [B]^{-1}[K_{ij}] \tag{26}$$

$$[B] = \begin{bmatrix} 1-A_{41}K_{14} & -A_{42}K_{14} & -A_{43}K_{14} & 0 & -A_{45}K_{14} & -A_{46}K_{14} & -A_{47}K_{14} & -A_{48}K_{14} \\ & 1-A_{42}K_{24} & -A_{43}K_{24} & 0 & -A_{45}K_{24} & -A_{46}K_{24} & -A_{47}K_{24} & -A_{48}K_{24} \\ & & 1-A_{43}K_{34} & 0 & -A_{45}K_{34} & -A_{46}K_{34} & -A_{47}K_{34} & -A_{48}K_{34} \\ & & & 0 & 0 & 0 & 0 & 0 \\ & & & & 1-A_{45}K_{54} & -A_{46}K_{54} & -A_{47}K_{54} & -A_{48}K_{54} \\ & & & & & 1-A_{46}K_{64} & -A_{47}K_{64} & -A_{48}K_{64} \\ & & & & & & 1-A_{47}K_{74} & -A_{48}K_{74} \\ \text{SYM.} & & & & & & & 1-A_{48}K_{84} \end{bmatrix} \tag{27}$$

and

$$[A_{ij}] = [K_{ij}]^{-1} \tag{28}$$

Similarly, the corresponding stiffness matrix for the Case 3 can be established by eliminating the displacement component u_8, namely

$$[\underset{\sim}{F}_i] = [\underset{\sim}{K}_{ij}][\underset{\sim}{u}_j] \tag{29}$$

where

$$[\underset{\sim}{K}_{ij}] = [C]^{-1}[K_{ij}] \tag{30}$$

and

$$
[C] = \begin{bmatrix}
1-A_{81}K_{18} & -A_{82}K_{18} & -A_{83}K_{18} & -A_{84}K_{18} & -A_{85}K_{18} & -A_{86}K_{18} & -A_{87}K_{18} & 0 \\
 & 1-A_{82}K_{28} & -A_{83}K_{28} & -A_{84}K_{28} & -A_{85}A_{28} & -A_{86}K_{28} & -A_{87}K_{28} & 0 \\
 & & 1-A_{83}K_{38} & -A_{84}K_{38} & -A_{85}A_{38} & -A_{86}K_{38} & -A_{87}K_{38} & 0 \\
 & & & 1-A_{84}K_{48} & -A_{85}A_{45} & -A_{86}K_{48} & -A_{87}K_{48} & 0 \\
 & & & & 1-A_{85}K_{58} & -A_{86}K_{58} & -A_{87}K_{58} & 0 \\
 & & & & & 1-A_{86}K_{68} & -A_{87}K_{68} & 0 \\
 & & & & & & 1-A_{87}K_{78} & 0 \\
\text{SYM.} & & & & & & & 0
\end{bmatrix} \tag{31}
$$

The modal characteristics of some lower beam (n=1) modes of the present model have been generated by the program ASHSD2. Similar results have also been produced by another program which was developed according to the beam theory. When results are compared, it was found that the correlations are quite good, except that the frequencies for shell elements having large radius-to-thickness ratios generated by the latter are relatively higher. It should be noted that these results are based on the lumped mass discrete systems.

Finally, it can be concluded that the axisymmetric model established by the present technique can be used not only equally well as those established by the conventional technique for seismic analyses due to horizontal motions, but also is capable of generating both the system dynamic characteristics and response due to vertical ground excitations (n=0 modes). It is apparent that the present model is relatively superior since the later capability is lacking in most of the presently available techniques which are established essentially by the beam theory.

REFERENCES

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TABLE I - FREQUENCY RATIOS

m	n	a_m	$\widetilde{\omega}$
1	1	π	1.17
2	1	2π	1.11
3	1	3π	1.03

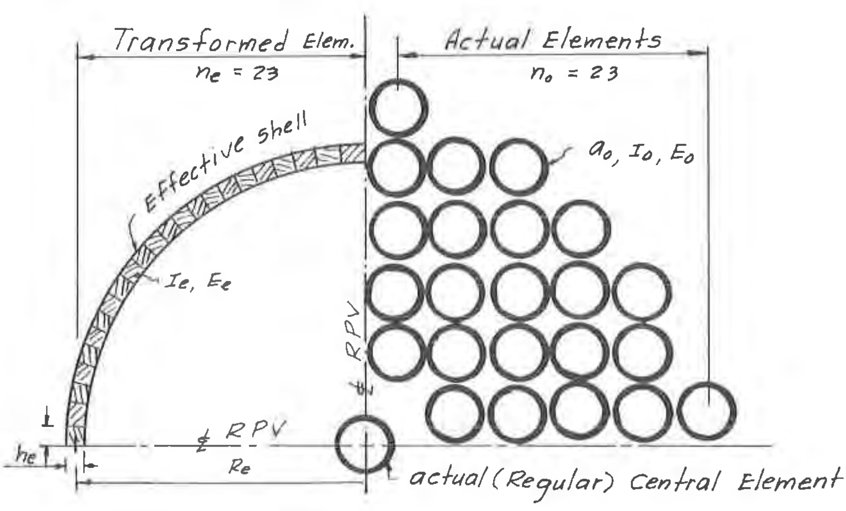
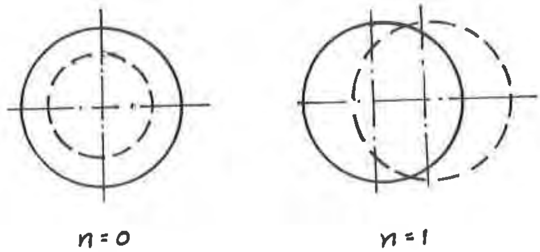
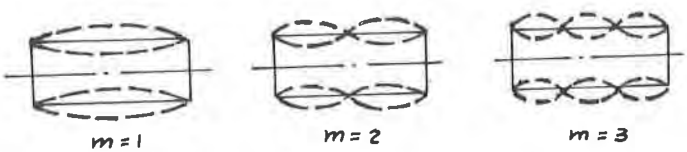


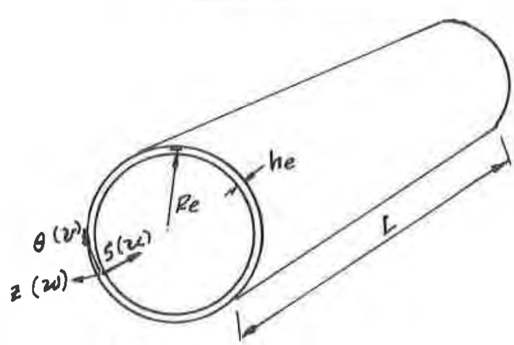
Fig. 1 CROSS-SECTION OF REGULAR AND TRANSFORMED ELEMENTS



a. CIRCUMFERENTIAL NODAL PATTERNS



b. AXIAL NODAL PATTERNS



c. GEOMETRY

Fig. 2 NODAL PATTERNS AND GEOMETRY OF SHELL

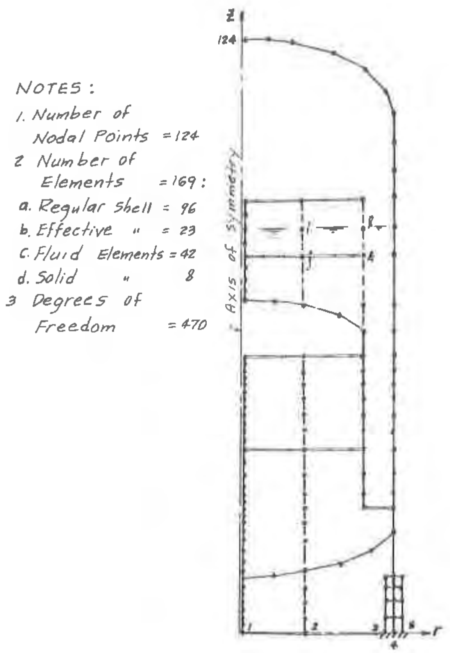
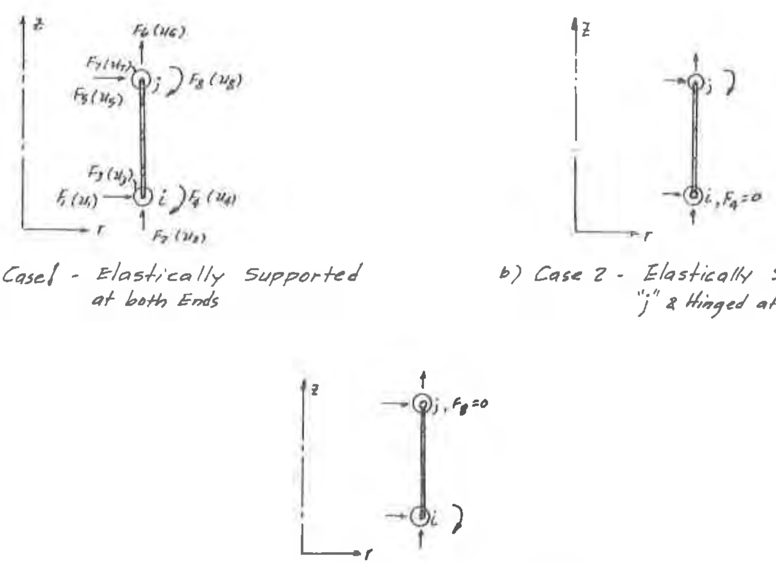


Fig. 3 MATHEMATICAL MODEL FOR REACTOR AND INTERNALS



a) Case 1 - Elastically Supported at both Ends

b) Case 2 - Elastically Supported at "j" & Hinged at "i"

c) Case 3 - Elastically Supported at "i" & Hinged at "j"

Fig. 4 BOUNDARY CONDITIONS OF EFF. SHELL ELEMENTS