

## Effects of Phase Characteristics of Simulated Earthquake Ground Motions on the Response Spectra

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### Abstract

The effects of phase characteristics of the simulated earthquake ground motions on the response spectra are examined by use of the phase difference distribution concept and the random theory.

The simulated motions which have the power spectra of the recorded earthquake motions are generated for three cases of the breadth of the phase difference distribution. It is found that when the standard deviation of the phase difference distribution of the simulated motions is 0.75 of the recorded one, the response spectra of the simulated motions are almost at the same level of the recorded ones. The equivalent duration time and the equivalent r.m.s. values are also close to the recorded ones.

### 1. Introduction

Many works have been carried out in order to examine the important role of phase characteristics of earthquake ground motions such that the probability distribution of phase differences approximately determines the envelope function of time history [1]. And the relationship between the breadth of the phase difference distribution and the variation in response spectra with various damping ratios was parametrically studied. An effective application of this relationship was successfully attempted to improve a generation technique of simulated earthquake ground motions as proposed by the authors [2].

However the comparison of the recorded earthquake motions and the simulated ones shows many discrepancies regarding phase characteristics. For example, the simulated motions which meet a target response spectrum with a specified damping ratio (5% in general) tend to produce greater values of response spectrum with lower damping ratio than those of the target response.

The purpose of this paper is to examine fundamentally the effects of phase characteristics of earthquake ground motions on the response spectra and then to explore additional factors of phase characteristics such as the equivalent duration time and the equivalent r.m.s. value by means of the random approach. The simulated motions are generated by utilizing the power spectra of the recorded earthquake motions and the phase difference distributions with their various breadth as parameters.

## 2. Generation of Simulated Earthquake Ground Motions

The power spectra of four recorded earthquake motions were used in this study. The recorded motions are as follows ;

- 1940 Imperial Valley earthquake, ElCentro NS ( EL1 )
- 1952 Kern County earthquake, Taft EW ( TFT )
- 1957 San Francisco earthquake, Golden Gate ( GGT )
- 1979 Imperial Valley earthquake, Differential array 360( EL2 )

The duration time of these records ( $T_d$ ) was determined as the recorded length from 5% to 95% of the amplitude energy sum plus additional 10% of this length at the front of the wave. The time history of EL1 in the duration time  $T_d$  is shown in Fig.1.

The phase difference ( $\Delta\phi$ ) is defined as the difference of Fourier phase angles for two consecutive frequencies [1].

$$\Delta\phi_k = \phi_{k+1} - \phi_k \quad (1)$$

The phase differences are expressed in negative values between 0 and  $-2\pi$ (rad). The phase difference distribution shape was postulated to be rectangular with breadth  $B$  in this study, for simplification. Since the breadth  $B$  equals  $\sqrt{12}$  times the standard deviation  $\sigma_{\Delta\phi}$ , the equivalent phase difference breadth for the recorded motion defined as  $B_o$ , i.e.,

$$B_o = \sqrt{12} \sigma_{\Delta\phi} \quad (2)$$

The breadth values for the simulated motions are selected equal to  $B_o$ ,  $0.75B_o$  and  $0.5B_o$ . The mean value of the phase difference distribution was set to  $-\pi$ , i.e. the large amplitudes are located at the middle of the duration time. Ten sample motions were generated in each case.

Sample time histories for the simulated motions with various  $B$  values are shown in Fig.2. These are the cases where the characteristics of EL1 were used. From the comparison of Fig.1 with Fig.2 it is found that in the case of  $B=0.75B_o$  the simulated motion has an envelope similar to the recorded one, while in cases of  $B=B_o$  and  $B=0.5B_o$  the simulated motions have longer and shorter envelope shapes than the recorded motion, respectively.

## 3. Application of Random Theory to Response Spectra Estimation

An earthquake motion is generally a nonstationary process; however, under certain conditions, it can be considered as an equivalent stationary process that has the equivalent root mean square (r.m.s.) value and the equivalent duration time determined by the random theory. Applying the relationship between the maxima and r.m.s. value [3], the equivalent r.m.s. value  $\sigma_e$  and the equivalent duration time  $T_e$  are expressed as follows [4],

$$\sigma_e = X_{max} / \{(2 \ln v_0 T_e)^{1/2} + \gamma (2 \ln v_0 T_e)^{-1/2}\} \quad (3)$$

and

$$T_e = E / \sigma_e^2 \quad (4)$$

where  $X_{max}$ ,  $v_0$ ,  $\gamma$  and  $E$  are the maximum amplitude, the dominant frequency, Euler constant ( $=0.5772$ ) and the summation of squared values of amplitudes, respectively.  $T_e$ ,  $\sigma_e$  and other parameters of the recorded motions are shown in Table 1.

Considering the transient response, the expected maximum response of the system is estimated as follows [5],

$$E[Y_{max}] = \sigma_y \left\{ (2 \ln \nu_0 T_e)^{1/2} + \gamma (2 \ln \nu_0 T_e)^{-1/2} \right\} [1 - \exp(-2h\omega_0 T_e)] \quad (5)$$

where  $\sigma_y$  is the r.m.s. value of the response process,  $h$  and  $\omega_0$  are the damping ratio and natural frequency of the system. For the light damping system the transient term is modified according to the damping ratio [6].

As an example of the application of this approach, the response spectra estimated by eq.(5) together with eqs. (3) and (4) are shown in Fig.3, in which those obtained by the step-by-step method are plotted for the comparison. As we see that the estimated values are very close to those by the step-by-step method,  $T_e$  and  $\sigma_e$  should be considered to be useful factors to identify some phase characteristics of earthquake motions.

#### 4. The Response Spectra, $T_e$ and $\sigma_e$ of Simulated Ground Motions

The velocity response spectra with 5% damping ratio were investigated to clarify the relationship between the phase differences and the response spectra. The response spectra of the simulated motion ( $SV_s$ ) were normalized by the recorded one ( $SV_o$ ) as the spectrum ratio  $R(T)$  which is expressed as,

$$R(T) = SV_s(T) / SV_o(T) \quad (6)$$

The mean values of  $R(T)$  for three cases of breadth  $B$  are shown in Fig.4. The  $R(T)$  values of the four earthquakes are scattered with periods; however the difference among the earthquakes for each cases of  $B$  is not significant.

when $B = B_o$	$R(T) = 0.6 \sim 1.1$	( 0.8 as an average)
$B = 0.75B_o$	$R(T) = 0.8 \sim 1.3$	( 1.0 )
$B = 0.5B_o$	$R(T) = 1.0 \sim 1.6$	( 1.2 )

The maximum acceleration ( $A_{max}$ ),  $T_e$  and  $\sigma_e$  of the simulated motions are shown in Table 2. When  $B$  equals  $0.75B_o$ ,  $A_{max}$ ,  $T_e$  and  $\sigma_e$  of simulated motions are fairly close to the recorded ones. In the case of  $B$  equal to  $B_o$ ,  $T_e$  is longer and  $\sigma_e$  is smaller, so the response spectra of the simulated motions are less than the recorded one, and the opposite tendency is recognized in the case of  $B$  equal to  $0.5B_o$ .

#### 5. Conclusion

The effects of phase characteristics of the simulated earthquake ground motions on the response spectra were examined by use of the phase difference distribution concept and the random theory.

A parametric study was performed to investigate the influence of phase difference distribution breadth upon the 5% response spectra. The simulated motions with the specified power spectra of the recorded earthquake motions were generated by utilizing the rectangular phase difference distribution with breadth  $B$ . The breadths  $B$  were selected parametrically equal to  $B_o$ ,  $0.75B_o$  and  $0.5B_o$ , in which  $B_o$  is that obtained for the existing recorded motions. The case of  $B$  equal to  $0.75B_o$  makes the 5% response spectra of the simulated motions be almost at the same levels as those of the recorded motions. And in this case  $T_e$  and  $\sigma_e$  are found to be also close to the recorded ones. It can be concluded that consistent phase characteristics for the simulated earthquake ground motions could be generated by the appropriate choice of the phase difference distribution breadth.

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Table 1  $A_{max}$ ,  $T_e$ ,  $\sigma_e$  and other parameters of the recorded earthquake motions

	$A_{max}$ (Gal)	$T_d$ (s)	$\sigma_{\Delta\phi}$ (rad)	$B_o$ (rad)	$T_e$ (s)	$\sigma_e$ (Gal)
EL1	341.7	26.86	1.192	4.125	7.23	119.1
TFT	175.9	31.90	1.197	4.147	8.92	61.7
GGT	127.0	3.00	1.435	4.971	1.05	54.6
EL2	477.14	7.24	1.166	4.039	3.45	188.4

Table 2  $A_{max}$ ,  $T_e$  and  $\sigma_e$  of the simulated earthquake motions ( mean values of ten motions )

	$B=B_o$			$B=0.75B_o$			$B=0.5B_o$		
	$A_{max}$ (Gal)	$T_e$ (s)	$\sigma_e$ (Gal)	$A_{max}$ (Gal)	$T_e$ (s)	$\sigma_e$ (Gal)	$A_{max}$ (Gal)	$T_e$ (s)	$\sigma_e$ (Gal)
EL1	247.6	17.49	76.6	310.1	9.74	102.6	460.0	3.35	175.0
TFT	134.6	19.09	42.2	174.5	9.69	59.2	238.7	4.22	89.7
GGT	101.5	2.23	37.5	126.1	1.23	50.4	158.6	0.55	75.4
EL2	464.6	4.06	173.7	593.7	2.18	237.0	735.4	1.02	346.5

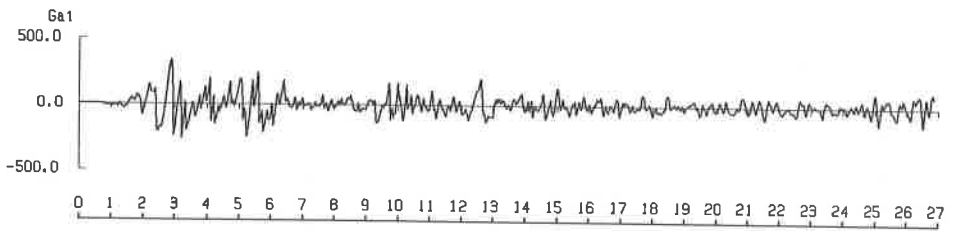


Fig.1 Time history of the recorded earthquake motion ( EL1 ) (s)

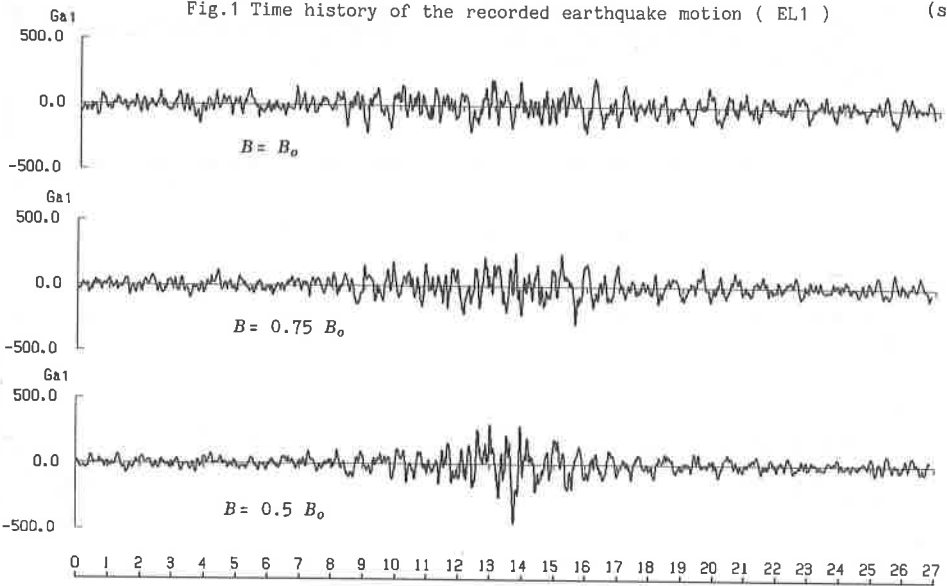


Fig.2 Sample time histories with various phase difference distribution breadths based on the characteristics of EL1 (s)

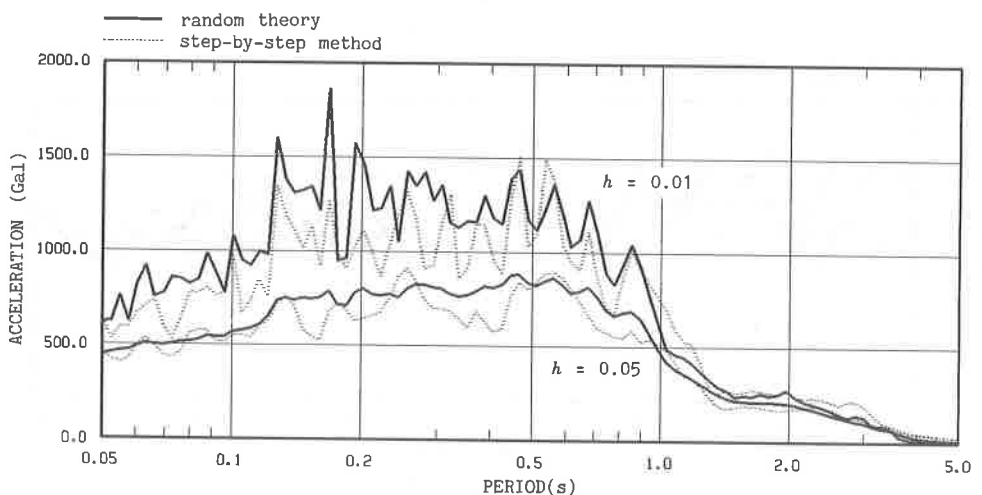
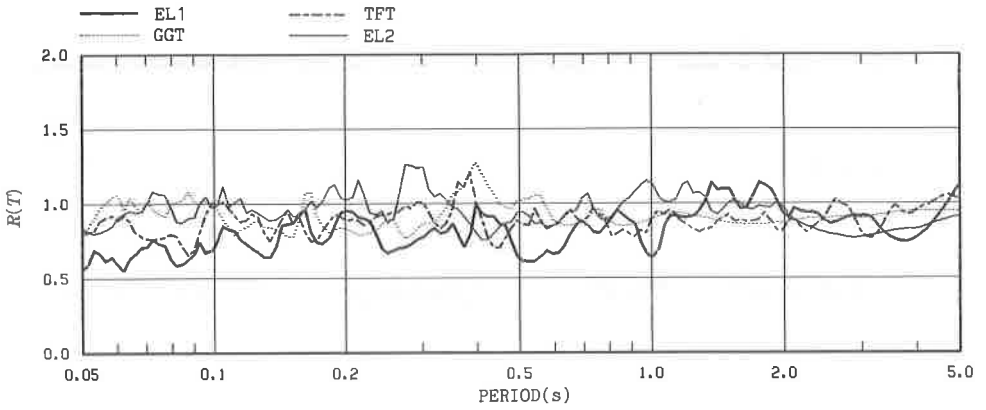
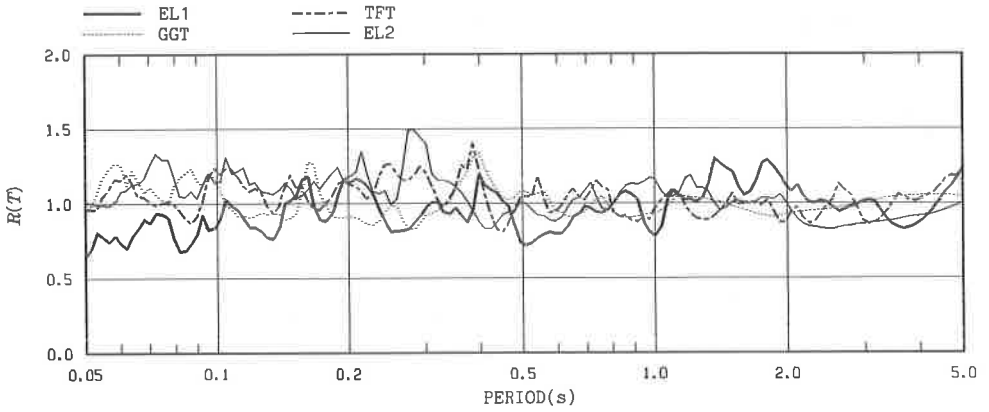


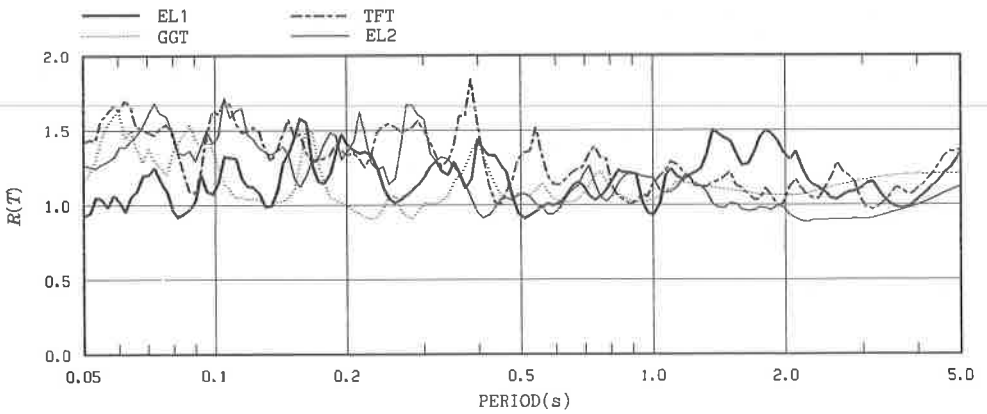
Fig.3 Comparisons of the response spectra calculated by the step-by-step method and the random theory



(a)  $B = B_0$



(b)  $B = 0.75 B_0$



(c)  $B = 0.5 B_0$

Fig.4 Response spectrum ratio  $R(T)$  for three cases of breadth  $B$