Energy Theory for Instability Analysis of Crack Growth

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ABSTRACT

A energy criterion for instabitity analysis of crack growth is prensted in the paper, it can be exxpressed in J, $\frac{dJ}{da}$ and $\frac{dP}{d\triangle}$, K_M. The experiments with larger loading system rigidity, K_M, show the criterion is correct while tearing modulus theory fails to analysis. Also the experiments show that the crack extensions would affect the instability and the material curves in stability analysis shall be $T_{Ym}-\triangle a$.

1 INTRODUCTION

Over the past few years, the "tearing instability criterion" have been proposed and improved the use of the J–R curves to predict the instability in structure with extensive plastic deformations and study the stable crack growth (Paris et al. 1979). It can be

$$T_{APP} = \frac{E}{\sigma_0^2}\left(\frac{dJ}{da}\right)_{\triangle tot} \geqslant T_{MAT} = \frac{E}{\sigma_0^2}\left(\frac{dJ}{da}\right)_m \quad \text{instability} \tag{1}$$

Where E is the elastic modulus and $\sigma_0$is flow stress. The material tearing moudlus T_{MAT}can be measured by J–R curves directly. For the general specimens acted on a single load P. the applied tearing modulus $T_{App}$to the system and T_{MAT}can be written in parallel forms as (Ernst et al. 1981)

$$T_{MAT} = \frac{E}{\sigma_0^2}\left\{\frac{\partial J}{\partial a}\Big|_\triangle + \left(\frac{\partial J}{\partial \triangle}\right)^2\Big|_a \frac{1}{\frac{\partial P}{\partial \triangle}\Big|_a - \frac{dP}{d\triangle}}\right\} \tag{2}$$

$$T_{APP} = \frac{E}{\sigma_0^2}\left\{\frac{\partial J}{\partial a}\Big|_\triangle + \left(\frac{\partial J}{\partial \triangle}\right)^2\Big|_a \frac{1}{\frac{\partial P}{\partial \triangle}\Big|_a + K_M}\right\}$$

Where the total displacement is $\triangle_{tot}=\triangle+C_M P \quad C_M=1/K_M$ (3)

For compact specimens and three–points–bending specimen, they are

$$T_{MAT} = \frac{E}{\sigma_0^2}\left\{-\gamma\frac{J}{b} + \frac{\eta^2 P}{b^2 B}\left(\frac{1}{\frac{H'}{WH} - \frac{1}{P}\frac{dP}{d\triangle}}\right)\right\} \tag{4}$$

$$T_{APP} = \frac{E}{\sigma_0^2}\left\{-\gamma\frac{J}{b} + \frac{\eta^2 P}{b^2 B}\left(\frac{1}{\frac{H'}{WH} + \frac{K_M}{P}}\right)\right\}$$

SMiRT 11 Transactions Vol. G (August 1991) Tokyo, Japan, © 1991

Where B is thickness, W is specimen width, $\eta=2+0.522b/w$, $\gamma=1+0.76b/W$ for CT specimens, $\eta=2$, $\gamma=1$ for 3PB specimens, and H is defined according to:

$$\frac{P}{BW(b/W)^2}=g(a/W)\cdot H(\triangle/W) \tag{5}$$

The forms for T in Eq(4) shows that the Paris instability criterion of Eq(1) is equivalent to the requirement that

$$\frac{-dP}{d\triangle}\geqslant K_M \quad \text{instability} \tag{6}$$

The eq(6) gives a simple form to analysis the instability directly from p–$\triangle$ curves. But the values of $-\frac{dP}{d\triangle}$ in unstable points observed in our experiments vary with the materials, specimens geometry, and usually are less than the value of K_M. Also the wide range of limited instability behavior observed in Joyce's works(Joyce, et al. 1981), which occured as $T_{APP}=0.6\sim0.8T_{MAT}$ even $T_{APP}<0.1\sim0.2T_{MAT}$

2 ANALYSIS OF ENERGY IN THE PROCESS OF CRACK GROWTH

Crack instability phenomena occur in brittle and ductile materials with different stable crack extension while the Latter usually have more stability, which show that the plastic energy including in far–field and in crack tip would effect on crack instability. In the process of crack growth, three types energy including the work dissipated by crack growth, U, to shape new crack growing faces, intense strain region in the crack tip and to open the crack, the elastic–plastic internal energy, $U_1(U_{1e}+U_{1p})$, and the work, W, done by applied P exist.
Crack growth needs the energy provided by W and released by elastic internal energy U_{1e} where the crack instability occur as the suppling energy exceeding energy required. If the increment $dU_1=dW-dU$ is presented to describe the crack instability, $dU_1>0$ implies the crack would grow outomatically without any work dw, and the existing dw would force the crack grow acceleratively until the specimens failure or return to stable crack growth again because of the excessive increasing of plastic internal energy dU_{1p} and the reducing of forcing energy. So a new criterion of instability can be written as

$$dU_1>0 \quad \text{stable} \tag{7}$$

$$dU_1\leqslant 0 \quad \text{instability} \tag{8}$$

dU_1 can be obtained by numerical method, but the convenient expression in load P, displacement $\triangle$ and J–integral would be more usefull.

$$dW=Pd\triangle_{tot} \tag{9}$$

$$dU=\alpha BJda \tag{10}$$

dU is expressed in J–integral and crack growing area dA=Bda, where α is about 1.0 given in the paper for considering the tearing typical crack growth and the intense strain region (Saka et al. 1983). So eq(8) can be written as

$$-\frac{dP}{d\triangle}\geqslant K_M\left(1-\frac{\alpha BJ}{P}\frac{da}{d\triangle}\right) \quad \text{instability} \tag{11}$$

Eq(11) is equivalent to another form using $\frac{dJ}{da}$, it is

$$T_{Yapp}\geqslant T_{Ym} \quad \text{instability} \tag{12}$$

where

$$T_{Ym}=\frac{b}{J}\left(\frac{dJ}{da}\right)_m \tag{13}$$

and

$$T_{Yapp}=\frac{b}{J}\left(\frac{dJ}{da}\right)_{tot}\left(1+\beta^*K_M\right)+\gamma^*\beta^*K_M \tag{14}$$

where

$$\beta^*(P,a)=\frac{\alpha BJ}{P}\frac{1}{\left(\frac{\partial J}{\partial\triangle}\right)_a} \tag{15}$$

$$\gamma^*=\frac{b}{J}\left(\frac{dJ}{da}\right)_\triangle$$

For CT specimens and 3 – PB specimens.

$$\beta^{\circ} = \frac{\alpha J B b}{\eta P^{2}} \qquad \gamma^{\circ} = \gamma \tag{16}$$

It is found that the material curves selected are not J_R curves or T_{Ym}–J curves, but the T_{Ym}–△a curves (see Fig.1), where the J_R curves and the T_{Ym}–J curves have larger divergences, and T_{Ym}–△a have smaller divergences.

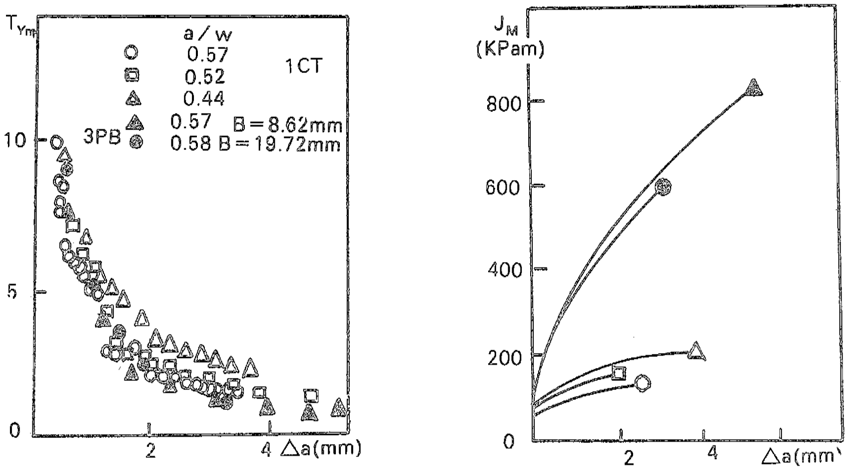


Fig1 (a) J–[R] curves of steel 45#

Fig1 (b) T_{Ym}–△a curves of steel 45#

3 EXPERIMENTS AND INSTABILITY ANALYSIS

The materials used in the experiments are PCrNiMo and 40CrNiMoA with unstable crack growing in 3PB specimens with B=20mm, W=60mm and L/W=4.0. The 1CT specimens and 3PB specimens with B=8.62mm, 19.72mm, W=23.60mm and L/W=4.0 for steel 45# are used for selecting the materials curves in the analysis of instability. Their chemical compositions are shown in Table. 1, and the mechanical properties are shown in Table.2.

Table.1 Chemical compositions(wt%)

Material	C	Si	Mn	S	P	Ni	Cr	Mo
steel 45#	0.45	0.20	0.60	0.04	0.04	0.25	0.25	
PCrNiMo	0.37	0.27	0.35			1.5	1.5	0.15
40CrNiMoA	0.40	0.30	0.65	0.75		1.5		

Table.2 Mechanical properties

Material	Modulus E(GPa)	Yield strength σ_s(MPa)	Tensile strenght σ_b(Mpa)	Elongation %	Ra %
steel 45#	200	350	650	17	35
PCrNiMo	210	950	1150		
40CrNiMoA	210	850	1000	12	55

3.1 Instability analysis from P–△ curves.

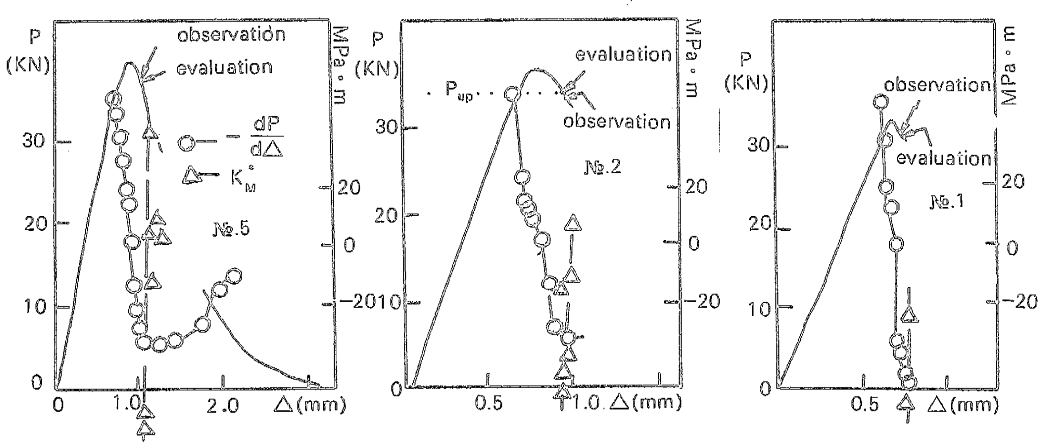
For the crack extensions △a can be caculated using IKCM method (Yin, another paper in SMIRT11), eq(11) can be written as

$$K_M^{\circ} = -K_M\left(1-\frac{\alpha BJ}{P}\frac{da}{d\triangle}\right) \geqslant \frac{dP}{d\triangle} \qquad \text{instability} \tag{17}$$

and be used to evaluated the ustable points and the loads in ustable points P_{up}. Fig.2 show the experimental P–△ curves and the results of instability analysis from P–△ curves.
It can be seen that eq(6) cann't evaluate the instability in these specimen. and the K_M° varies with the displacement △. The loads P_{up} are shown in Table.3 with accurate evaluations.

Table.3 The evaluations of the parameters in unstable points

Material	specimen	a_0/w	observation Pup(KN)	Pup(KN)	Evaluation J(KPam)	△(mm)	a(mm)
40CrNiMoA	№.5	0.530	34.4	32.3	194	1.13	14.4
	№.6	0.533	39.3	36.5	240	1.13	14.0
PCrNiMo	№.1	0.518	28.8	30.6	95	0.74	13.1
	№.2	0.543	30.3	33.0	170	0.96	14.1



(a) spcimen №5 (K_M=160Mpa·M) (b) spcimen №2 (K_M=140Mpa·M) (c) spcimen №1

Fig.2 Instability prediction diagrams using P–△ curves

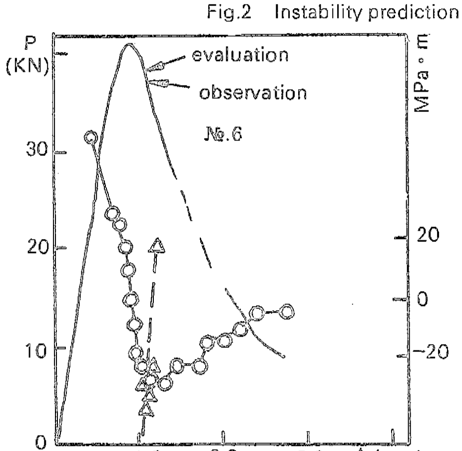


Fig.2(d) specimen №.6(K_M=160MPa·M)

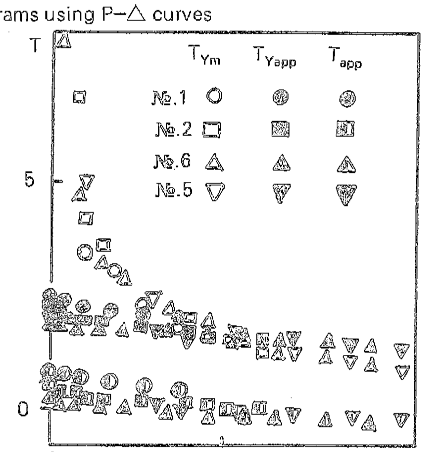


Fig.3 T_{Ym}–△a diagram

3.2 Instability analysis in $T_{Ym}$and J.

Also the instability condition can be obtained use eq(12). Fig.3 gives the T_{Ym}—△a diagram from which the instability points are given, but $Tapp=\frac{b}{J}\left(\frac{dJ}{dt}\right)_{\Delta tot}$ gives us incorrect conclusions for predicting the instability of crack growth. In order to evaluated the J, it can be converted from the crack lenght in T_{Ym}—△a diagram, or given directly from T_{Ym}—J diagram (see Fig.4).All these test results evaluated by Paris tearing modulus for instability predict stable crack extension.

3.3 Discussion

The specimens of alloys PCrNiMo and 40CrNiMoA exhibite unstable crack extension with lower tearing modulus in unstable points and higher system rigidity(K_m=140MPa~160MPa, T_{MAT}=4~10). T_{APP}—values are less than 0.4 in unstable points. Comparing the T_{aPP}—values with T_{Ym}—values, it can be seen that T_{app}—values are much less than T_{Ym}—values, it can be seen that Tapp—values are much less than T_{Ym}—values and would predict stable crack extension. In eq(11) the second term caused by crack growing is of importance to predict instability. As the specimens being tested with higher K_M—values and high crack growing rate da/d△ the eq(6) cannot be used to predict instebility.

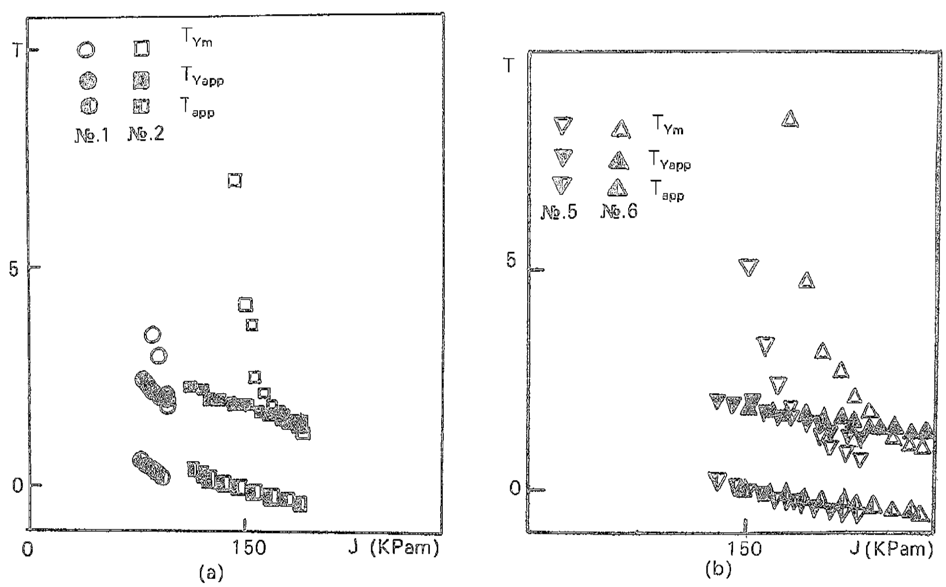


Fig.4 T_{Ym}—J dingrams

For high toughness and high crack grwoing rate existing in the materials, the energy dU_1would be greater even exceed the work done in the specimen. The effect of dU_1on crack extension instability would be obvious, and the rate of load—displacement at the unstable point is less than K_M—Values. In this time the tearing modulus would give the consclusions of crack stable growth to the specimens exhabiting the unstable and limited instability behaviors. In Joyce's experiments (Joyce et al. 1981) some limited instability behavior was found where T_{APP}—values were less than T_{MAT}—values, the T_{APP}—values were less than 25 and the possibility of limited instability was increased as T_{MAT}—values decreas. But no instability behavior was found as T_{APP}—values less than T_{MAT}—values.
In the researches of successful prediction in instability behaviors by tearing modulus, the second term in eq(11)

was small (McCabe et al. 1983, Joyce 1983)or with smaller K_M(Paris et al 1979). The tearing modulus theory is included in energy theory presented by eq(11), it would be useful as second term in eq(11) being small or the effect of the enery rate dU / d△ being smaller.

Another conclusion given in the paper is that T_{Ym}–△a curves may be the materials curves used to predict the instability behavior, it is independ on specimen geometries and crack lenght. Other curves such as T_{MAT}–J curves, T_{Ym}–J curves are depend on specimen geometries and crack length.

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