STRESS DISTRIBUTIONS OF AN ELBOW WITH STRAIGHT PIPES

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SUMMARY

Elbows are the most flexible components in the thin-walled, large diameter coolant piping systems of a nuclear power plant. Prediction of the elbow behavior is very important in understanding the response of a piping system. Many theoretical and experimental studies have been reported on the stress and displacement analysis of the elbow. Recent development of the finite element method, in particular, enables one to solve even inelastic problems such as elastic-plastic and creep behavior of an entire piping system under complex loading conditions. Their proper analysis, however, requires an adequate selection of finite elements with respect to their characteristics, and a thorough understanding of elastic behavior of elbows under several basic conditions.

The present paper investigates three-dimensional elastic analyses of an elbow with straight pipes under in-plane or out-of-plane bending moment, using three different types of models in commercially available general purpose computer programs. A post-processor for output plotting is developed to assist intuitive understanding of the stress distributions over the surfaces of the elbow and straight pipes.

The elbow with straight pipes is fixed at one end, and, at the other end, subjected to loads corresponding to in-plane or out-of-plane bending moment. Three different models (quadrilateral thin shell elements in the ANSYS program, quadrilateral higher order solid elements in the ASKA program, and the combination of elbow and beam elements in the MARC program) were used to obtain reasonably accurate solutions of these problems and also to assess the validity of these approximations. Analyses were performed with well-refined meshes in all cases.

With respect to stress distributions, fairly good agreements were obtained between the shell and solid element idealizations. Computed stresses by the elbow and beam element model were relatively large compared with those obtained by the other two models, particularly near the connecting sections of the two segments. These differences may be ascribed to the fact that the tying condition of elbow and beam elements cannot satisfactorily simulate actual behaviors. However, the last model is sufficient for stress and strain evaluation at the central section of an elbow, and may provide an effective tool for elastic and/or inelastic analyses of the piping system at the present state-of-the-art. A post-processor was devised for a two-dimensional representation of the stress distribution of the entire pipe bend. With this plotting, the complex stress distribution caused by the interaction of the elbow and straight pipes can be understood completely. This post-processor is also valid for plotting the inelastic stress and strain distributions.

1. Introduction

As important structural components of piping systems, elbows are used widely in various industries. This type of piping structural component has so high a flexibility that it is conveniently used also for absorbing thermal expansions of a piping system. It is also widely practiced to adjust the locations and number of elbows to be used in an attempt to acquire a reasonable piping design.

It is very important to understand the stress distributions and deformation behaviors of elbows in the evaluation of the responses of such piping systems. Many theoretical and experimental studies have so far been made on the problem of the elbows subjected to bending moments or internal pressures. Such studies have lately been putting emphasis on the investigation of inelastic deformations rather than elastic deformations. In the analytical studies of such problems, inelastic analyses under complex loading conditions have also come to be performed particularly due to the advances in the finite element numerical method (1-5).

However, the elastic deformation behavior of elbows has not yet been fully understood. The known analysis results contain various assumptions concerning the geometry and boundary conditions. What are the true stress distributions and deformations after all such assumptions are eliminated ? This is a problem we have to try to find answers from now on. This task involves many problems that need analytical studies in the future such as, for instance, whether the uneven wall thickness and initial imperfection of sectional geometry of elbows have important effects on stresses and deformations, what are the stresses in the joints of elbows, what are effects of the geometrical and material discontinuity of the welded joints, and what evaluation should be made of the flexibility factors and stress indices considering the interaction between elbows and straight pipes.

This paper is intended to obtain the entire picture of the stress distributions of the elbow with straight pipes and describes the analysis results by the finite element method in the cases where they are subjected to in-plane or out-of-plane loading. The analytical study uses three different types of finite element models, that is, three-dimensional solid elements, thin shell elements, and the combination of elbow and beam elements. The characteristics of thus obtained solutions are compared with one another, thereby to make reference to the validity of such approximations. The paper also describes the computer plotting that has been devised for intuitive understanding of the overall stress distributions of the elbow with straight pipes.

2. Elbow with Straight Pipes and its Finite Element Idealizations

The elbow with straight pipes that is treated in this paper is fixed at one end and subjected to in-plane or out-of-plane loading at the other end as illustrated in Fig.1. The finite element method is considered most effective in performing the detailed analysis of this problem, taking into consideration the interaction between the straight pipes and elbow. The stress distributions of the elbow has become fairly well known owing to the several analytical studies by the finite element method (1,4,5) and experimental results, but not to the extent that the entire picture thereof has been clarified. In the analysis results by the finite element method, for instance, localized disturbances of stresses are observed in the ends of the elbow in some cases and it is still not known whether they are real physical phenomena or the errors inherent in the finite element method arising due to the element characteristics and the employed finite element modeling. There are also few experimental and analytical works which have clarified the discontinuities of the stress distributions occurring in the elbow-straight pipe intersections. In this analysis, therefore, paying attention to the above point, solutions were obtained by use of the following three different types of finite element models, thereby to make a comparative study of thus obtained results. In performing the analysis, considerably fine meshes were used as compared with ordinary cases.

(a) Three-dimensional solid element

The three-dimensional solid element used was HEXEC-27 (6) of the ASKA program, that is, the hexahedral element formulated by the quadratic Lagrange interpolation function and 27 degrees of freedom per element. As seen from Fig. 2, the elbow with straight pipes was divided into sixteen sections in the circumferential direction, one section in the direction of thickness, and twenty-two sections in the axial direction (divided by the odd-numbered lines in the figure). The idealization by solid elements is generally uneconomical but it will be interesting to compare this type of idealization with the idealization by thin shell elements having some assumptions.

(b) Thin shell element

The shell element used in this analysis was STIF 43 (7) of the ANSYS program, that is, the 4-node quadrilateral flat plate element (20 degrees of freedom). The total structure was divided into thirty-two sections in the circumferential direction and forty-four sections in the axial direction as shown in Fig.2. The interpolation function of this element was bilinear for the in-plane displacements and incomplete bicubic for the out-of-plane displacement.

(c) Combination of elbow and beam elements

Some of the finite elements have been developed specially for the purpose of idealizing the geometry of the elbow. They include the elbow element (1) made up of a deformation mode of a torus that is an axisymmetric shell combined with a deformation mode of a curved beam, and the ring element (9) in which a ring sliced out of the elbow is assumed to be a finite element and the Fourier series expansion was used for the circumferential shape function and the Hermite interpolation for the axial one.

In this analysis was used the MARC program and the elbow structure was idealized by elbow elements (ELEMENT 17 (8)) and the straight pipes by beam elements (ELEMENT 14 (8)). The elbow elements were used partly for the purpose of confirming the fundamental characteristics of this element prior to the inelastic analysis of the complete piping system performed by the research group including the authors. Fig.3 shows the finite element idealization by use of the combination of elbow and beam elements. The elbow was divided into thirty-two sections in the circumferential direction and eight sections in the axial direction. In this type of idealization, the elbow elements are joined with the adjacent elbow and beam elements only via the nodes located at the centers of the cross sections. These nodes represent the beam mode deformations of elbow elements.

3. Analysis Results and Discussion

Figs. 4 and 5 respectively show the stress distributions on the inner surface at the center of the elbow (Section B) and at the joint between elbow and straight pipe (Section C) in the case of an in-plane bending load. Solutions were obtained by the three finite element models. Figs. 6 and 7 show the stress distributions in the case where these models were subjected to an out-of-plane bending load.

The stresses induced in the elbow subjected to the in-pane load (Fig. 4) are greatly different from those which are obtained by the curved beam theory. The elbow analyzed in this paper was a thin-walled, large diameter, long elbow (Fig.1), 823.9 mm in outside diameter (2r), 1219.2 mm in radius of curvature (R) and 11.1 mm in thickness (t) and the pipe factor was $\lambda = Rt/r^2 \fallingdotseq 0.08$. According to the curved beam theory, the main stress is the axial one σ_ℓ, the maximum compressive stress occurs at the location of $\theta = -90^\circ$, and the maximum tensile stress at the location of $\theta = 90^\circ$, but the calculated stress distributions were utterly different from these solutions, that is, they represented the behavior peculiar to a pipe elbow as is well known. When a bending load is applied to an elbow, unlike a beam whose end rotates due to the axial strain, the elbow is easily ovalized to cause the end to rotate, thus yielding the maximum stress in the circumferential direction.

Comparing the numerical solutions by the three types of finite element idealization, it was found that the maximum stress took the highest value in the case of elbow elements. This is presumably due to the fact that the elbow elements are joined with the adjacent elements only via the central positions of the sections and the angle of rotation, and the individual elbow elements are ovalized independently, thereby causing a rather larger degree of ovalization.

In contrast, when the shell elements were used, since the elements were connected with one another, the ovalization becomes continuous in the axial direction due to the restraints by all adjacent elements. The effects of the straight pipe that would not easily ovalized appeared in membrane stresses, bending and torsional moments of each shell element, and extended to near the central section. The ovalization is weakened in this way, thereby slightly lower maximum stress than in the case of elbow elements.

The analysis results by the solid elements were fairly well in agreement with the results obtained by use of shell elements although the maximum stress took a value somewhat lower. In Fig.4 showing the stress distributions in the joint (Section C) between elbow and straight pipe, the elbow elements caused considerably high stresses as compared with other finite element idealizations. This tendency is also due to the similar reasons stated before. It is also presumably due to the fact that since the straight pipes were modeled by beam elements incapable of expressing the ovalization, the effects were further pronounced by the discontinuity of the stiffness properties in the joints. In these joints, when shell and solid elements were used, there occured considerably high shear stresses $\tau_{\theta l}$ and these values were well agreement between both cases. On the other hand, the elbow elements that were used here are incapable of expressing such shear stresses.

A similar tendency was observed when subjected to an out-of-plane load as when subjected to an in-plane load (Figs. 6 and 7).

From the above results, it can be said that the use of elbow elements is not necessarily sufficient for obtaining the stress distributions in the elbow with straight pipes but they show almost accurate solutions at the central section of the elbow and therefore this type of idealization will be useful in performing the elastic or inelastilc analysis of the entire piping system in order to evaluate the stresses and strains in the central portions of the elbows.

4. Stress Distributions on the Surfaces of the Elbow with Straight Pipes

In the preceding section, we selected a typical section of the elbow and made a comparative study of the analysis results performed by use of three different types of finite element idealizations. Speaking only about the elbow with straight pipes, the shell element type can be said to be most effective in terms of calculation time, the expression of analysis results and accuracy of the obtained solutions although it is somewhat troublesome to generate the finite element mesh.

In order to investigate the overall stress distributions of the elbow with stright pipes on the basis of the results by the shell elements, we prepared the postprocessor, that is, the surfaces of the elbow with straight pipes were expanded into a reactangular form to achieve a two-dimensional representation as shown in Fig.8 through 11 in order to explicitly show not only the stress distributions in specific sections but also the global stress distributions on the inner and outer surfaces.

For example, Fig. 8 shows the circumferential stress distribution on the outer surface due to the in-plane bending load. The maximum stress occurred near the central section and decreased sharply in the areas nearer the straight pipes. There occurred considerable stresses also in the straight pipes due to the ovalization influenced by the elbow.

Fig. 9 shows the shear stress distribution on the inner surface due to the in-plane bending load. The shear stresses $\tau_{\theta\ell}$ were large only in the areas near the elbow-straight pipe intersections and reached a peak in the neiborhood of $\theta = 0$ and $\pm 45°$. These stresses are presumably caused due to the combination of structural elements having the different fundamental deformation modes.

Figs. 10 and 11 show the stress distributions due to the out-of-plane load. This case was characterized by the fact that there are two peaks in the circumferential stress. Since the out-of-plane bending moments are necessarily applied accompanied with torsional moments, the total sum of shear stress in the section does not become zero and their distribution is not uniform and complicated. However, it was found that discontinuous stresses occurred only in the elbow-straight pipe intersections.

5. Conclusion

We performed the elastic stress analysis of the elbow with straight pipes by use of three different types of finite element idealizations. As a result of comparing thus obtained solutions, we have reached the following conclusion.

1) The analysis results by flat plate elements were fairly well in agreement with the results by higher-order solid elements, showing the effectiveness of this element in the stress analysis of the pipe elbows from an overall point of view including the calculation cost.
2) When elbow elements are used, the ovalization will be overestimated in the areas near the elbow-straight pipe intersections. With this element, there still remains a problem that torsional and shear stresses are not taken into consideration for the out-of-plane bending load. However, the overall stress distribution seems to be reasonably evaluated although the normal stresses in the central section of the elbow are somewhat overvalued.
3) There occur complicated discontinuous stresses in the elbow-staight pipe intersection. However, the maximum stress in the elbow subjected to the bending moment still occurs generally in the locations near the central section. This fact suggests that the stresses are evaluated in safety by an approximate method disregarding the interaction between the elbow and straight pipes.
4) The elbow-straight pipe intersection appears to be continuous but this is a location where discontinuous stresses occur as in the case of a pressure vessel consisting of a hemispherical shell connected with a cylindrical one. Such being the case, when performing detailed stress analysis of this particular portion, it is necessary to use a refined mesh, considering the discontinuity of this section.

5) The two-dimensional representation of the stress distributions is effective to show explicitly the fundamental patterns of stress distributions in the elbow with straight pipes.

Acknowledgement

The authors are grateful to the cooperation extended by Messrs. H. Narita, M. Kimura and S. Uesugi of Century Research Center Corporation in performing these analyses and express their gratitude to Prof. Y. Ando of the University of Tokyo who gave us a direct motive as to "the availability of the data by which the stress distributions of the elbow with straight pipes can be understood with a single glance."

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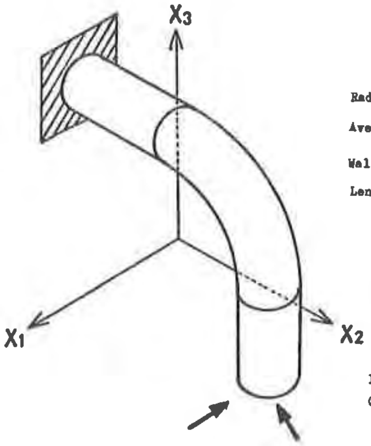
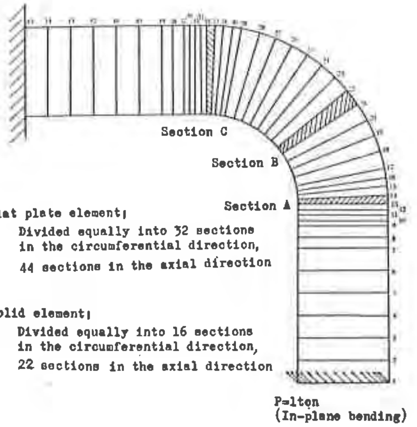


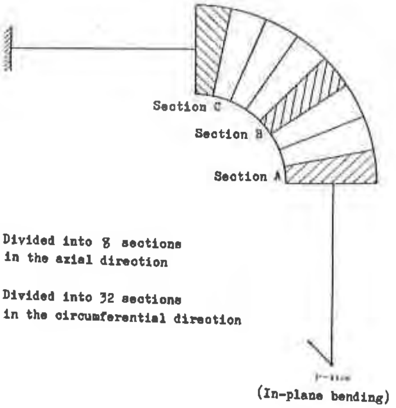
Radius of curvature of elbow(R)	121.92cm
Average diameter of pipe(D)	81.28cm
Wall thickness of pipe(t)	1.11cm
Length of straight pipe(L)	203.2 cm
Young's modulus(E)	1.58×10^6kg/cm²
Poisson's ratio(ν)	0.304
In-plane load	1.0ton
Out-of-plane load	1,0ton

Fig. 1 Geometry of the Elbow with Straight Pipes



Flat plate element;
Divided equally into 32 sections
in the circumferential direction,
44 sections in the axial direction

Solid element;
Divided equally into 16 sections
in the circumferential direction,
22 sections in the axial direction



Divided into 8 sections
in the axial direction

Divided into 32 sections
in the circumferential direction

Fig. 2 Finite Element Idealization by Solid Elements or Flat Plate Elements

Fig. 3 Finite Element Idealization by Elbow and Beam Elements

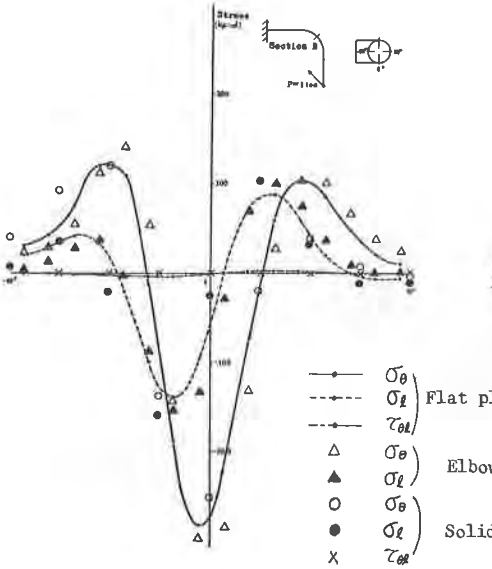


Fig. 4 Stresses due to the In-Plane Bending Load (Inner Surface of the Section B)

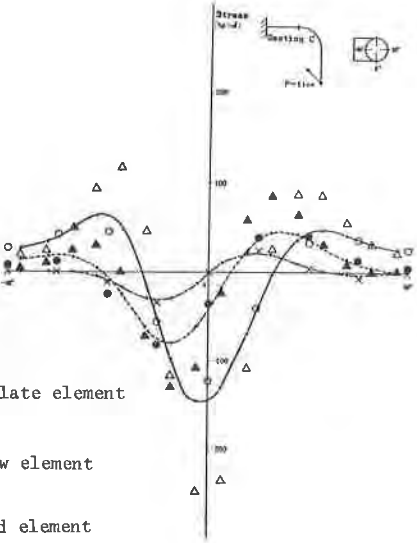


Fig. 5 Stresses due to the In-Plane Bending Load (Inner Surface of the Section C)

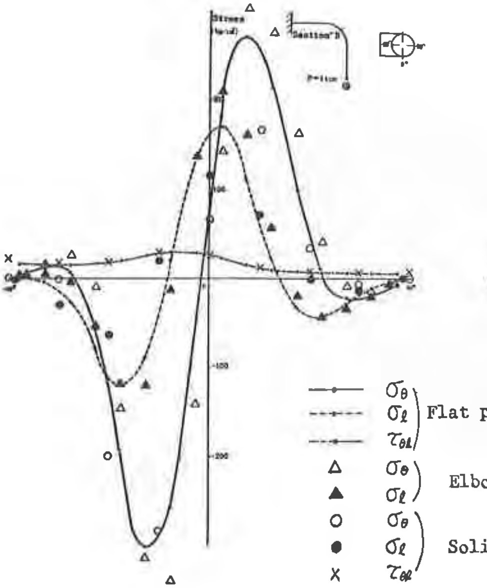


Fig. 6 Stresses due to the Out-of-Plane Bending Load (Inner Surface of the Section B)

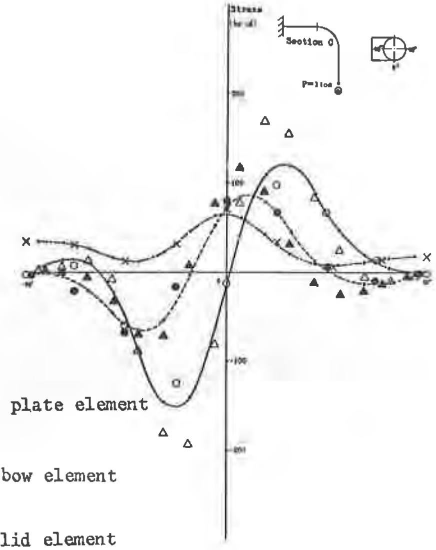


Fig. 7 Stresses due to the Out-of-Plane Bending Load (Inner Surface of the Section C)

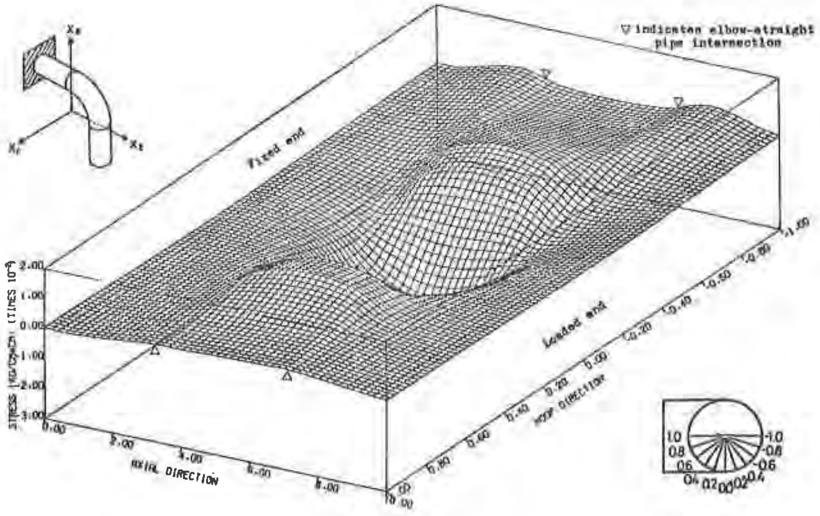


Fig. 8 Hoop Stress Distribution on the Inner Surface due to the In-Plane Bending Load

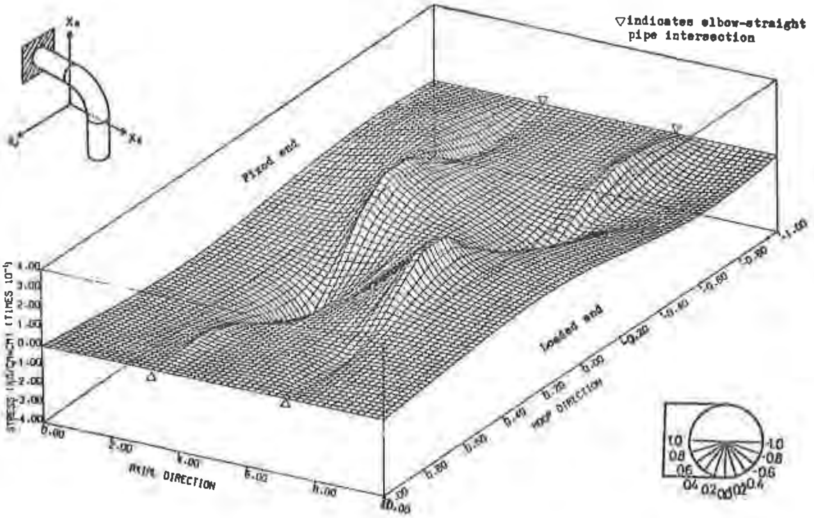


Fig. 9 Shear Stress Distribution on the Inner Surface due to the In-Plane Bending Load

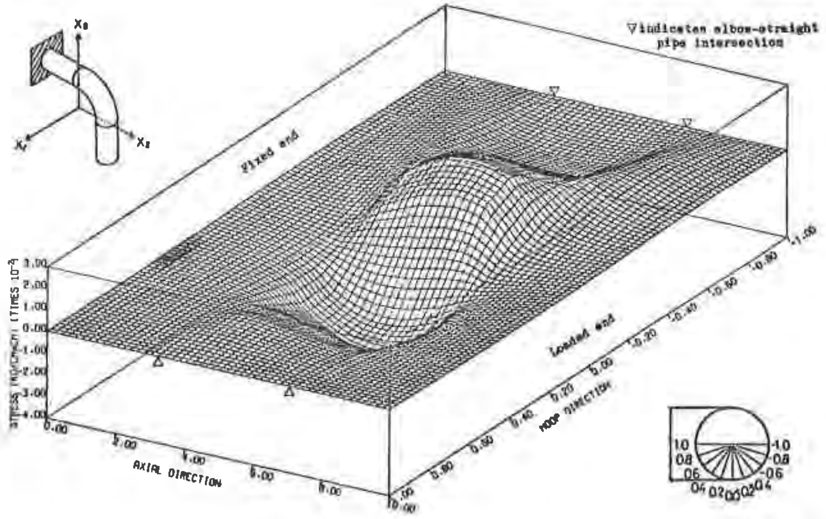


Fig. 10 Hoop Stress Distribution on the Inner Surface due to the Out-of-Plane Bending Load

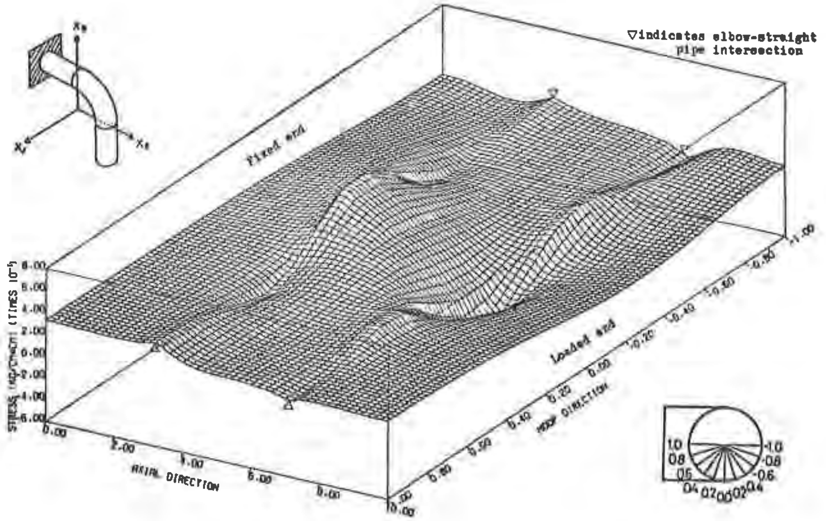


Fig. 11 Shear Stress Distribution on the Inner Surface due to the Out-of-Plane Bending Load