Seismic Response of Elastic-Plastic MDOF Structures

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ABSTRACT

A procedure for estimating the seismic response of elastic-plastic multi-degree-of-freedom systems is presented. The formulation uses random vibration and extreme value theories to obtain the distribution of maximum response. Ground motion is described by an average power spectral density function and the nonlinearity is treated by a series of equivalent linear subsystems. The response of a three-degree-of-freedom system is obtained using the proposed procedure and the results are compared with a time-history integration.

INTRODUCTION

The analysis procedure used in earthquake resistant design depends to a large extent on structural and functional requirements. For critical structures, such as nuclear power plants, dams, offshore platforms, and LNG facilities, a dynamic analysis is usually performed. If a linear analysis is desired, the mode-superposition procedure together with a design spectrum is often used to compute the response. However, if a nonlinear analysis is warranted, the time-history integration is the only alternative. The difficulty with the time-history integration, particularly for a nonlinear analysis, is that it is expensive, and unless the input motion is generated from a smooth design spectrum, the computed response reflects the characteristics of a single record. In such cases, the time-history analysis may have to be repeated many times for different base excitations in order to arrive at representative response values for design.

The random vibration theory has also been used for computing the response of structures and equipment. However, the applications have primarily been concerned with linear systems and only a few studies have extended it to nonlinear analysis.

This paper uses random vibration and extreme value theories to obtain the distribution of the response of elastic-plastic multi-degree-of-freedom (MDOF) systems. The ground motion is described by an average power spectral density function and the structural nonlinearity is treated by a series of equivalent linear subsystems. Numerical solutions from the proposed procedure are compared with those from the time-history integration for a three-degree-of-freedom (3-DOF) system.

FORMULATION

Peng et al. (1987) have shown that for a stationary process, the root-mean-square (rms) response computed from the power spectral density of recorded accelerograms can be used to estimate the maximum response of an elastic-plastic SDOF system. For a MDOF system, one needs to compute the rms response of each mass. Expressing the rms response of mass i ($\bar{\sigma}_i$) in terms of the occurrence probability corresponding to the linear or a nonlinear state j ($P^{(j)}$) and the rms response of mass i in the state j (σ_{ij}), one obtains

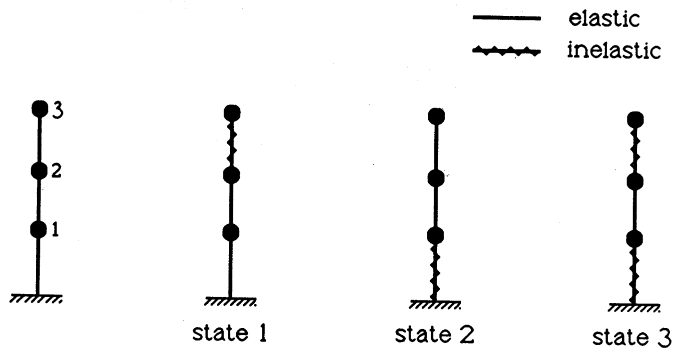

Fig. 1 Different linear and nonlinear states for computing the rms response of mass 2.

$$\bar{\sigma}_i = \sum_{j=1}^{M} P^{(j)} \sigma_{ij} \tag{1}$$

where M is the number of all possible linear and nonlinear states (column yieldings) which influence the determination of $\bar{\sigma}_i$. For an N-DOF system

$$M = 2^{N-1} \tag{2}$$

The occurrence probabilities describe all possible linear and nonlinear conditions for mass i. Thus,

$$\sum_{j=1}^{M} P^{(j)} = 1 \tag{3}$$

Defining P_i as the probability of column i yielding within the strong motion duration t, and t_y as the sum of the time intervals during which the response fluctuation includes yielding, we have

$$P_i = \frac{t_y}{t} = \frac{n_y}{n_0} \tag{4}$$

where n_y is the number yield level crossings and n_0 is the number of zero crossings within the duration t. Defining $P_{\bar{i}j}$ as the probability of column i remaining elastic while column j yields (a subscript with a bar denotes elastic and the one without a bar indicates yielding), the occurrence probabilities for the linear and different nonlinear states for a typical example shown in Fig. 1 can be computed as

$$P^{(1)} = P_{\bar{1}3} = P_3 (1 - P_1) \tag{5}$$

$$P^{(2)} = P_{1\bar{3}} = P_1 (1 - P_3) \tag{6}$$

$$P^{(3)} = P_{13} = P_1 P_3 \tag{7}$$

$$P^{(4)} = P_{\bar{1}\bar{3}} = 1 - P^{(1)} - P^{(2)} - P^{(3)} \tag{8}$$

Substituting equations 5-8 into equation 1, the rms response $\bar{\sigma}_2$ of mass 2 is

$$\bar{\sigma}_2 = P^{(1)} \sigma_{21} + P^{(2)} \sigma_{22} + P^{(3)} \sigma_{23} + P^{(4)} \sigma_{24} \tag{9}$$

It should be noted that σ_{24} in the above equation represents the rms response of mass 2 when the system is elastic.

YIELD LEVEL CROSSINGS

The expression for the mean level crossing rate (Vanmarcke, 1976) was used in this study to estimate the number of yield level crossings n_y within the strong-motion duration t as

$$n_y = u_{yo}\, t \tag{10}$$

where

$$u_{y_0} = 2f_c \left[1 - \exp\left(-\sqrt{\pi/2}\; k\delta \right) \right] \exp\left(-k^2/2 \right) \tag{11}$$

and

$$k = \frac{y_0}{\sigma_y} \tag{12}$$

In the above equations σ_y is the rms response in general and f_c, k, and δ are the central frequency, yield factor, and shape factor, respectively. Equations 10-12 can also be used to compute the number of zero crossings by setting y_0 equal to zero. Therefore,

$$n_0 = 2 f_c\, t \tag{13}$$

Equations 10-12 show that n_y is a function of the rms response and equations 4-9 indicate that the rms response depends on the ratio of n_y to n_0. Therefore, an iterative procedure is needed to obtain n_y and the rms response.

EQUIVALENT LINEAR SUBSYSTEM

For a linear MDOF system, the rms response σ_i may be estimated using the CQC mode combination technique (Der Kiureghian, 1981)

$$\sigma_i = \left(\sum_n \sum_m \psi_{in}\, \psi_{im}\, \rho_{0,nm} \sqrt{\lambda_{nn}\lambda_{mm}} \right)^{1/2} \tag{14}$$

where ψ_{in} and ψ_{im} are the effective participation factors of modes n and m, $\rho_{0,nm}$ is the cross-modal coefficient, and λ_{nn} and λ_{mm} are the mean square values for n-th and m-th modes, respectively. The rms response corresponding to the linear state in equation 9 (σ_{24}) can be computed using equation 14. For obtaining the rms responses corresponding to the nonlinear states (such as σ_{21}, σ_{22}, and σ_{23}), one may utilize the equivalent linear subsystem technique (Gates and Vanmarcke, 1976). The equivalent subsystem is a substitute structure which models the motion of the structure at the instant of yielding. Using the results from the time-history integration of several accelerograms, it was found that the portion of the structure below the yielding column tends to vibrate as an isolated system while the portion above it tends to vibrate as a lumped system with a lower frequency. The decrease in frequency results from the stiffness change caused by yielding. Based on these observations, the following two approximations were made for linearizing the system:

1. The rms response of a mass below the yielding column is computed from an equivalent linear subsystem which neglects the masses and column above the yielding column.
2. The rms response of a mass above the yielding column is computed from an equivalent linear subsystem with a lumped mass $\bar{m}$ and stiffness $\bar{s}$ obtained from

$$\bar{m} = \sum m_i \tag{15}$$

$$\bar{s} = \left(\frac{\sum m_i}{\sum i\, m_i} \right)^3 \sum s_i \tag{16}$$

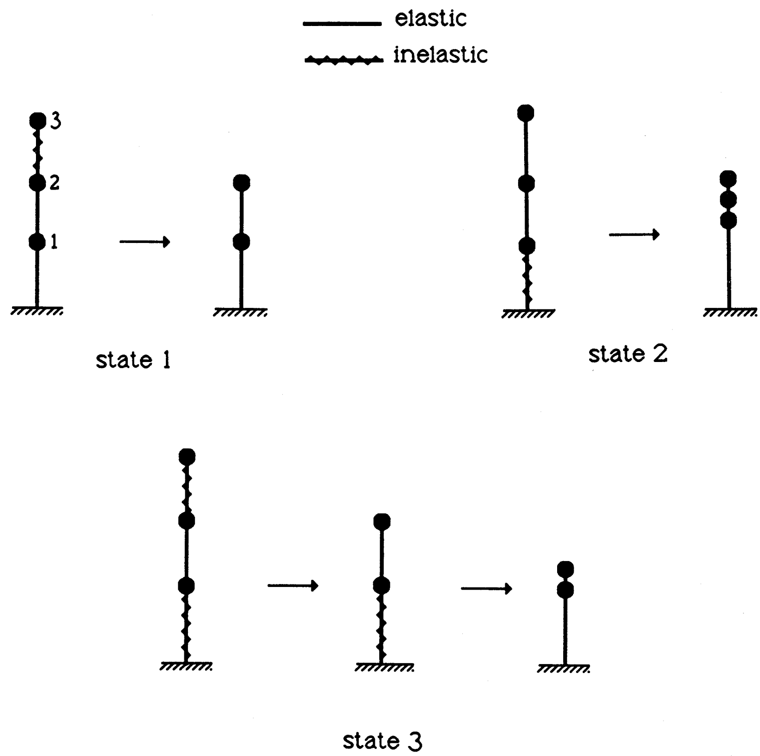

Fig. 2. Equivalent linear subsystems corresponding to nonlinear states in Fig. 1.

where i refers to those masses and column stiffnesses above the yielding columns. The above two approximations were also used for cases where more than one column experienced yielding. The equivalent linear subsystems corresponding to the nonlinear states 1, 2, and 3 in Fig. 1 are shown in Fig. 2.

Peng et al. (1988) have shown that the maximum inelastic response can be expressed as the sum of a nondeterministic maximum drift and the specified yield deformation. Thus, for mass i

$$y_{max,i} = e_{max,i} + y_{0,i} \tag{17}$$

where $y_{max,i}$, and $e_{max,i}$ are the maximum response and maximum drift of mass i, and $y_{0,i}$ is the yield deformation of column i. The rms response $\bar{\sigma}_i$ obtained from equation 1 is used to compute the rms deformation $\sigma_{d,i}$ of column i as

$$\sigma_{d,i} = (\bar{\sigma}_i^2 + \bar{\sigma}_{i-1}^2)^{1/2} \tag{18}$$

The average inelastic deformation D_i may be computed (Karnopp and Scharton, 1965) from

$$D_i = \frac{\sigma_{d,i}^2}{2y_{0,i}} \tag{19}$$

and used together with the number of yield level crossings $n_{y,i}$ to obtain the distribution of the maximum drift for each mass (Peng et al., 1987 and 1988). Thus,

$$e_{max,i} = \begin{cases} D_i \,(e^{n_{y,i}} - 1)/(e-1) & n_{y,i} \le 1 \\ D_i & 1 \le n_{y,i} \le 2 \\ D_i \,[\,6.719 + 1.017m\,)\ln \dfrac{n_{y,i}}{2} + 1\,] & 2 \le n_{y,i} \le 20 \\ D_i \sqrt{n_{y,i}} \,\left(\sqrt{2 \ln n_{y,i}} + \dfrac{0.5772 + 1.28m}{\sqrt{2 \ln n_{y,i}}} + 1\right) & n_{y,i} \ge 20 \end{cases} \quad (20)$$

where m is the σ-level for a specified probability (Peng et al., 1988).

RESULTS

The proposed procedure was used to compute the maximum response of a 3-DOF system with the properties given in Table 1. The input motion consists of the ensemble power spectral density of eight recorded accelerograms (two horizontal components each of El Centro, 1934 and 1940, Olympia 1949, and Taft 1952). The rms acceleration of the S00E component of El Centro, 1940 was used to specify the intensity of the motion. A strong motion duration of 10 sec was assumed in obtaining the solutions. The results for 2 percent damping for three different combinations of the yield deformation are given in Table 2 and are compared with those from the time-history integration of the eight records. The maximum displacements recorded in the table for the time-history integration were computed by averaging the responses from the eight records (each record was normalized to the peak acceleration of the S00E component of El Centro, 1940, before the responses were computed).

Table 1 Structural Properties and Yield Deformations Used in the Example Problem

Level	m $\frac{\text{kip - sec}^2}{\text{in.}}$	s (kip/in.)	Yield deformation y_0 (in.)		
			Case 1	Case 2	Case 3
3	1.0	60	1.0	1.0	1.1
2	1.5	120	1.0	0.9	0.9
1	2.0	180	1.0	0.8	0.7

CONCLUSIONS

A procedure for estimating the response of elastic-plastic MDOF systems is presented. The procedure is based on random vibration and extreme value theories and treats the nonlinearity as a series of equivalent linear subsytems. The response of a 3-DOF system using the formulation presented herein is in close agreement with the response computed from a time-history integration. Although a 3-DOF system is used to describe the formulation and the numerical example, the procedure provides a systematic approach for computing the response of elastic-plastic structures with any number of degrees of freedom.

Table 2 Comparison of Maximum Displacement Response for a 2 Percent Damped 3-DOF System Using Eight Recorded Accelerograms at the Intensity of S00E Component of El Centro, 1940 as Input Motion.

		y_{max} (in.)	
Case	Level	This study	Time-history
	3	7.93	8.73
1	2	5.06	6.11
	1	2.93	3.07
	3	7.78	7.07
2	2	5.69	5.96
	1	4.25	3.83
	3	6.98	6.96
3	2	5.67	6.08
	1	5.50	4.89

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