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Advances in software development in computational reliability

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ABSTRACT: The fast development of computational facilities and new computational methodologies allow the efficient implementation of stochastic structural analysis. Thus, a quantification of the probability of results of structural analyses in addition to pure deterministic prediction of their magnitudes is possible. Especially the evaluation of the reliability of structures expressed in terms of failure probabilities becomes feasible. To apply these methods in research as well as in industry, efficient programming concepts are necessary, i.e. convenient user interaction, modular programming, comprehensive macro language, etc. These topics are exemplified by utilizing a concept denoted as COSSAN (*Co*mputational *S*tochastic *S*tructural *An*alysis)

1 INTRODUCTION

Nowadays structural analysis is by and large still based on deterministic reasoning, despite the fact that it was recognized already fifty years ago (see e.g. [2]) that the quantification of structural reliability is only feasible by taking into account the statistical and probabilistic uncertainties of loads and structural parameters respectively. It may well be that well known procedures are often continued to be applied, as new, probabilistic procedures are often difficult to be handled in the everyday process of structural analysis and design. However, the fast development of computational facilities (hardware) and new computational methodologies (software) open the door for efficient numerical procedures to taking into account structural uncertainties and quantifying the results in terms of probabilities. Of course, these methods have to be embedded in respective software environments such that they are easily accessible for the engineering community. Some of these codes already developed so far are stand-alone programs with fixed input/output sequences or they provide programming interfaces (see e.g. ISPUD^{TM} [1]). Others already allow modular usage and/or have a user interface (PROBAN^{TM} [10], NESSUS^{TM} [5]). An assessment of the capabilities of these methods is given e.g. in [6]. The developments in this direction continue in a fast pace (see e.g. [4]), where it is attempted that the created software is both a tool for solving industrial problems as well as for the development of new methods and procedures in the field of *Co*mputational *S*tochastic *S*tructural *An*alysis (COSSAN). Hence, the development of such a code follows two basic directions:

(1) A Stand-Alone Toolbox allows the efficient development of new algorithms in theoretical and applied research.

(2) A communication part with other software is partially established (FE-Software IDEAS^{TM} , Matlab^{TM}) or is currently under development.

In the following, the concept of the development of a probabilistic structural analysis code is established by referring mainly to [4]. The applicability of the concept for industrial as well as theoretical problems is demonstrated by some numerical examples.

2 METHODS OF ANALYSIS

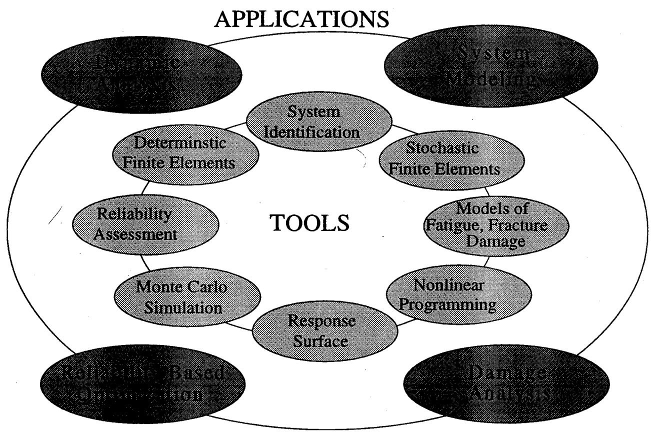

Figure 1: Structure of a modular Concept for Stochastic Structural Analysis

2.1 General

A program that is developed to perform computational stochastic structural analysis has to include both the possibility of modeling and handling a structure as well as the management of systems with uncertain properties and loads in probabilistic terms. In addition, it is clear that only a *modular concept* is feasible to fulfill the requirement of being convenient and easy to use and, furthermore, to be conveniently expendable. This modular concept is illustrated briefly in Fig. 1. Furthermore, a macro language with comprehensive commands is needed to allow access to the modules. Using this macro language, new procedures can be developed (research). Once these procedures are established, convenient so called script files can be produced which can then be extended and updated to industrial needs. These topics are described in detail in the following subsections.

2.2 Modular Programming

In general a program is developed by different groups and for different capabilities. Therefore, a natural modular structure is automatically given. However, in many cases this structure is violated by programmers or, even worse, modules are available, but different data handling procedures within the modules do not allow an efficient transfer of data. In this case often additional routines are required as interface to the modules.

Within the field of modern stochastic structural analysis the use of different calculation techniques is inevitable. Obviously, within a software environment these calculation techniques can be implemented into different modules. Then one major task during the development of such software environments is to keep these modular programming transparent for the user as well. A second task is, to allow the user to access these modules at any time of the current analysis. Finally, the third task is to organize the easy transfer of data from one module to the other, i.e. input data as matrices, vectors as well as parameters for the calculations within the distinct modules. This concept allows the combined usage of finite element structures together with random parameters, reliability analysis together with optimization tasks etc.

Having described the requirements to programs for stochastic structural analysis, a possible way of implementation is now shown, based on the implementation of COSSAN [4]. The discussed transparency of the modular structure can be obtained by using *command groups*. Each group represents one module (identified by a group name) and contains several commands which allow to work within the module. A comprehensive macro language (denoted as S_Lang, The *S*tructural *Lang*uage) is used, i.e. by allowing commands consisting of *group name* and *command name* the developer and user can follow the flow of the macros easily. Finally, *standardized data types* (matrices, vectors, etc.) combined with powerful *basic operation tools* allow the efficient transfer of data between the modules. This allows researchers, and of course engineers, to handle the data of interest without the necessity of being directly familiar with involved programming details.

2.3 Development of New Procedures for Research Purposes

For the development of new procedures, especially with respect to reliability analysis, a number of modules are required to be accessible. E.g. it is often necessary to handle structural models defined by finite elements (FE), modules are needed that allow the identification of certain parameters of structures based on tests of existing prototypes, etc. Of course, the possibility of describing random variables, random processes in time (random vibration) and space (stochastic finite elements) are inevitable for the quantitative description of structural reliability, and therefore these modules have to be contained in a modern program. But as space is limited, only some of the most recently developed procedures are described in more detail, i.e. *Monte Carlo Simulation Procedures* and *Reliability Based Optimization.*

Controlled Monte Carlo Simulation (MCS) Procedures

It has been shown that MCS methods are a very powerful tool to analyze the reliability of structures expressed by the probability of failure

$$p_f = \int_{D_f} f_{\boldsymbol{X}}(\boldsymbol{x})\,\mathrm{dx}, \qquad (1)$$

where D_f is the failure domain and $f_{\boldsymbol{X}}(\boldsymbol{x})$ is the joint probability density function of the random variables applied. This integration is performed by MCS numerically, i.e. random numbers are generated according to $f_{\boldsymbol{X}}(\boldsymbol{x})$ and samples in the failure domain D_f are summed up. However, as p_f usually ranges from 10^{-8} to 10^{-4}, in direct MCS hundreds of thousand of samples are needed, and the computational effort becomes unatractable. For this purpose, so called variance reduction techniques like Importance Sampling, Adaptive Sampling, etc., have been developed and applied successfully for structural reliability assessment.

Unfortunately, the numerical effort grows quite fast with respect to the number of random variables (RV) involved. For this reason, the mentioned variances reduction techniques are mainly employed for static analysis for which the number of RVs can be kept reasonably small. In case the loading is represented by a stochastic process, it might be acceptable in some cases to represent the stochastic process by a small number of RVs using the Karhunen-Loeve expansion and utilize the well developed standard variance reduction methods. Since the efficiency of the variance reduction techniques are adverse affected by the number of RVs, only a limited set of the most important RVs is usually considered, which are determined by prior sensitivity studies.

None of these concepts, however, is applicable for a "white noise - type" of stochastic loading, for which only recently so called Controlled Monte Carlo simulation procedures as "Double & Clump" [7] and "Russian Roulette & Splitting" [8] have been developed. Contrary to analytical and accurate numerical procedures based on the Fokker-Planck equation, which are restricted to state space dimensions less than four, Controlled MCS is applicable to MDOF systems (FE - models) as required for engineering practice. The purpose of these procedures is to increase the density of realizations in the region of interest, i.e. the low probability domain where failures might occur. Applying such Controlled MCS procedures, the assessment of failure probabilities of the order $p_f \approx 10^{-5}$ becomes feasible using a sample size $N \approx 1000$, corresponding to an enhancement of efficiency by a factor hundred.

Controlled MCS procedures are completely independent of the integration scheme of the stochastic differential equation. Hence any suitable scheme can be employed. The only requirement is the simultaneous generation all realizations, since Controlled MCS procedures manipulates the sample at frozen instants of time. In COSSAN Monte Carlo simulation is carried out by calling a sequence of modules to generate the stochastic loading and to integrate the equation of motion. Employing Controlled MCS, the command sequence is just extended by a few additional commands, such as "Double & Clump" and "Russian Roulette & Splitting", etc.

Reliability Based Optimization

There is also the possibility to use reliability analysis to optimize structures, e.g. with respect to minimization of their weight or life cycle costs. In this case reliability measures are introduced into the so called objective function (see e.g. [3]). In order to evaluate an objective in terms of costs, the objective function has to include three general types of costs, the initial construction costs as well as consequence costs related to partial and total failure respectively. Thus, a general reliability-based optimization problem can be defined as follows:

$$\mathrm{E}[C(\boldsymbol{x}, \boldsymbol{Y})] = c_I(\boldsymbol{x}) + \sum_j \mathrm{E}[C_{f,j}(\boldsymbol{x}, \boldsymbol{Y})] \rightarrow \min \qquad (2)$$

where $c_I(\boldsymbol{x})$ denotes the initial costs, and $C_{f,j}(\boldsymbol{x}, \boldsymbol{Y})$ are the expected failure costs related to partial and total system failure. The vector $\mathbf{Y}$ contains the random variables which are used to describe the statistical uncertainties of the structural system. Considering one single particular failure criterion related to a specific structural damage state the objective function i.e. eq. (2) rewrites in terms of failure probabilities

$$\mathrm{E}[C] = c_I + c_s(1 - p_{fs})p_{fs} + c_c p_{fc} \rightarrow \min \qquad (3)$$

where p_{fs} and p_{fc} are the occurrence probabilities with respect to partial and total failure respectively. The factors c_s and c_c are the respective consequence cost factors. At this point it should be noted that the failure probabilities depend on the respective design state and therefore, they have to be evaluated repeatedly during the iterative design optimization process. For the highly nonlinear optimization problem as defined by eqs. (2), (3), the well known NLPQL-algorithm [9] is used to control the optimization procedure. This algorithm again can be provided in modular form. It is also a very instructive example to show the use of different modules, i.e. FE-analysis, reliability analysis and optimization, as it will be shown below in more detail.

2.4 Industrial Utilization

It is important that probability based software is readily applicable in engineering practice. This implies that for the software to be used for structural reliability analysis in day to day work a standardization of procedures is necessary. Therefore the following requirements need to be fulfilled:

1. *Comprehensive commands* need to be available to write standard macros, which can be updated to the current needs. It is important, that required changes are restricted to the very necessary inputs, e.g. by using macro-subroutines.

2. Instead of repeatedly editing input files, as it is often done in software utilized for research purposes as described above *user interaction by graphical output and menu-like input* is needed. This can be accomplished by basic dialogs for input and output as well as by graphical outputs for the current status of the calculation.

3 NUMERICAL EXAMPLES

3.1 General

Two examples, as described in the previous section are analyzed, one for the demonstration of the capabilities of modern simulation techniques for nonlinear stochastic dynamics, the second is concerned with reliability based optimization (RBO).

3.2 Nonlinear Stochastic Dynamics

COSSAN contains several module groups for determining the stochastic response. The most important classes are (1) linear stochastic response of MDOF-Systems under stationary and non-stationary white noise excitation in the time domain, (2) the stationary response of FE-model in the frequency domain, (3) Equivalent linearization modules with cooperate with the module group (1), (4) Monte Carlo Simulation and (5) Controlled Monte Carlo simulation.

The latter has been mentioned in the previous section, wherein Fig.2 shows a typical result obtained by this module group. It can be seen from Fig.2 that the low probability domain $1.0 \cdot 10^{-6} < p_f(t) < 0.5 \cdot 10^{-3}$ can be assessed by the Controlled Monte Carlo procedure. Note that no estimates can be obtained by the standard MCS procedures which naturally can not estimate a $p_f(t) < 1/N$ where N denotes the sample size. To assess the the statistical errors introduced by MCS, several estimates are generally computed.

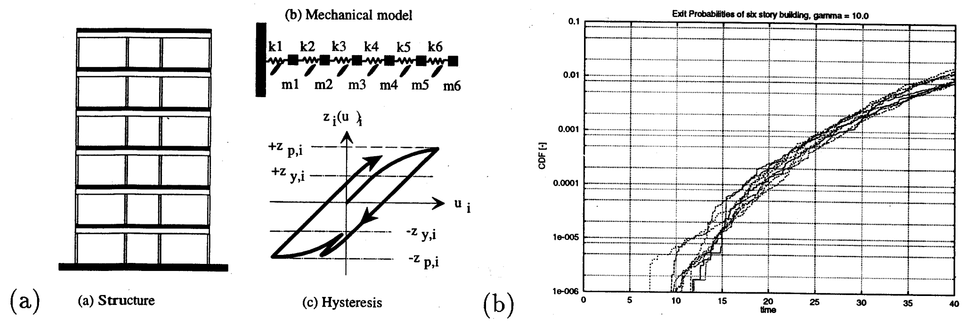

Figure 2: (a) Structural model with Hysteresis (b) Nine estimates of the First Passage distribution of a damage function using Controlled MCS and a sample size N = 2000.

3.3 Reliability Based Optimization (RBO)

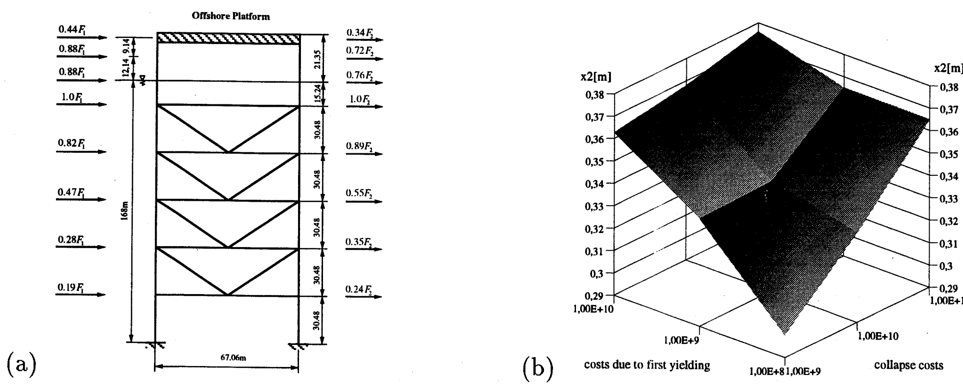

Figure 3: (a) Model of fixed offshore structure (b) Optimal Radius of horizontal and diagonal stiffeners dependent on the failure consequence costs

A template, steel made platform as sketched in Fig. 3(a) is numerically analyzed using RBO procedures. The structure has a quadratic cross section and consists of elements with tubular cross-sectional areas. Uncertainties in the structural resistance are taken into account by a fully correlated random yield strength of the structural elements. A coefficient of variation of 0.05 is assumed for the used steel type. First yielding (partial failure) in any structural component and complete plastification (collapse) are the considered two failure criteria. It is assumed that the main part of the loading to the offshore structure results from extreme waves due to severe storms.

From the computational point of view, many different tasks are required to solve this RBO problem:

1. A mechanical model of the structure is required. Here, a finite element (FE) model is used, provided in form of a module group of the used software. This *FE-module* is applied to calculate the required input for a reliability analysis, namely sectional forces and stresses as a result of highly developed load models.

2. The above described uncertainties require a probabilistic description. This is achieved by applying a module group, that is primarily designed for the description and handling of *random variables.* In this case the yield strength is such a random variable.

3. To avoid the high computational efforts which would be required for the estimation of failure probabilities by direct MCS, the response surface method (RSM) is applied. The required points on the limit state function are calculated iteratively using the *FE-module* and further processed within the *RSM-module.* By application of the *random variable module* and different *MCS modules* the evaluation of the failure probability can be obtained most efficiently.

4. Finally, the *optimization module* is applied. In the used software, this module can be operated in reverse communication mode as well, i.e. the values of the cost function as well as the equality and inequality constraints can be provided from the interactive user level, e.g. as results from other modules.

These topics are summarized in Fig. 4. It shall be mentioned, that a detailed description of this example from the point of view of an analyst is given in [3].

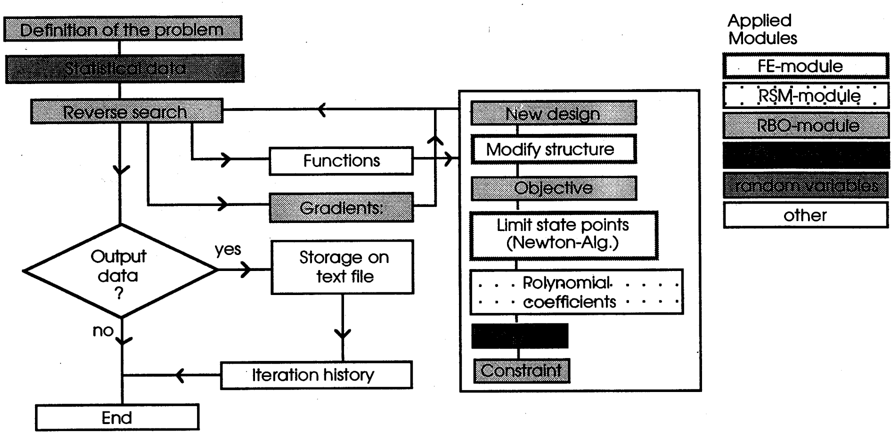

Figure 4: Interaction of modules needed for RBO

Using this procedure the strong influence of the individual cost factors respectively the failure criteria on the structural design becomes apparent (see Fig. 3(b)). It should be noticed, that partial failure as well as total failure are of almost equal importance in reliability-based decision making. Both, increasing collapse costs and partial failure costs lead to larger cross-sectional dimensions. Thus, neglecting partial failure criteria would lead to a weaker structural design.

4 CONCLUSIONS

In this paper an approach to stochastic structural analysis is described. The advantages of this approach can be summarized as follows:

- A transparent modular programming technique is used, where the modules allow standardized data transfer.

- The interaction between the user and the calculation routines is organized by a comprehensive macro language. The macros can be organized in input file, which can be conveniently edited.
- It is possible to establish standardized input files with menu-like structures for the day to day industrial application. This is facilitated by user definable graphical outputs and various kinds of interactive input boxes.

However, there should be a continuous improvement of the software, e.g by faster computational algorithms. Moreover, these algorithms also should facilitate the improving hardware environment as well.

Acknowledgment

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