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Evaluation of Solutions for Axial Cracks in Ovalising Pipes

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Abstract

The characterisation of cracks in shells subject to predominantly bending loading was investigated. Experimentation provided a suitable means for evaluating the accuracy of available solutions, particularly for axial cracks in cylindrical shells. Reissner - based theory was shown to be accurate whilst Kirchoff - based theory was grossly inaccurate for bending loading. Serious implications arise in relation to a leak-before-break strategy for if bending predominates, a very long part-through crack growth is promoted with a greater potential for a break-before-leak mode. A simple adjustment allows through-crack solutions to be applied in part-through crack situations.

1. Introduction

Predominant bending loading of shells occurs frequently in pipe bends, pressurised pipes which are accidentally or deliberately non circular, and at shell intersections. Valid fracture mechanics characterisation of cracked shells in bending is especially important with respect to the prediction of alternative leak/break modes for pressure vessels.

If bending loading in a shell wall predominates over membrane loading, a part-through crack may extend lengthwise without penetrating the thickness. A catastrophic 'break-before-leak' failure mode is thereby promoted. Solutions for through cracks in shells, based on a Kirchoff approximation have been produced by FOLIAS (1), and ERDOGAN and KIBLER (2), among others, and have been verified experimentally under membrane loading.

In the case of bending loading no tests are known and the Kirchoff-type solutions diverge considerably from the less approximate Reissner-type solutions as formulated by SIH (3), and KRENK (4) for example. The work reported here was therefore designed to:

i) determine the validity and accuracy of existing solutions with respect to crack tip stress field intensity factor in a bending case;
ii) provide a basis for adjusting solutions for a part-through crack;
iii) check that the analytical approach gave realistic prediction of crack propagation behaviour notwithstanding the two-dimensional simplification embodied in the analysis.

The evaluation was carried out experimentally using axially cracked cylinders of low alloy pipe steel.

2. Experimentation

Cylindrical pipe specimens were subjected to cyclic transverse stretching on a diameter resulting in ovalisation and repeated bending in the cracked region. Arrangement of the apparatus is shown in Fig 1. The test series included uniform length through-cracks and cracks varying in length through the thickness representing part-through geometries. Milled and saw cut notches were pre-test sharpened by high cycle fatigue. The crack sizes throughout the test covered a range of the shell parameter λ corresponding to $1.2 < \lambda < 2.6$ where

$$\lambda = (12(1 - \nu^2))^{\frac{1}{4}} \cdot \frac{a}{(RT)^{\frac{1}{2}}} \qquad \text{eq (1)}$$

a is the half crack length, and R and t are the cylinder radius and thickness respectively.

The cylinder material had previously been calibrated with respect to fatigue crack propagation characteristics in a plane SEN specimen configuration. The methods of crack length measurement employed in this calibration were the travelling microscope and the potential drop technique. Using a Paris Law formulation for ΔK in the range 19.6 - 60 $MN/m^{3/2}$ gave:

$$\frac{da}{dN} = 8.85 \times 10^{-13} \ \Delta K^{3.34} \quad \text{(m/cycle)} \qquad \text{eq (2)}$$

for P.D. equipment, and

$$\frac{da}{dN} = 10.44 \times 10^{-13} \ \Delta K^{3.32} \quad \text{(m/cycle)} \qquad \text{eq (3)}$$

for the travelling microscope.

The 'effective' level of crack tip stress intensity factor in the ovalising pipe could therefore be inferred from careful measurement of crack growth rates and substitution in the above equations. Testing was carried out in a servo-hydraulic machine and the crack length monitored by means of an optical arrangement. The axial cracks were located centrally and both crack tip extremities monitored independently. Load levels were continuously adjusted to achieve a constant range of stress intensity factor at the crack tip. The loading produced a combined membrane and bending stress configuration such that the ratio of membrane to bending was 1:24.6.

3. Results

Typical crack growth measurements from the test series are shown graphically in Fig 2 for a particular cyclic loading range. The linear behaviour of both crack tip extremities indicates a constant cracking rate $\frac{da}{dN}$ using a least squares analysis. The careful reduction of alternating load levels as the crack extended incrementally provided the required constant stress range intensity factor (ΔK). The magnitude of which could be inferred from equations (2) and (3).

After testing, the cylinders were split apart to reveal the crack growth pattern and a 'spring back' model was used to ascertain the level of residual stress present. This showed levels of approximately ± 30 N/mm^2 being compressive at the cylinder bore where cracking occurred. Comparison with the membrane stress levels over the test series (7 - 16 N/mm^2 tensile) indicates that a retarding effect on crack growth might be incorporated in the experimental results. The crack growth driving force is however due primarily to the bending stress (180 - 400 N/mm^2) with retardation effects being of less significance. The theoretical values in table I were derived from refs (2) and (4), being appropriate Kirchoff and Reissner based formulations respectively.

The scaled drawing of Fig 3 illustrates the typical crack growth pattern obtained throughout the test series.

The tendency was for part-through defect growth becoming progressively more longitudinal and reducing the aspect ratio ($^d/2a$). Beachmarks were found at the boundary between the different constant ΔK levels (at the bore) due to the significant increase in peak load levels. Fatigue cracking originated from the tensile corner of the notch spreading longitudinally, with crack growth through the thickness gradually decreasing.

The crack growth delay model suggested by McCARTNEY (5) was incorporated into a computer program to assess the effects of the step-wise reduction in test load levels required to attain a constant ΔK level. This suggested an insignificant overload effect on crack growth of between 1-3% throughout the test series. More detail of the test series can be obtained from ref (6).

4. Discussion

In the test series, very long part-through cracks were developed along the tensile fibres on the inner wall of the shell. Such a pattern is potentially dangerous in practice as critical crack sizes may be developed without leakage indications, thereby promoting a catastrophic break-before-leak failure mode. The danger of relying on a leak-before-break criterion is readily apparent for those areas of pressurised shells where bending effects dominate, unless the material exhibits the necessary toughness to contain very large defects. A comparison of bending and extensional SIF's would allow alternative leak/break modes to be identified. For this reason accurate crack tip characterising parameters need to be available.

The results were assessed relative to the two types of analytical solution described previously. The 'effective' stress range intensity factors determined experimentally were typically much smaller than those predicted by Erdogan and Kibler's theory for through cracks with the difference increasing as the part-through crack profile became more pronounced. Comparison with Krenk's formulation however suggests a much closer agreement although this also deteriorated slightly with changing crack shape. One of the authors has confirmed the results of ref (4) using a finite element analysis (6), by assuming a through thickness crack shape throughout. Residual stress and the part-through crack profile are probably responsible for the lower apparent effective SIF in the tests.

The retarding effects from residual stress are not significant since they are of similar magnitude to the membrane stress levels. Also the retardation effect would, in fact, decrease since load levels were increased in stages after the crack growth rate was evaluated. The changing crack profile is therefore a more significant factor. The through-crack solutions can be simply adjusted to account for this effect, in part, by superposing appropriate models as shown in Fig 4. Forces developed in the uncracked area on the tension side are applied to close the crack.

The stress intensity factor for a pair of such forces is given by

$$K = 2\sqrt{\frac{a}{\pi}}\ \frac{P}{\sqrt{a^2-x^2}} \qquad \text{eq (4)}$$

Using the actual crack profile of Fig 3 an allowance for the reduction in SIF can be evaluated by estimating the shaded area and hence determining the force magnitude from the stress distribution attained during that particular test. This reduction varied between 4 and 11 $MN/m^{3/2}$ over the test series and suggested approximately a 13% decrease in Krenk's results when corrected for part-through effects. This considerably improved the agreement with experiment as shown in table I. The above superposition provides a simple adjustment to through-crack solutions for application to problems containing part-through crack situations with reasonable results.

The ligament on the compression side has been ignored in this treatment and this may have some effect at the larger crack sizes. Further work is aimed at improving this aspect of the model.

5. Conclusion

The work presented here provides much needed experimental evidence to clarify several areas of uncertainty.

i) The leak-before-break concept cannot be relied upon in those areas of pressurised shells where bending predominates. Failure to appreciate this could have serious consequences since break-before-leak conditions are likely to prevail.

ii) The theoretical LEFM solutions based on Kirchoff approximations do not provide accurate SIF for predominant bending of cracked shells. Those solutions based on a Reissner approach are considered to give an accurate representation of crack tip conditions provided they are adjusted to suit part-through crack geometry.

iii) A suitable model was suggested which allowed 2-D solutions, using a through-crack idealisation, to be applied satisfactorily in predicting 3-D crack growth.

6. References

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Table 1 - Experimental and Theoretical Results

λ	R/t	Average da/dN ($x10^{-8}$ m/cycle)	Experiment ΔK(eq.2) ($MN/m^{3/2}$)	Experiment ΔK(eq.3) ($MN/m^{3/2}$)	Theory ΔK(ref 4) ($MN/m^{3/2}$)	Theory ΔK(ref 2) ($MN/m^{3/2}$)	Theory ΔK part-through reduction	Theory ΔK(ref 4) part-through correction
1.2	11.2	4.323	25.3	24.7	28.6	43.6	4.1	24.5
1.4	11.2	8.416	30.8	30.2	40.4	63.2	6.6	33.8
1.7	11.2	32.866	46.3	45.6	64.7	103.5	8.9	55.8
2	11.2	48.462	52	51.2	72.9	123.7	11.2	61.7
1.3	11.5	4.544	25.7	25.1	33.1	53	4.4	28.7
1.4	11.5	11.952	34.2	33.6	48.8	78.3	6.5	42.3
1.7	11.5	30.317	45.2	44.4	69.3	113.8	7.2	62.1
1.9	10.6	3.934	24.5	24	32.6	53.1	4.3	28.3
2.2	10.6	18.875	39.3	38.5	54.1	90.4	5.9	48.2
2.6	10.6	32.635	46.2	45.5	72.6	123.1	7.6	65

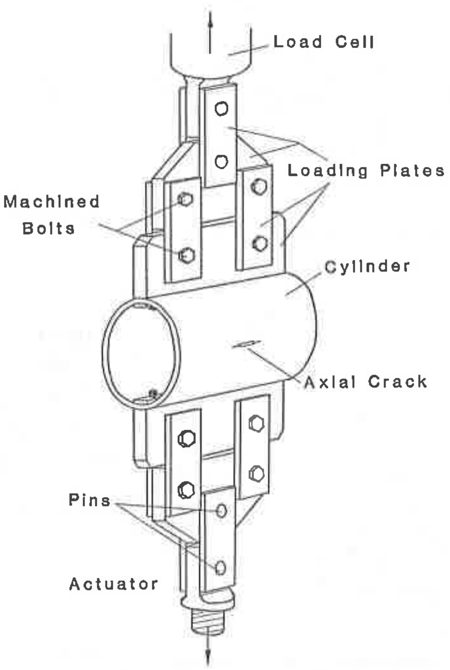

Fig 1. Loading Arrangement for Cylinder Test Series

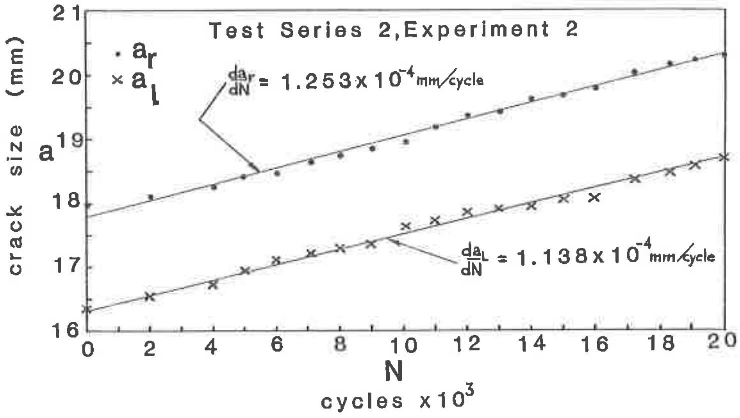

Fig 2. Typical Crack Growth Data

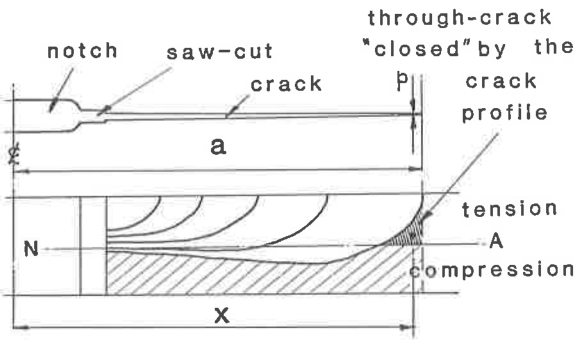

Fig 3. Typical Crack Growth Profiles (Test Series 1)

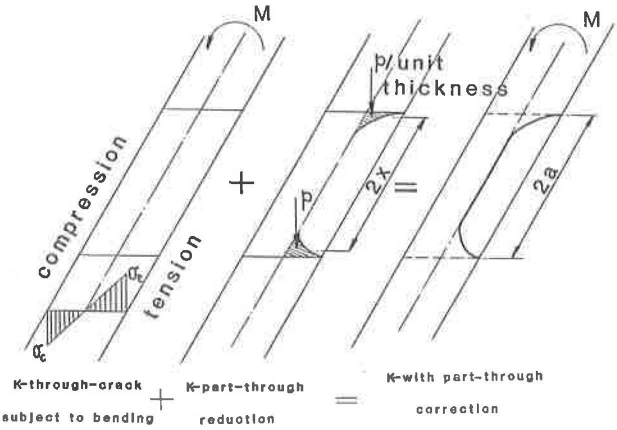

Fig 4. Superposition of Models for Part-Through Effects