

## **ABSTRACT**

WANG, YANG. Estimation of Recreation Demand Models Using Dual Approaches.  
(Under the Direction of Daniel Phaneuf and Matthew Holt).

The purpose of this dissertation is to investigate dual approaches to corner solutions that are both tractable and flexible in recreation demand. Two structural models are estimated using data from the Iowa wetlands survey in the incomplete demand system context. This dissertation expands existing economic literature on recreation demand in several respects, including 1) combining incomplete demand system framework with corner solution estimation, 2) advancing knowledge on estimating utility consistent recreation models allowing for corner solutions, 3) presenting feasible computational techniques for the use in recreation demand estimation. The results show that parameter estimates under the models studied have a similar pattern with the expected signs. However, the two models predict different correlation patterns among error terms across recreation sites and have different behavioral interpretations. I also construct welfare measures under various scenarios and find that the welfare measures are consistent with the expected signs and are of reasonable magnitudes.

**ESTIMATION OF RECREATION DEMAND MODELS  
USING DUAL APPROACHES**

by  
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A dissertation submitted to the Graduate Faculty of  
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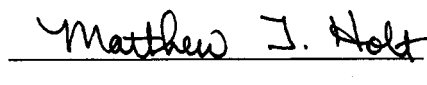
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**APPROVED BY:**

Co-chair of Advisory Committee - Co-chair of Advisory Committee

献给我敬爱的父亲王国生母亲赵宏！

感谢你们多年来对我的理解与支持、鼓励与鞭策。

*I will remember what I was taught at home:*

*You may never become a hero, but you can always be a man!*

## Biography

I was born on June 30, 1972 in Tangshan, Hebei Province, P.R.China to Guosheng Wang and Hong Zhao. In 1991, I graduated from Tangshan No. 1 High School and enrolled in Chongqing University after a successful performance in the competitive National Entrance Examination.

As a student in the School of Business Administration I was active in academic and extracurricular activities that include serving as President of Student Body of Chongqing University and Chairman of Chongqing Student Union, and I was also honored as the pacesetter of Chongqing University in the year of 1993. I received a Bachelor of Engineering degree in Engineering Economics in July 1995.

After graduation I worked as an assistant economist at China National Beijing Contracting & Engineering Institute for Light Industry in Beijing. In recent years, with the establishment of the market economy in China, it became necessary to further my knowledge in economics as I have taken economics as my lifetime career. In August 1998 I became a student in the Economics Department at East Carolina University. After ten months' busy studies I earned a Master of Science in Economics in June 1999. In July 1999 I started my graduate studies in economics as a Ph. D. student at NC State.

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I thank my parents for providing a “stressful” environment from which I started an exciting life and for their understanding my decisions although some are unimaginable. Finally, I would like to thank my sister for her aspirations and for taking care of mom and dad while I am not around.

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## Chapter 1: Introduction

When estimating systems of demand functions, it is common for household level data to have zero consumption of one or more goods. In this case the econometric model should account for the positive probability of zero consumption. Ignoring corner solutions in the demand system estimation will result in biased estimation, since the individual's response to price depends on the set of goods she consumes at corners. Exclusion of observations with zero expenditures on one or more goods will result in both significantly reduced sample size and inconsistent estimators.

In estimating recreation demand models using seasonal and multiple site data, it is typical to find that most individuals visit only a small subset of available sites, and a significant portion of sites are not visited. Considerable progress had been made in dealing with corners in recreation demand literature. However, many approaches rely on restrictive assumptions, or are not utility consistent. For example, count models and count models with zero inflation do not provide an economic interpretation for corners, even though the models account for the nonnegative integer nature of the recreation demand. Share models do not allow zeros as a solution and can only handle the site selection decision. The popular random utility maximization (RUM) model paradigm typically has zero parametric cross price effects and a restrictive error structure. The repeated RUM model assumes a fixed finite number of independent choice occasions and has welfare measures conditional on the per choice occasion income. Although the linked models address corner solutions in an intuitive manner, they are not utility consistent and treat the simultaneous participation

and site selection decisions in two separate steps. In contrast, the Kuhn-Tucker model (starting with a stochastic direct utility function) addresses many of these weaknesses. However, its reliance on explicitly specified first order conditions and simple error structure restricts the cross price effects and the stochastic substitution patterns that can be characterized.

The virtual price approach (starting with an indirect utility function) explicitly states demand equations for demand regimes, and can potentially address several weaknesses in the KT model. In a recreation demand context, the virtual price approach is theoretically appealing since it is the virtual price rather than market price which characterizes corner solutions. However, estimation and interpretation challenges have prevented its wide application in both recreation and general demand contexts.

An open research task is the development of models that maintain the theoretical advantages of the virtual price approach, while allowing more tractable and flexible estimation and welfare calculations in the recreation demand context. Specifically, to best model seasonal multiple site recreation behavior, structural models are needed that 1) are utility consistent, 2) allow estimation using seasonal and multiple site data, 3) allow rich parametric and stochastic substitution effects, 4) incorporate both the discrete and continuous aspects of recreation demand decision. This research represents such an effort. In this dissertation I investigate two dual approaches (in particular, the Amemiya-Tobin model and the virtual price model) to corner solution estimation that are tractable and flexible in the incomplete demand context. The incomplete demand system is attractive in

the recreation demand context in that we can construct exact welfare measures conditional on the household's total income rather than the predetermined recreation expenditure. Both dual approaches have the desirable characteristics described above. Both the Amemiya-Tobin (AT) model and the virtual price (VP) approach start with the specification of the demand equations from which the quasi-indirect utility functions are recovered if weak integrable conditions are met. The major difference separating them, and a main theme of this dissertation, is whether the reservation price plays a role in the individual's decision making process.

Though the appealing attributes of these two models are well known, there are several estimation difficulties that have prevented wider applications. First, the likelihood function is highly non-linear, and estimation requires multiple dimension integration with difficulties increasing exponentially with increases in the number of goods of interest. The second problem associated with estimating the dual approaches in the incomplete demand system context is "whether the system is consistent with a rational individual maximizing her utility subject to a linear budget constraint" (von Haefen [2002]). The integrability conditions must be met to generate consistent Hicksian welfare measures. In this study Gibbs sampling, along with the data augmentation and rejection sampling (Pitt and Millimet [2001]), are employed to solve both the dimensionality and integrability problems. Given the simple latent structure of the demand system, augmenting the data removes the need to directly evaluate any probability integrals. Rejection sampling allows me to only collect parameter estimates that satisfy the integrability conditions.

I apply the two approaches to a common data set on the recreational use of wetlands in Iowa. I find that estimation of both models is feasible using Gibbs sampling, but there are still complexities, particularly for the VP approach. The two models provide similar parametric price and quality effects. However, they have somewhat different patterns of stochastic substitution effects. In addition, the two models also have very different interpretation and magnitude of welfare measures.

The remainder of this dissertation is organized as follows. Chapter 2 reviews the recreation demand literature dealing with corner solutions and the development of the virtual price method literature. Chapter 3 describes the data used in this dissertation and discusses the previous studies that also used this same data. The Amemiya-Tobin approach and its welfare measurement methodology are developed and implemented in Chapter 4. As part of the discussion, this chapter also presents the general ideas of the incomplete demand system, the Gibbs sampler, and other econometric techniques used in this dissertation. Chapter 5 describes the virtual price approach and its welfare measurement methodology, and provides the details on estimation strategies and the estimated results. Chapter 6 concludes with final observations and suggestions for future research.

## Chapter 2: Literature Review

In estimating recreation demand models using seasonal and multiple site data, it is typical to find that most individuals visit only a small subset of available sites, and a significant portion of sites are not visited. In the recreation demand literature there are several approaches for dealing with the corner solution problem. In this chapter I first review the evolution of corner solution approaches in recreation demand literature, and then discuss the literature surrounding the original Lee and Pitt [1986] virtual price approach article.

### 2.1 Development of Corner Solution Approaches in Recreation Demand Literature

When estimating recreation demand using general population surveys, researchers need to investigate two decisions facing recreators: whether to participate, and conditional on participation, how many trips to make. This decision-making process can be resolved separately or simultaneously, and structurally or statistically.

#### 2.1.1 *The Single Equation Tobit Models*

The Tobit model is the best known econometric model for dealing with censored data. Smith [1988] and Bockstael et al. [1990] give several applications in the recreation demand context. The Tobit model specifies the demand equation that is derived from the individual's utility maximization, and attaches a stochastic error term to the demand equation. Formally, let the number of trips be

$$y_j = \begin{cases} X_i \beta + \varepsilon_i & \text{if } X_i \beta + \varepsilon_i > 0 \\ 0 & \text{otherwise} \end{cases} \quad (2.1)$$

for  $i = 1, \dots, N$ , where  $X_i$  is a vector of characteristics of individual  $i$  (including travel cost), and  $\varepsilon_i \sim \phi(0, \sigma^2)$ .

The parameters are estimated by truncated variable methods. Under this statistical setup the probability of participation is

$$P(y > 0) = \Phi\left(\frac{X_i \beta}{\sigma}\right), \quad (2.2)$$

where  $\Phi(\cdot)$  is the standard normal cumulative density function. For the Tobit model the participation and quantity decisions are formulized in a single behavioral model, i.e., the same the error terms and explanatory variables explain both participation and quantity decisions.

### 2.1.2 The Single Equation Count Models

Because the observed recreational trips are non-negative integers, count models such as Poisson or negative binomial distributions are natural candidates to account for this characteristic. Failure to account for this distributional issue could lead to biased estimates and hence inaccurate welfare measures. Smith [1988] and Hellerstein and Mendelsohn [1993] suggest a variety of count models in the recreation demand context.

Formally, the Poisson model assumes trips to a single site  $y_i$  is distributed

$$f(y_i) = \frac{e^{-\lambda_i} \lambda_i^{y_i}}{y_i!} \quad y_i = 0, 1, 2, \dots, \quad (2.3)$$

where  $\lambda_i = \exp(X_i \beta)$  is the expected number of trips.

The probability of participation is

$$P(y_i > 0) = 1 - \exp(-\exp(X_i \beta)). \quad (2.4)$$

Also note that the probability of nonparticipation is  $P(y_i = 0) = \exp(-\exp(X_i \beta))$ . As in the Tobit model, the same error term and explanatory variables explain both the participation and quantity decisions.

In contrast, Haab and McConnell [1996] and Shonkwiler and Shaw [1996] argue that observing zero consumption could result from either demanding zero visits (the person is priced out) or no utility for the site (the site is not in choice set). If a model correctly distinguishes between participants and nonparticipants, including the zero observations in estimation will more effectively use information contained in the zero observations and achieve higher efficiency. The basic count-data models accommodate zeros. However, if the sample includes both recreators and nonrecreators, it is likely that many individuals would never take trips to a recreation site. Since the basic count-data models may not place enough probability mass on zero in these cases, the augmented count distribution with a “spike” or zero inflator was developed. Formally, assume  $y_i$  has a nonnegative discrete p.d.f.

$$P(Y = y_i) = f(y_i) \quad y_i = 0, 1, 2, 3, \dots \quad (2.5)$$

An augmented p.d.f. of  $y_i$  to account for the two types of nonparticipant is

$$f_A(y_i) = \begin{cases} w_i + (1 - w_i)f(0) & y_i = 0 \\ (1 - w_i)f(y_i) & y_i = 1, 2, 3, \dots \end{cases} \quad (2.6)$$

where  $w_i$  is an individual specific weight parameter that is a function of variables which affect only the participation decision. This augmented count model separates the factors which influence the participation and quantity decision and allows both decisions to drive individuals out of the market. The hurdle models are based more on intuition than a utility consistent model, since one can not write a traditional neoclassical model that matches the empirical specification. The double-hurdle models do allow exploration into nonuse, as opposed to potential use (Shonkwiler and Shaw [1996]). The hurdle models also allow investigation of other variables that may explain participation.

### *2.1.3 Multiple-Site Demand Models*

It is well recognized that single and isolated sites rarely exist, and the failure to incorporate substitute sites' prices could lead to biased estimation results, and hence inaccurate welfare measures. Caulkins, Bishop and Bouwes [1985, 1986] show that the biases could be upward or downward depending on the correlations among alternative sites.

To account for the influence that existing alternative sites have on a newly developed recreational site, Burt and Brewer [1971] suggest a multi-site demand model to assess the net social benefits from development of this new site. Burt and Brewer address this problem in two ways. First, they include substitute site variables (such as price) in each demand equation. Second, they jointly estimate a system of equations and impose cross-equation restrictions. Formally, the linear demand equations are

$$Y = \alpha + \beta P, \tag{2.7}$$

where  $Y$  is a  $J \times 1$  quantity vector,  $P$  is a  $J \times 1$  price vector,  $\alpha$  is the intercept with income as a component, and  $\beta$ 's are parameters with symmetry restrictions.

The statistical model corresponding to equation (2.7) is

$$y_{ij} = \alpha_j + \sum_{k=1}^J \beta_{jk} p_{ik} + \delta_j m_i + \varepsilon_{ij} \quad i = 1, 2, \dots, N; \quad j = 1, 2, \dots, J, \quad (2.8)$$

where  $i$  and  $j$  index individuals and sites, respectively,  $m$  is income, and  $\varepsilon_{ij} \sim \phi(0, \Sigma)$  with  $\Sigma$  having the following structure:

$$\text{cov}(\varepsilon_{ij}, \varepsilon_{hk}) = \begin{cases} = \sigma_{jk}, & \text{if } i = h \\ = 0, & \text{if } i \neq h, \quad j \neq k \\ = \sigma_{jj}, & \text{if } i \neq h, \quad j = k, \end{cases} \quad (2.9)$$

which gives a block diagonal covariance matrix for the error vector.

A large proportion of corner solutions in the recreation demand data tend to cause heteroscedasticity across households for a given commodity. The iterative approach suggested in this paper is employed to cope with the heteroscedasticity in the context of system of demand equations. Although Burt and Brewer consider the qualities of sites in the sense of site aggregation, the quality variables are not explicitly included in the estimation. Any differences due to quality characteristics of sites will show up in the estimated coefficients of the different demand equations.

Cichetti et al. [1976] estimate a similar model to Burt and Brewer [1971] with zonal data to analyze the Mineral King Project in California. A system of demands for alternative sites or site groups are regressed against the travel cost to each site without including

quality variables and individual income. Because this model does not include the characteristics of the sites or individuals, this model assumes that the proposed site is a perfect substitute site, in terms of characteristics, for an existing site. If there is no existing similar site, the social benefits cannot be assessed in this model.

Both Burt and Brewer and Cichetti et al. made significant contributions to recreation demand estimation, but their models are not derived explicitly from a well-specified model of consumer behavior, which may lead to problems such as theoretically implausible function forms, or incorrectly defined cost of producing recreational activities (Morey [1981]).

#### *2.1.4 The System of Counts Models*

Although Burt and Brewer and Cichetti et al. jointly estimate a system of demand equations to include substitute sites, the non-negative integer aspect of recreation trips is not imposed in their applications. Ozuna and Gomez [1994] and Englin et al. [1998] suggest the system of count models to estimate recreation demands. Ozuna and Gomez consider the following demand functions:

$$Y_{ij} = \alpha_i + \sum_{k=1}^J \beta_{ik} p_{ik} + \varepsilon_{ij}, \quad (2.10)$$

where  $i$  and  $j$  index individuals and sites, respectively,  $i=1,2,\dots,N$  and  $j=1,2,\dots,J$ , and  $Y$  and  $p$  are quantity and prices of the interested sites,  $\alpha$  and  $\beta$  are parameters to be estimated, and the  $\varepsilon_{ij}$ 's are error terms. The seemingly unrelated Poisson regression model provides parameter estimates that are asymptotically more efficient than equation-by-equation Poisson estimates and overcomes the bias and inconsistency problems that result when

using the SURE model with count variables as dependent variables. Englin et al. [1998] develop a utility theoretic truncated Poisson demand system and apply it to recreational trips to four wilderness park in central Canada. The results show that the imposition of restrictions on parameters resulting from economic theory (Lafrance and Hanemann[1989]) is important when estimating a system of demand equations, particularly when welfare measurement is the goal.

Von Haefen and Phaneuf [2002] compare continuous and count demand system models both conceptually and empirically. The continuous models specify the individual's demand functions up to an unobserved heterogeneity term while the count models specify expectations of individual demand. The economic and statistical characteristics of both models are explored with an application to a common data set. The empirical results show that the continuous and count demand models have qualitatively similar policy implications when comparable preference structures and prediction strategies are employed. Although both the continuous and count data demand system frameworks integrate the individual's site selection and total trips demand decisions, the restrictive preference specifications limit the ability to address the policy issues.

#### *2.1.5 Share Models*

Both Burt and Brewer [1971] and Cichetti et al. [1976] are significant contributions to the recreation demand literature, but the demand functions used are not directly derived from a well specified model of consume behavior. To address this, Morey [1981,1984,1985] suggests systems of share functions for site-specific skiing activities

which explicitly include the characteristics of the alternative recreation sites. Inclusion of the attributes of the sites allows this model to be used to estimate the demand for not yet existing sites as a function of their proposed attributes and prices. Although share equation systems are popular in the general demand estimation literature, there are only a few applications in recreation demand literature.

Assume the representative individual faces the following problem

$$U = U(Y, A, \varepsilon), \quad \text{s.t. } \tau = \Gamma'Y, \quad (2.11)$$

where  $Y=[y_j]$  is the number of trips to site  $j$ ;  $\Gamma=[\gamma_j]$  is the cost (measured in units of time) to site  $j$ ;  $\tau$  is the individual time allotment to recreation activities;  $A=[a_{kj}]$  is the characteristic  $k$  of site  $j$ . Note that this model assume that the recreation subutility function is weakly separable from other activities.

Maximizing the utility function subject to the budget constraint gives the demand functions

$$y_j = y_j(\tau, \Gamma, A, \varepsilon) \quad j = 1, 2, \dots, J, \quad (2.12)$$

and the system of share equations are obtained as

$$s_j = \frac{y_j}{T} = s_j(\tau, \Gamma, A, \varepsilon) \quad j = 1, 2, \dots, J, \quad (2.13)$$

where

$$T = \sum_{j=1}^J y_j.$$

Although the functional forms are identical for all sites, the value of the independent variables varies across sites' share functions. Note both the demands and shares are random

variables from the researcher’s perspective. Because shares must lie between zero and one the multinomial distribution is chosen to represent the individual’s density function for shares

$$f(s_1, s_2, \dots, s_j; T; \theta) = \left( \frac{T!}{\prod_{j=1}^J y_j!} \right) \left( \prod_{j=1}^J (s_j)^{y_j} \right). \quad (2.14)$$

If the shares are independent across individuals, the parameters can be recovered through the maximum likelihood method , where the likelihood function is

$$L = \prod_{i=1}^N f(s_{1i}, s_{2i}, \dots, s_{ji}; T_i; \theta), \quad (2.15)$$

and i index individual  $i=1,2,\dots,N$ .

An empirical application of Colorado skiers shows that this share model predicts the skier’s choice of alternative sites better than that of the multinomial logit model. Share models assume the weakly separable structure that is consistent with an underlying theory of utility maximizing behavior. One limitation of share models is that the underlying multinomial distribution of shares doesn’t admit shares that are strictly zero. Also, share models predict the allocation of trips but not the total number of trips. The limitation of this is that share models explain only the choice of sites given the decision to participate. Morey et al. [2001] generalize share models to an “alternatives” model to explain both participation and site choice. The alternatives model assumes that the year consists of a finite number of choice occasions to participate in recreation, and that nonparticipation is an activity with a cost which varies across individuals and is equal to the individual’s

expenditure on everything else divided by the number of times of nonparticipation. The alternatives model estimates the choices among recreational sites and nonparticipation treating the nonparticipation as an alternative choice. The alternatives model avoids assuming that the choice occasions are independent from each other, and allows complements among sites, which are attractive attributes of the alternatives model.

#### *2.1.6 The Repeated RUM Model*

The repeated RUM model (Morey et al. [1993]) builds the participation decision directly into the RUM model. The repeated RUM model assumes that a recreational season consists of a fixed number of independent choice occasions such that on each occasion the individual takes no trip or at most one trip. If a trip is taken, they also choose the site. The individual chooses the option that provides the greatest utility. The utility from choosing site  $i$  during choice occasion  $t$  is

$$U_{jt} = V_j + \varepsilon_{jt} \quad j = 0,1,2,\dots,J; \quad t = 1,2,\dots,T, \quad (2.16)$$

where  $j = 0$  means no trip was taken at choice occasion  $t$ ,  $J$  is the total number of alternative sites and  $T$  is the total number of choice occasion.  $V_j$  is the deterministic part of utility function which is a function of the individual and site attributes, and  $\varepsilon_{jt}$  represents unobserved taste that is known to the individual but random to researcher. If  $\varepsilon_{jt}$  follows the generalized extreme value (GEV hereafter) distribution, conditional on the structure of the GEV c.d.f. the participation and site choice can be estimated in the framework of the multi-level nested logit models. The upper level is a binary decision, i.e., whether to take a trip or not; the next level, conditional on participating in recreation, is the choice among

aggregated sites; and so on, until the lowest level is to choose an alternative site within the aggregated group. There is no nesting structure when extreme value errors are assumed. In this simple case the expected maximum utility on a choice occasion is

$$IV = \ln\left(\sum_{j=0}^J \exp(V_j)\right) + 0.5722, \quad (2.17)$$

where the nonparticipation option is a component in the calculation. The seasonal welfare measure in the linear case is

$$CV = T \times ((IV^0 - IV^1) / \lambda), \quad (2.18)$$

where T is the total number of choice occasions and  $\lambda$  is the marginal utility of income.

In a later paper Morey et.al [1995] note that they “are not comfortable with” the assumption that a season consists of a fixed number of independent choice occasions without considering the dynamic impacts of the previous trips and the possible further trips. The model doesn’t assume that preferences are defined over the season and hence doesn’t allow diminishing marginal utility for an alternative. The other limitation of the repeated RUM model is that it doesn’t allow complements among sites. The repeated RUM model is consistent with utility maximization theory, and the GEV error structure makes the model easy to estimate and can handle a large number of alternative choices. However, EV and GEV errors lead to restrictive patterns of stochastic substitution.

#### *2.1.7 The Linked Site Selection and Participation Models*

The multinomial logit models have dominated the recreation demand literature in modeling choices among alternative sites. However, the site selection model alone can not predict the total number of trips over a season. This led to the development of the linked

site selection and participation models. Although the various linked models (Bockstael, et al. [1986, 1987], Creel and Loomis [1992], Hausman, et al. [1995], Feather, et al. [1995], Parsons and Kealy [1995]) differ from each other in the way the models link the site selection step and seasonal participation step, they all have the ability to predict the number of trips over a season. The linked models consider two components of individual's recreation decision sequentially: one is whether to participate, and if so, how many trips to take; the other is which site to visit on each choice occasion conditional on participation. In this way seasonal welfare measures include both the site substitution effects (through the site selection step) and changes in the total number of trips over a season (through the participation model). The estimation of linked models follows two steps. The allocation of trips to alternative sites is determined based on site attributes and relevant costs under the random utility framework in the first step, and in the second step the number of trips is modeled as a function of individual attributes and the price and/or quality index calculated based on the estimation of the first step.

The site-allocation decision is estimated first in the random utility model framework. The conditional indirect utility function for a visit to site  $j$  is

$$u_j = v_j + \varepsilon_j = v_j(m - p_j, z_j) + \varepsilon_j \quad j = 1, 2, \dots, J, \quad (2.19)$$

where  $j$  index sites and  $J$  is the total number of alternative sites,  $z_j$  is a vector of characteristics of site  $j$ ,  $p_j$  is the travel cost to site  $j$ , and  $m$  is per choice occasion income.

The term  $v_i(\cdot)$  is the deterministic part of the indirect utility function, and  $\varepsilon_j$  is the

unobserved variation in tastes and omitted variables. Estimation of the first stage is straight forward if the errors are extreme value or generalized extreme value variables.

In the participation stage the papers share the same basic idea, but vary in the way the price and/or quality indices are constructed and how the individual's participation decision is modeled. Let the general participation model have the following form

$$y = h(x, I, \varepsilon), \quad (2.20)$$

where the  $x$  is a vector of explanatory variables, such as individual attributes, income etc; index  $I$  is the price and/or quality index calculated from the site-allocation step, which is where the papers differ from each other; and  $\varepsilon$  is the error term.

Bockstael, et al. [1987] estimate the individual's participation decision as the Tobit model, i.e.,

$$y_j = \begin{cases} h(x_i, I) + \varepsilon_j & \text{if } h(x_i) + \varepsilon_j > 0 \\ 0 & \text{otherwise} \end{cases} \quad (2.21)$$

where  $j = 1, \dots, J$ ,  $\varepsilon_j \sim \phi(0, \sigma^2)$ ,  $y_j$  is the observed trips to site  $j$ , and  $x_i$  is vector of characteristics of individual  $i$ . The index is the inclusive value variable calculated from the first stage, which reflects the average value of recreational opportunities. Define the inclusive value as

$$IV = \ln\left(\sum_{j=1}^J \exp(v_j)\right), \quad (2.22)$$

which represents the value of different alternative sites weighted by the probability of being chosen.

The per choice occasion compensating variation (CV) for the multinomial logit model is

$$CV = (IV^0 - IV^1) / \lambda, \quad (2.23)$$

where 1 and 0 represent the inclusive value under new and initial level and  $\lambda$  is the marginal utility of income. Seasonal welfare measures are the product of per choice occasion compensating variation and the number of trips. Whether the observed number of trips or the predicted number of trips is used varies in different applications (Creel and Loomis [1992]).

Parsons et al. [1999] compare the linked models [Hausman, et al. [1995], Feather, et al. [1995], Parsons and Kealy [1995]] as well as repeated RUM model [Morey et al. [1993]] using a common application to fishing at Tennessee Reservoirs. Although the Hausman, et al. [1995] and Morey et al. [1993] models have quite different motivating theories, the authors find that they are nearly the same mathematically and have nearly identical seasonal welfare measures. Feather, et al. [1995] and Parsons and Kealy [1995] construct the expected prices and quality variables instead of the inclusive value as the link between site selection step and the seasonal participation step, and interpret the seasonal participation model as a demand equation. The seasonal welfare measures are calculated by integrating the demand equation over the range of variation in the price and/or quality index. Thus, their welfare measures are quite different from that of Hausman, et al. [1995] and Morey et al. [1993].

The uses of multinomial logit or nested logit models in the linked model in the site selection step make this model tractable for a large number of alternative sites without having to aggregate recreation sites into groups. Although the linked models have many advantages, such as dealing the corner solutions in an intuitive way and accounting for both the discrete and continuous nature of recreation decision, the models do have some unappealing features, such as non utility consistency and modeling a single agent's decision in two steps, which can not explain how the individual's utility adds up over time.

### 2.1.8 The Kuhn-Tucker Models

The methods discussed above suggest considerable progress has been made in dealing with corners in recreation demand. Many of the methods, however, rely on restrictive assumptions or are not derived from a consistent consumer choice problem. Wales and Woodland [1983] suggest the Kuhn-Tucker (KT hereafter) approach which characterizes the occurrence of corner solutions in a unified and internally consistent context. Phaneuf et al. [2000] apply this model in the recreation demand context. Application of the KT model requires specification of the direct utility function and a distribution for the error vector.

Assume each individual faces the following problem

$$\underset{y,z}{Max} U(y, z; q, \beta, \varepsilon) \quad s.t. \quad p'y + z \leq m, \quad z \geq 0, \quad y_j \geq 0, \quad j = 1, 2, \dots, J, \quad (2.24)$$

where  $U(\cdot)$  is a quasi-concave, increasing, and continuously differentiable function of  $(y, z)$ ,  $y = [y_j]$  is a vector of goods of interest,  $z$  is the numeraire good with price normalized to 1,  $p = [p_j]$  is a vector of prices,  $m$  is the individual's income,  $\varepsilon = [\varepsilon_j]$  is random error term which is known to the individual and unknown to the researcher. The KT model

incorporates the stochastic term into the utility function (i.e., assume the preferences are random over the population) rather than attaches the error to the demand equations ad hoc.

The first order necessary and sufficient Kuhn-Tucker conditions are

$$U_j(y, z; q, \beta, \varepsilon) \leq \lambda p_j, \quad y_j \geq 0, \quad y_j U_j(y, z; q, \beta, \varepsilon) = 0, \quad j = 1, 2, \dots, J, \quad (2.25)$$

$$U_z(y, z; q, \beta, \varepsilon) \leq \lambda, \quad z \geq 0, \quad z U_z(y, z; q, \beta, \varepsilon) = 0, \quad (2.26)$$

and

$$p'y + z \leq m \quad \lambda \geq 0, \quad (m - p'y - z)\lambda = 0, \quad (2.27)$$

where  $U_j(\cdot)$  and  $U_z(\cdot)$  are the partial derivatives of  $U(\cdot)$  with respect to  $y_j$  and  $z$ , and  $\lambda$  is the lagrange multiplier .

If the numeraire good is strictly positive, the following equation will hold

$$\lambda = U_z(y, z; q, \beta, \varepsilon). \quad (2.28)$$

Since  $U(\cdot)$  is increasing in  $(y, z)$ , the budget will be exhausted, i.e.,

$$z = m - p'y. \quad (2.29)$$

Substituting equations (2.28) and (2.29) yields the  $J$  first order conditions for utility maximization:

$$U_j(y, m - p'y; q, \beta, \varepsilon) \leq p_j U_z(y, m - p'y; q, \beta, \varepsilon), \quad y_j \geq 0, \quad y_j [U_j(\cdot) - p_j U_z(\cdot)] = 0, \quad (2.30)$$

where  $j = 1, 2, \dots, J$ . Assuming

$$U_{z\varepsilon} = 0, \quad \frac{\partial U_j}{\partial \varepsilon_k} = 0 \quad \forall k \neq j, \quad \text{and} \quad \frac{\partial U_j}{\partial \varepsilon_j} > 0 \quad \forall j = 1, 2, \dots, J, \quad (2.31)$$

the first order conditions, after some manipulation, become:

$$\varepsilon_j \leq g_j(y, m, p; q, \beta), \quad y_j > 0, \quad y_j[\varepsilon_j - g_j(y, m, p; q, \beta)] = 0 \quad j = 1, 2, \dots, J. \quad (2.32)$$

The specification of the joint density function  $f_\varepsilon(\varepsilon)$  for  $\varepsilon$  and equation (2.32) provide the necessary information to construct the likelihood function for estimation. Without loss of generality, assume the individual only visits the first  $k$  sites. This individual's contribution to the likelihood function is

$$\int_{-\infty}^{g_{k+1}} \dots \int_{-\infty}^{g_J} f_\varepsilon(g_1, \dots, g_k, \varepsilon_{k+1}, \dots, \varepsilon_J) |J_k| d\varepsilon_{k+1} \dots d\varepsilon_J, \quad (2.33)$$

where  $J_k$  is the Jacobian for the transformation from  $\varepsilon$  to  $(y_1, \dots, y_k, \varepsilon_{k+1}, \dots, \varepsilon_J)'$ . The maximum likelihood method is used to recover the parameters of the utility function.

The KT model has several advantages over many of the recreation demand models discussed above. First, the KT approach estimates the demand for trips to multiple sites over a season instead of a choice occasion. Second, the KT model maintains the aspects of a traditional system of demand equations while emphasizing the substitutability between sites and their attributes in the random utility model. Third, the KT model allows construction of welfare estimates in an internally consistent and utility theoretic context. Finally, the Kuhn-Tucker model simultaneously models both the site selection decision and participation decision.

Although the KT model is theoretically attractive and allows a smooth integration of the behavioral and econometric model, few applications exist in the literature. The difficulties arise from the complexity of estimation and welfare calculation for large numbers of sites, and the use of the restrictive utility functions which rule out flexible functional forms and

interesting substitution patterns for the recreation demands. Wales and Woodland [1983] estimate a KT demand system with non-negativity constraints on Australian meat consumption (i.e., beef, lamb, and other meats). In their application a random quadratic utility function is estimated with maximum likelihood. Because they observed no variation in prices, they estimated only the variation in demand due to differences in demographic characteristics. Other limitations in their analysis include relatively short time period surveyed and relatively limited geographic area surveyed. That is, the corner solutions may emerge as the result of infrequency-of-purchase or short-term deviations from equilibrium conditions.

Bockstael et al. [1986] suggest the KT model in the recreation demand context without estimating an empirical model. Phaneuf, Kling, and Herriges [1998, 2000] estimate the Kuhn-Tucker model using data on recreation in the Great Lakes regions. The authors also develop a methodology for estimating compensating variation, relying on Monte Carlo integration to derive expected welfare changes. The authors demonstrate the attractiveness of the KT model in recreation demand, and suggest that with improvement available computing power the KT model can be used to address questions of policy interest. Von Haefen, Phaneuf and Parsons [2002] demonstrate estimation with a large number of sites.

Although the KT model has the advantage of treating corner solutions in a theoretically consistent manner, its reliance on explicitly specified first order conditions restricts the utility functions that can be used in application. In practice, additive separable utility functions are employed in the KT model, which have restrictive cross price effects. The KT

model includes limited stochastic substitution if the generalized extreme value distribution is assumed to be the underlying distribution of the error terms. In contrast, Lee and Pitt [1986] follow up the Wales and Woodland paper by showing how the estimator can be derived with greater flexibility by beginning with the indirect utility function and use the virtual prices rather the actual prices to characterize corner solutions. The dual approach is theoretically equivalent to the Kuhn-Tucker model.

## **2.2 Development of Lee & Pitt Literature**

Wales and Woodland [1983] relax the regularity conditions to the direct utility function and then impose the non-negativity (i.e. Kuhn-Tucker conditions) as part of the defining optimization problem. Lee and Pitt [1986] start with the indirect utility function and use the derived virtual price to transform the binding non-negativity constraints into nonbinding constraints. These approaches employ theoretically equivalent techniques to recover the parameters of the utility function that endogenously allow for corner solutions while consistently accounting for unobserved heterogeneity. The dual approach makes it possible to explicitly state demand equations for each demand regime, which makes it easy to investigate “how switching occurs in response to changes in prices, income, or household characteristics.” (Lee and Pitt [1986, p.1239]), as well as allowing the use of more interesting functional forms.

In the recreation demand context, the dual approach is theoretically appealing in that it starts with a utility maximization problem that automatically satisfies the restrictions of utility theory. The same error structure and variables are employed to account for

nonconsumption, unobserved heterogeneity, and to explain both the continuous and discrete aspects of recreation demand. Though these are appealing attributes of the dual approach, there are several estimation difficulties that have prevented wider application. The likelihood function is highly non-linear, and calculation of the likelihood function requires integration of multivariate normal variables, the difficulties of which increase exponentially with increases in the number of goods of interest. To date only a small number of choice variables have been investigated in this framework.

A second problem associated with estimating the dual approach is model coherency. For a model to be well-behaved, there must exist a one-to-one mapping between all feasible realizations of the stochastic components of the model and all possible vectors of demand (Pitt and Millimet 2001). Unfortunately, coherency is not guaranteed over the entire parameter space of the model. In such cases where the parameters of the model stray outside the coherent region, the sum of the probabilities for all demand regimes is not unity and maximum likelihood estimates are inconsistent (van Soest et al.1993).

In this section I will review Lee and Pitt's [1986] original paper which advocates the virtual price approach, then review the applications of the dual approach using the integration method (Lee and Pitt [1987], Winer and Srinivasan [1994] and Phaneuf [1997,1999]). The section follows with discussion of the alternative approaches to the dual method (Heien and Wessells [1990] and the Amemiya-Tobin approach), The section ends with a description of the newly proposed simulation method from Pitt and Millimet [2001].

### 2.2.1 Original Paper

Let  $H(v; \theta, \varepsilon)$  be an indirect utility function defined as

$$H(v; \theta, \varepsilon) = \max_y \{U(y; \theta, \varepsilon \mid vy = 1)\}, \quad (2.34)$$

where  $U(\cdot)$  is a strictly quasi-concave utility function defined on  $J$  commodities,  $v$  is a vector of income normalized market prices,  $\theta$  is a vector of unknown parameters, and  $\varepsilon$  is a vector of random components. The *notional* demand equations  $D(v; \theta, \varepsilon)$  for the  $J$  commodities, via Roy's Identity, are

$$y_j = \frac{\partial H(v; \theta, \varepsilon)}{\partial v_j} / \sum_{j=1}^J v_j \frac{\partial H(v; \theta, \varepsilon)}{\partial v_j} \quad j=1, \dots, J. \quad (2.35)$$

The *notional* demand equations are demand equations derived without account for non-negativity constraints. At this point the virtual prices are introduced. Virtual prices are the reservation prices at which the individual is indifferent between zero consumption and positive consumption. If the market price is higher than the virtual price for a good, the individual will consume zero; if the market price is lower than the virtual price for a good, the individual will end up with a positive consumption. Without loss of generality, assume the demands for the first  $r$  goods are zero. The virtual prices  $\pi_j(v_{r+1}, \dots, v_J)$  are solved by setting the first  $r$  demand equations to zeros:

$$0 = \partial H(\pi_1(\bar{v}), \dots, \pi_r(\bar{v}), \bar{v}; \theta, \varepsilon) / \partial v_j \quad j=1, \dots, r, \quad (2.36)$$

where  $\pi_j(\bar{v})$  is the virtual price of the  $j^{\text{th}}$  good and  $\bar{v}$  is the set of market prices of the positively consumed goods  $r+1$  to  $J$ . The remaining actual demands for  $j=r+1, \dots, J$  are

$$y_j = \frac{\partial H(\pi_1(\bar{v}), \dots, \pi_r(\bar{v}), \bar{v}; \theta, \varepsilon)}{\partial v_j} / \sum_{j=1}^J v_j \frac{\partial H(\pi_1(\bar{v}), \dots, \pi_r(\bar{v}), \bar{v}; \theta, \varepsilon)}{\partial v_j}. \quad (2.37)$$

The demand regimes are determined by comparing virtual prices and market prices. The regime in which the first  $r$  goods are not consumed is characterized by the conditions

$$\pi_j(\bar{v}) \leq v_j \quad j=1, \dots, r, \quad (2.38)$$

which are theoretically equivalent to the Kuhn-Tucker conditions.

#### *Derivation of econometric model*

Most applications of the Lee and Pitt approach assume that the indirect utility function is weakly separable in the goods of interest and all other goods. A two-stage budget process is the natural result, where in the first stage individuals allocate expenditure between the goods of interest and all other goods, and in the second stage the expenditures on the goods of interest are allocated among the available goods in this group. Applications focus on the second stage of the budgeting process and assume the expenditure is predetermined.

Consider the translog indirect utility function of Christensen et al. [1975] for the subutility function

$$H(v; \theta, \varepsilon) = \sum_{j=1}^J (\alpha_j + \varepsilon_j) \ln v_j + 1/2 \sum_{j=1}^J \sum_{k=1}^J \beta_{kj} \ln v_k \ln v_j, \quad (2.39)$$

where  $v$  is a vector of normalized market prices,  $\theta$  is a vector of unknown parameters, and  $\varepsilon$  is a  $J$ -dimensional vector of normal variables with distribution  $\phi(0, \Sigma)$ , constrained such that

$$\sum_{j=1}^J \varepsilon_j = 0. \quad (2.40)$$

Following Christensen et al. [1975], the normalization

$$\sum_{j=1}^J \alpha_j = -1, \quad (2.41)$$

is maintained which results in the expenditure shares adding up to 1.

As usual the equality and symmetry conditions are enforced, i.e.,

$$\sum_{j=1}^J \beta_{IJ} = \sum_{k=1}^J \beta_{IK} \quad j, k = 1, \dots, n \text{ (equality)}, \quad (2.42)$$

and

$$\beta_{jk} = \beta_{kj}, \quad \forall k, j; k \neq j \text{ (symmetry)}. \quad (2.43)$$

With the additional assumption of homogeneity of the utility function, the following restriction is also enforced:

$$\sum_{j=1}^J \beta_{kj} = 0. \quad (2.44)$$

From (2.42) and (2.44) we get

$$\sum_{k=1}^J \beta_{kj} = \sum_{j=1}^J \beta_{kj} = 0 \text{ (homogeneity)}. \quad (2.45)$$

By applying Roy's Identity to above sub-utility function, we obtain the *notional* share functions as:

$$v_j y_j = \frac{\alpha_j + \sum_{k=1}^J \beta_{kj} \ln v_k + \varepsilon_j}{-1 + \sum_{j=1}^J \sum_{k=1}^J \beta_{kj} \ln v_j}. \quad (2.46)$$

With the homogeneity restriction, the denominator drops out and the above equation reduces to

$$v_j y_j = -(\alpha_j + \sum_{k=1}^J \beta_{kj} \ln v_k + \varepsilon_j). \quad (2.47)$$

Without loss of generality, assume the first  $r$  goods are not consumed, and set the above *notional* shares for the first  $r$  goods to zeros and solve the virtual prices as

$$\begin{bmatrix} \ln \pi_1 \\ \cdot \\ \cdot \\ \cdot \\ \ln \pi_r \end{bmatrix} = -B_r^{-1} \begin{bmatrix} \alpha_1 + \sum_{j=r+1}^J \beta_{1j} \ln v_j + \varepsilon_1 \\ \cdot \\ \cdot \\ \alpha_r + \sum_{j=r+1}^J \beta_{rj} \ln v_j + \varepsilon_r \end{bmatrix} \quad (2.48)$$

where

$$B_r = \begin{bmatrix} \beta_{11} & \cdots & \beta_{1r} \\ \vdots & & \vdots \\ \beta_{r1} & \cdots & \beta_{rr} \end{bmatrix}. \quad (2.49)$$

Since the first  $r$  goods are not consumed their market prices must exceed their virtual prices. That is

$$\begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_r \end{bmatrix} \geq \begin{bmatrix} t_1 \\ \vdots \\ t_r \end{bmatrix}, \quad (2.50)$$

where

$$\begin{bmatrix} t_1 \\ \cdot \\ \cdot \\ t_r \end{bmatrix} = - \begin{bmatrix} \alpha_1 + \sum_{j=r+1}^J \beta_{1j} \ln v_j \\ \cdot \\ \alpha_r + \sum_{j=r+1}^J \beta_{rj} \ln v_j \end{bmatrix} + B_r \begin{bmatrix} \ln v_1 \\ \cdot \\ \cdot \\ \ln v_r \end{bmatrix} \quad (2.51)$$

For the positively consumed  $J-r$  goods, substitute the virtual prices into the remaining Roy's identity equations (2.47) to get the observed expenditure shares for the positively consumed goods:

$$v_j y_j = -(\alpha_j + \sum_{k=1}^r \beta_{jk} \ln \bar{\pi}_k + \sum_{k=r+1}^J \beta_{jk} + \varepsilon_j + \eta_j), \quad j=r+1, \dots, J-1, \quad (2.52)$$

where

$$\eta_j = B_r^j B_r^{-1} \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_r \end{bmatrix}, \quad (2.53)$$

$B_r^j = (\beta_{j1}, \dots, \beta_{jr})$ ,  $s_j$  is the observed expenditure share for the  $j^{\text{th}}$  good,  $\bar{\pi}_j$  is the deterministic component of the virtual price in (2.48), and  $j=r+1, \dots, J-1$  since  $J^{\text{th}}$  share is not independent, i.e. the sum of shares should be equal to unity.

Rearranging (2.52) we can obtain

$$\varepsilon_j = t_j, \quad j=r+1, \dots, J-1, \quad (2.54)$$

where

$$t_j = s_j - (\alpha_j + \sum_{k=1}^r \beta_{kj} \ln \bar{\pi}_j + \sum_{k=r+1}^J \beta_{kj} \ln v_j + \eta_j). \quad (2.55)$$

Equation (2.50) and equation (2.54), along with the joint density function  $f_\varepsilon(\varepsilon)$  for  $\varepsilon$ , the likelihood function can be derived.

The contribution to likelihood function for an individual who positively consumes  $J-r$  goods is

$$\int_{t_1}^{\infty} \dots \int_{t_r}^{\infty} J(x, \varepsilon_1, \dots, \varepsilon_r) g(\varepsilon_1, \dots, \varepsilon_r) h(\varepsilon_{r+1}, \dots, \varepsilon_r) d\varepsilon_1, \dots, d\varepsilon_r, \quad (2.56)$$

where  $J(\cdot)$  is the Jacobian of the transformation from  $(\varepsilon_{r+1}, \dots, \varepsilon_r)$  to  $(y_{r+1}, \dots, y_r)$ , the  $g(\cdot)$  is a marginal distribution and  $h(\cdot)$  is the conditional distribution.

Because the  $J^{\text{th}}$  share is not independent, there are  $2^{J-1}$  possible demand regimes from which equation (2.56) can be constructed. The likelihood function for an independent sample with  $N$  observation is

$$L = \prod_{i=1}^N \prod_c [I_c(y_i; \theta)]^{I_i(c)} \quad (2.57)$$

where  $I_i(c) = 1$  if demand regime  $c$  is observed for individual  $i$ , zero otherwise, and  $I_c(y_i; \theta)$  is the contribution of individual  $i$  with demand regime  $c$  to likelihood function. The maximum likelihood method can be used to estimate this model.

### 2.2.2 Applications of Traditional Integration Method

One issue in implementing the dual approach is the potential problem of model coherency. For a model to be well-behaved, there must exist a one-to-one mapping between all feasible realizations of the stochastic components of the model and all possible vectors of demand (Pitt and Millimet [2001]). Unfortunately, coherency is not guaranteed over the entire parameter space of the model. In such cases where the parameters of the model stray outside the coherent region, the sum of the probabilities for all demand regimes is not unity and maximum likelihood estimates are inconsistent (van Soest et al. [1993]). The econometric model above cannot guarantee that the model is coherent, i.e. the sum of probabilities of different demand regimes is not always equal to one. Van Soest and Kooreman [1990] propose the following sufficient conditions required for model coherency in the context of translog indirect utility function:

- B positive semidefinite
- $1 - \sum_{j=1}^J \sum_{k=1}^J \beta_{kj} \ln v_j > 0$
- $B\tau \geq 0$

where  $\tau$  is a J by 1 vector with each component 1. The last two conditions are satisfied by model construction, the first condition should be tested to assure the model coherency.

Srinivasan and Winer [1994] use the dual approach to estimate the spatial representation of objectives and preference vectors from actual choice behavior of individuals. A maximum likelihood method for estimating parameters is described and illustrated using choice data of three major ketchup brands collected by the use of electronic supermarket scanners. Wolak [1996] estimate a five good model to analyze the welfare impacts of the price changes in local residential phone service and/or long distance phone service using the dual approach. Since no observations in the sample have more than two zeros, computing the likelihood function for this sample only requires a univariate or bivariate integration, and this is really a two dimension problem.

The virtual price approach was first introduced in the recreation demand literature by Phaneuf [1997, 1999] to estimate fishing demand in the Great lakes regions. The virtual price approach allows a more flexible utility function and general error structures than the KT approach. A translog weakly separable indirect utility function was applied to a four site recreation demand model. To make the estimation feasible an exchangeability assumption is assumed in this application, so the full correlation matrix is not estimated.

Since the weakly separable subutility for the recreation activities is assumed, the expenditure for the recreation activities is predetermined in this application. The calculated welfare measures are all conditional on the predetermined expenditure. Since the expenditure share on recreation is only a small part of the individual's total income, the welfare measures are not very useful to policy makers.

Both the above applications impose restrictions on the covariance matrix. As Arndt et al. (1999, p1007) point out “derivations of the reservation prices and then substitution of the analytical solution for the reservation price(s) into the remaining demand system equations implies that the associated covariance matrix,  $\Omega$ , is dense and is a function of the behavioural parameters to be estimated. Consequently, structure on the covariance matrix implies structure on the estimates of behavioural parameters. To avoid this restriction, fully general integration methods are required”.

Lee and Pitt [1987] estimate a translog energy cost function for two Indonesian manufacturing sectors with a sample of firms who may not employ one or more fuels using firm-level data. The authors assume that the production function is weakly separable in energy inputs (purchased electricity, fuel oils and other fuels) and the energy aggregate is homothetic, which leads to a two stage budget procedure. Because the translog cost function does not globally satisfy the concavity property, the sum of the seven demand regime probabilities may not be one. The authors discuss and impose the restrictions on parameters to guarantee the translog cost function is coherent.

### *Summary of implementation challenges*

As mentioned in the introduction section, the Lee and Pitt approach is theoretically attractive since it is consistent with the utility maximum theory, and allows use of more flexible forms than the Wales and Woodland primal approach. Several issues associated with this approach have prevented wider application. The following lists the challenges and unsolved issues of the Lee and Pitt approach:

1. Dimension challenges, i.e. the requirement of multivariate integration to construct the likelihood functions;
2. Potential problem of model coherency, i.e. the Lee and Pitt approach can not guarantee the sum of probabilities for all demand regimes is equal to unity;
3. No income effect, i.e. due to the use of homogeneous utility or cost function in applications, the prices are normalized by the associated expenditures;
4. Assumption of weak separability in the two-step budgeting process, i.e. in applications assuming the expenditure of goods to be investigated is predetermined;
5. Interpretation of results of estimated models.

When applying the Lee and Pitt approach to the recreation demand, there are some specific issues to be resolved:

1. Welfare calculation;
2. incorporating quantities into model consistent with weak complementarity.

Due to the challenges and issues above the Lee and Pitt approach has not been seen wide application. Many papers describe attempts to maintain the attractiveness of the

model while allowing more tractable estimation. There are two notable alternative approaches in literature to dealing with the corner solutions. The modified Heckman two step estimator is discussed in the following section and the Amemiya-Tobin model will be presented in chapter 4.

### 2.2.3 The Modified Heckman Two Step Estimator

The modified Heckman two step method (Heien and Wessells [1990]) accommodates corner solutions at the estimation stage, at which censored regressions or other related methods are employed to accomplish the goal. Binding non-negativity constraints distort the distribution of the error term and the sizes of the error terms are unknown. The basic idea behind the modified Heckman two-step estimator is to estimate the magnitude of this error term in the first step and include the inverse Mills ratio in the second step. Formally,

$$y_j \begin{cases} = \beta' x_j + \varepsilon_j & \text{if } \beta' x_j + \varepsilon_j > 0 \\ = 0 & \text{otherwise, for } j = 1, \dots, J, \end{cases} \quad (2.59)$$

where  $y_j$  is the dependent variable,  $x_j$  is a vector of independent variables,  $\varepsilon_j$  is the random term, and  $\beta$  is the parameter vector to be estimated. The conditional expectation is

$$E(y_j | y_j > 0) = \beta' x_j + \sigma \frac{\phi_j}{\Phi_j}, \quad (2.60)$$

where  $\phi_j$  and  $\Phi_j$  are the probability and cumulative density function of standard normal

evaluated at  $\frac{\beta' x_j}{\sigma}$  and  $\sigma$  is the variance to be estimated. Economic theory indicates that it is

the virtual price, not the market price, that determinates the quantity demanded. For the non-consumed goods the market prices are irrelevant. If the virtual price is the relevant

price in individual's decision making process, the virtual prices should be the prices in the estimation process. Because the modified Heckman's two step estimator does not take into account the virtual price, the estimation is not consistent with economic theory. Arndt et al.[1999] compare the Lee and Pitt approach and the modified Heckman's two steps estimator using simulated data. Their results show that the modified Heckman two step estimator performs poorly due to the failure to account for the virtual price.

#### *2.2.5 Pitt and Millimet Approach*

Two issues in implementing the dual approach are the requirement of multivariate integration and potential problem of model coherency. Pitt and Millimet [2001] propose an econometric treatment to reduce the computational burden that is close to the original Lee and Pitt approach but uses simulation and sampling approaches to solve the dimension challenges and potential problem of model coherency. If the indirect utility function is not concave at each observation the sum of probabilities for all demand regimes is not equal to one and hence the maximum likelihood estimate is not consistent. Pitt and Millimet [2001] suggest that Gibbs sampling, along with the data augmentation and rejection sampling can solve both the dimensionality and coherency problems. Given the simple latent structure of the demand system, augmenting the data removes the need to directly evaluate any probability integrals. With binding non-negativity constraints the shares for the non-consumed goods are augmented by simulating the latent shares conditional on the observed data and initial estimates of the model. The data augmentation algorithm for multinomial probit model is demonstrated in McCulloch and Rossi [1994]. The advantage of rejection

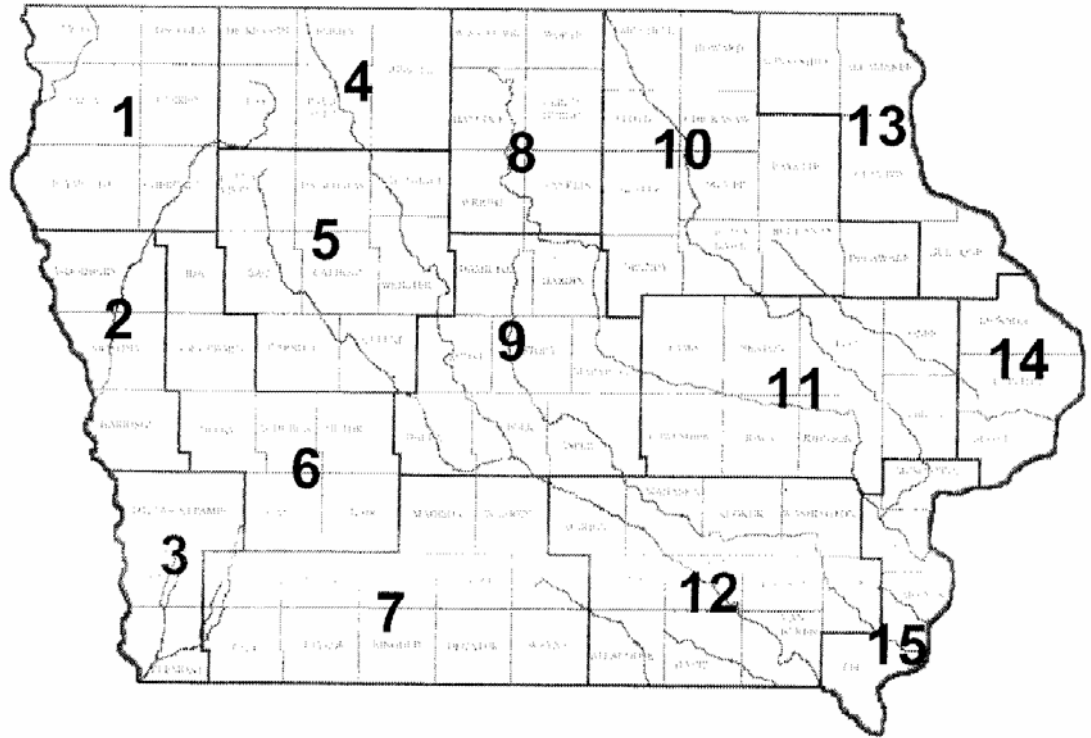
sampling is that local coherency rather than global coherency is imposed. The intuition behind imposition of the local coherency rather than global coherency is that the global coherency places unreasonable restrictions on the parameter space in many applications. As shown in Terrell [1996], rejection sampling imposes only the minimum set of restrictions required for model coherency in the sense that parameter draws are rejected only if one observation violates the concavity conditions.

Pitt and Millimet implement the simulation approach with both simulated data and household-level data on food consumption from Indonesia. The results confirm the accuracy of the Gibbs sampler estimates and highlight the importance of addressing the problem of coherency. In all of the models estimated, the parameter vector frequently enters the space of incoherent values and rejecting on the basis of global coherency significantly restricts the acceptable parameter space relative to the model imposing coherency locally. The application with the Indonesia food demand shows that important differences in measured price responses arise between imposing the local coherency and global coherency conditions, even in low dimensional problems with a small number of binding non-negativity constraints.

### **Chapter 3: Data**

The data analyzed in this dissertation come from the 1997 Wetlands Survey conducted by researchers at Iowa State University. The purpose of the survey was to understand how Iowans value services from wetlands and to examine the use and value of wetlands for recreation. The survey collected information about how residents use wetlands in the state, as well as information on the value that Iowans place on wetland preservation and restoration. Questions providing data on both actual and hypothetical uses of wetlands, as well as contingent valuation questions, are included in the survey. Socioeconomic information on income, gender, education, etc. was also collected.

To gather information both from visitors and non-visitors to the wetland area, a sample of the general population in the state and holders of Iowa hunting/fishing licenses was drawn. In this dissertation I focus only on questions soliciting information on actual visits to wetland areas in the state. The researchers divided Iowa into 15 zones with a map (see Figure 1) and asked respondents to list the number of visits to each zone during 1997. The 15 zones allow aggregation to 5 sites which exhibit similar geographical characteristics and form the basic choice site definition for much of this dissertation. Corresponding to the above map, the aggregate sites are grouped as {1,2,3}, {4,5,8}, {6,7,12}, {9,10,11} and {13,14,15}. Sites {1,2,3} and sites {13,14,15} are the east and west riverine wetland regions, and sites {4,5,8} is the Prairie Pothole Region. Of the 6000 surveys sent out 2891 households returned usable surveys. The information on respondents' trips to the 15 wetlands zones of Iowa during 1997 are used in the current application.



**Figure 1: Iowa Wetland Zones**

In the travel cost model the price of a visit to each site consists of the round trip travel costs and opportunity cost of travel time. The round trip travel cost was calculated as the estimated round trip distance to each site multiplied by \$0.21; the opportunity cost of travel time was measured as the estimated travel time valued at one third of the average wage rate. PCMiller was used to estimate separate travel distances and times to each of the 15 sites from each respondent's home.

Descriptive statistics for the five aggregate sites and the 15 disaggregate sites are provided in Table 3.1 and Table 3.2, respectively. On average the survey respondents took 8.51 trips to Iowa wetlands during 1997. Among them 924 respondents are non-users of the

sites and 74 respondents made more than 40 trips. These statistics show that participation in wetland visitation varies considerably among Iowans. The average travel cost to the five zonal sites is about \$115, and \$120 to 15 zonal sites.

Table 3.3 and Table 3.4 report the number of different sites visited by Iowans. There is an extremely large number of corner solutions in this sample. For the five site specification 55% of the Iowa wetlands users visit only one site, 31% visit two sites, 11% visit three sites, 3% visit four sites, and less than one percent visit all five sites. For the fifteen site specification no person visits more than ten sites during 1997.

Descriptive statistics for the socioeconomic variables are provided in the Table 3.5. It shows that the survey respondents have an average income of \$43,266. About three quarters of the respondents are male. The average age for the sample is about 49 years old. Other information shows that three quarters of the sample has a hunting or fishing permit and a quarter of the sample has a four-year college degree.

Site characteristic variables are used in recreation demand studies to gauge the importance of amenities in explaining visitation. The quality variables representing sites' characteristic in this application are pheasant counts collected by the Iowa Department of Natural Resources (DNR). The DNR conducts roadside pheasant counts regularly to evaluate the health of the population in the state. Each fall, the DNR surveys fixed-length routes for each county in Iowa, and aggregates the survey findings to form county level indices of pheasant sightings per mile. Since the indices are well publicized we expect them to influence visitation decisions, particularly for pheasant hunters, an important group

of wetland users. The county level indices are averaged to produce zonal indices for each of the fifteen sites. These zonal indices represent the quality variables in the current application. The site specific pheasant count indexes for the five site and the fifteen site choice sets are reported in Table 3.1 and Table 3.2, respectively. Since hunters/fishers are more likely to be influenced by game species such as pheasants, for estimation I construct an effective quality variable as the product of site pheasant counts and the hunting/fishing permit dummy variable. This provides variation in the quality variable across individuals.

Several researchers have analyzed this data previously under different modeling frameworks and under different site aggregation levels. Phaneuf and Herriges [1999] investigate the effects on estimates and welfare measures of choice sets representing various levels of site aggregation (i.e., five-site and fifteen-site model) and market scope (i.e., statewide sample and prairie pothole sample) using the Kuhn-Tucker model. The authors found that changing the choice set definition can cause significant differences in welfare measures. For a 20% increase in pheasant counts at the prairie pothole sites the welfare estimates are \$154 for the 15-site application and \$73 for the five-site application, using the full sample. For the loss of prairie pothole region the welfare estimates are \$208 for the 15-site application and \$156 for the five-site application, using full sample. Since weak complementarity is not imposed in these models the calculated welfare measures include both the use value and the nonuse value of the recreation sites.

In a later paper von Haefen and Phaneuf [2003] compare the continuous and count data demand system frameworks and conclude that although the two frameworks differ

significantly conceptually, if the analyst uses comparable preference structures and controls for the difficulties with respect to prediction of observed behavior, similar policy inference could be reached. The Kuhn-Tucker (continuous) model and negative binomial (count data) model are estimated for the 15-site application. Two welfare scenarios are considered, namely, a 20% increase in pheasant counts at prairie pothole sites and a \$50 increase in access fee at the east and west riverine wetland regions. For the pheasant count scenario the welfare estimates are \$14.22 for the Kuhn-Tucker model and \$11.45 for the negative binomial model. For the access fee scenario the welfare estimates are -\$37.66 for the Kuhn-Tucker model and -\$57.21 for the negative binomial model. Since the authors imposed weak complementarity on both models, the welfare measures calculated include only use values of the recreation sites, and therefore are not directly comparable to Phaneuf and Herriges [1999].

Herriges and Phaneuf [2002] examine the ability of repeated nested logit model (RNL) and repeated mixed logit model (RXL) to capture patterns of correlation and substitution in recreation demand applications. The authors use RXL model to address criticism of the RNL through introducing more general correlation structure among available alternatives and correlation across choice occasions. The authors also explore the economic implications of the alternative error specifications. The empirical results indicate significant increases in the richness of site substitution patterns captured via inclusion of richer patterns of error correlation. Both the RNL and the RXL models are estimated for a five-site application and estimated results and corresponding elasticities are reported.

Herriges, Kling and Phaneuf [2003] consider welfare estimates from revealed preference models when weak complementarity (WC) does not hold. Within the Kuhn-Tucker framework, which starts with individual's direct utility function, it is possible to specify a utility function without imposing the WC assumption. Through a decomposition of the total value associated with changes in the quality variables, use value can be recovered. Only the prairie pothole sites are included in this application. Welfare measures for a 20% increase in pheasant counts at all sites are considered. The use value associated with this change in site quality is \$103 for the utility function with WC imposed, and \$88 for the model where the use value is calculated as a component of total value.

The most frequently used model in the above applications is the Kuhn-Tucker model (Phaneuf et al. [2000]). As mentioned in literature review, although the KT model is theoretically consistent, the dependence on explicitly specified first order conditions restricts flexibility in the functional form used (eg., the additively separable utility function). The restrictive assumption on the error terms (i.e., GEV random variables) makes estimation feasible, but limits the stochastic substitution patterns across sites. The Amemiya-Tobin model and the virtual price model in this dissertation employ the incomplete demand system with the linear demand function and the normal distribution as the underlying distribution of the error terms, and hence allow both richer stochastic and parametric substitution possibility across sites than the KT model. This is demonstrated in the following two chapters.

## Chapter 4: Amemiya-Tobin Approach

In this chapter, the Amemiya-Tobin (AT hereafter) approach for the incomplete demand system is introduced. The section starts with the ideas behind the incomplete demand system and the restrictions imposed on the specification of demand equations to recover the quasi-indirect utility function. Then derivation of the estimable equations and the likelihood functions are presented. The strategy for estimation using simulation methods is outlined and implemented. The results of main model and its variations, as well as welfare measures, are reported.

### 4.1 The Incomplete Demand System

To consistently model the demand for a subset of consumed goods within a demand system context, researchers have several alternative approaches. The first approach assumes that the individual's utility function is weakly separable in the goods of interest and all other goods. A two-stage budget process is the natural result, where in the first stage individuals allocate expenditures between the goods of interest and all other goods, and in the second stage the expenditures of goods of interest are allocated among the available alternatives in this group. Applications of such a strategy focus on the second stage of the budgeting process and assume the expenditure is predetermined (Winer et. al [1994] and Phaneuf [1999]). The second approach constructs a single *Hicksian* composite good for all other goods and maximizes the utility subject to a linear budget constraint ( von Haefen and Phaneuf [2002]). The current application adopts the third alternative approach, the incomplete demand approach, which specifies “a demand system for the goods of interest

as functions of their own prices, total income, and the other goods' prices that are assumed quasi-fixed." (von Haefen [2002]).

The incomplete demand system is especially attractive in the recreation demand context. If parameter estimates of the incomplete demand system satisfy the weakly integrable conditions (see later explanations) the underlying quasi-indirect utility function can be recovered. Analysts can construct exact welfare measures based on the indirect utility function. Another desirable aspect of the incomplete demand system is that the calculated welfare measures are conditional on the household's total income instead of the predetermined share of expenditure of the goods interested (e.g. the weakly separable approach). This is especially important in the context of recreation demand analysis, since the expenses on recreation only account for a small share of total income.

The incomplete demand approach starts with specifying the system of Marshallian demand functions. Suppose that consumer demand for a set of  $J$  alternative choices with  $N$  individuals in the demand system could be expressed by the following system of Marshallian demand functions:

$$y_{ij} = \alpha_j(z_i) + \sum_{k=1}^J \beta_{jk} p_{ik} + \gamma_j m_i \quad j = 1, 2, \dots, J, \quad i = 1, 2, \dots, N, \quad (4.1)$$

where  $y_{ij}$  is the number of trips by individual  $i$  to site  $j$ ,  $p_{ik}$  is the travel cost for individual  $i$  to site  $k$ ,  $z_i$  is a vector of prices of all other goods for individual  $i$ , and  $m_i$  is the individual's total income. In the current application the intercept contains the quality variables in the above demand equations.

In order to construct consistent Hicksian welfare measures using equation (4.1), the system must be consistent with a rational individual who maximizes his/her utility subject to a linear budget constraint. This is the integrability problem in the incomplete demand system. Lafrance and Hanemann's [1989] concept of weak integrability requires the minimal set of assumptions to generate exact welfare measures for price changes. Four conditions for an incomplete demand to be weakly integrable are given in Lafrance and Hanemann (1989) as follows:

1.  $y_{ij}$  is homogeneous degree zero in prices and income;
2.  $y_{ij}$  is nonnegative,
3. Expenditures on the  $J$  goods explicitly specified are strictly less than total income,
4. The Slutsky matrix is symmetric, negative semidefinite.

Condition 1 is satisfied by the way the above incomplete demand system is constructed, and the condition 2 and 3 are generally true in the recreational demand context. The condition 4 requires, in an open neighborhood of prices and income, that the Slutsky matrix be symmetric and its eigenvalues are nonpositive.

In the context of the above specification these conditions can be expressed as

$$\beta_{jk} + \gamma_j y_{ik} = \beta_{kj} + \gamma_k y_{ij}, \quad k, j \in 1, \dots, J; k \neq j, i = 1, 2, \dots, N. \quad (4.2)$$

The parameter restrictions that satisfy Slutsky symmetry for the above incomplete demand system are given in Lafrance [1985, 1986, 1990] as follows

- $\beta_{jk} = \beta_{kj}, k, j \in \{1, \dots, J\}, k \neq j,$

- $\gamma_j = 0, j \in \{1, \dots, J\}$ .

That is, the incomplete demand system becomes

$$y_{ij} = \alpha_j(z_i) + \sum_{k=1}^J \beta_{jk} p_{ik}, \quad j \in \{1, \dots, J\}. \quad (4.3)$$

The quasi-indirect utility function (Hausman [1981]) can be recovered by solving a series of partial differential equations. The quasi-indirect utility functions for the above incomplete demand system is given by von Haefen [2002]:

$$\Phi(p, q, m) = m_i - \sum_{k=1}^J \alpha_k(z_i) p_{ik} - \frac{1}{2} \sum_{j=1}^J \sum_{k=1}^J \beta_{jk} p_{ij} p_{ik}. \quad (4.4)$$

With this quasi-indirect utility function, exact welfare measures can be constructed.

#### 4.2 General Structure of the Amemiya-Tobin Model

In recreation demand applications most individuals are observed to visit only a subset of the  $J$  alternative choices, i.e., corner solutions dominate recreation demand data. In this dissertation I investigate two alternative approaches to solve the corner solutions problem: the AT approach and the virtual price approach. The major difference separating the AT approach and the virtual price approach is whether the reservation or choke price plays a role in the individual's decision making process. Economic theory tells us that it is the comparison between the reservation price and the market price that determines whether an individual consumes a good or not. The AT approach ignores the behavioral explanation for corner solutions and address corners statistically in the estimation process, while the virtual price approach treats corner solutions in an economic manner, explaining behaviorally and statistically why an individual has zero consumption.

The following simple primal model illustrates the difference between the AT approach and the virtual price approach to modeling zeros. In the AT model an individual maximizes utility  $U(x)$  subject to a linear budget constraint

$$\text{Max}_x U(x) \quad \text{s.t.} \quad m = p'x, \quad (4.5)$$

where  $p$  is a vector of the prices and  $m$  is income. The first order conditions are equality constraints:

$$\frac{\partial U(x)}{\partial x_j} = \lambda p_j, \quad (4.6)$$

where  $\lambda$  is the marginal utility of income. The first order conditions are solved for an interior solution to get the corresponding demand functions

$$x = x(p, y). \quad (4.7)$$

Estimation proceeds by attaching an error term to the functional forms for an interior solution given in equation (4.7). Thus, no mention is given to corner solutions in the economic model. If demand is observed to be zero, the estimator is adjusted to ensure consistent parameter estimate. This statistical approach to dealing with corners is described below.

In contrast, in the virtual price approach an individual maximizes utility subject to a linear budget constraint and nonnegativity constraints:

$$\text{Max}_x U(x) \quad \text{s.t.} \quad m = p'x, \quad x \geq 0. \quad (4.8)$$

The first order conditions imply

$$\frac{\partial U(x)}{\partial x_j} \leq \lambda p_j, \quad (4.9)$$

where  $\lambda$  is the marginal utility of income. Define the virtual or reservation price by

$$\pi_j = \frac{\partial U(x)}{\partial x_j} / \lambda. \quad (4.10)$$

Thus, equation (4.9) becomes

$$p_j \geq \pi_j. \quad (4.11)$$

From equation (4.11) it is clear that non-consumption occurs when the market price is higher than the virtual price. After manipulating the first order conditions, we obtain the solution

$$\begin{aligned} x_j &= x_j(p, m), \quad \text{if } x_j > 0; \\ p_j &\geq \pi_j \quad \quad \quad x_j = 0. \end{aligned} \quad (4.12)$$

Estimation of the virtual price approach describes both components of the solution in equation (4.12), and thus behaviorally and statistically addresses corner solutions. This is the major difference between the AT and virtual price approaches. The details of the virtual price approach are given in chapter 5.

In this chapter the incomplete demand system is estimated in the context of the AT approach. As mentioned earlier we observe only nonnegative outcomes for recreation demand, i.e., the recreation demand data are the result of censoring. In the AT approach corners are simply censored variables which are treated as result of censoring in a statistical manner. If there are no correlations among the error terms of the available alternatives and

no cross-equation restrictions on parameters, the model could be estimated by fitting separate Tobit equations for each alternative. If the error terms are correlated across available alternatives and/or there are cross-equation restrictions on the parameters, which are commonly needed for structural models, the AT approach is more efficient since a system of equations is estimated simultaneously.

Following the notation in Huang [2001], suppose that there are  $J$  alternative choices with  $N$  individuals in the demand system. To simplify notation let  $x_{ij}$  represent the independent variables and  $\theta$  be the parameters to be estimated on the right hand side of equation (4.1). The latent demand of the  $i^{\text{th}}$  individual for the  $j^{\text{th}}$  alternative choice,  $y_{ij}^*$ , can be written as

$$y_{ij}^* = x_{ij}' \theta_j + \varepsilon_{ij}, \quad 1 \leq i \leq N, \quad 1 \leq j \leq J. \quad (4.13)$$

The relationship between the latent demand and the observed demand can be expressed as

$$y_{ij} = \begin{cases} y_{ij}^* & \text{if } y_{ij}^* > 0 \\ 0 & \text{otherwise,} \end{cases} \quad (4.14)$$

where  $y_{ij}^*$  is the latent variable, and  $y_{ij}$  is the observed variable.

The system of equations can be written as:

$$\begin{bmatrix} y_{1i}^* \\ y_{2i}^* \\ \cdot \\ \cdot \\ y_{Ji}^* \end{bmatrix} = \begin{bmatrix} x_{1i}' & 0 & 0 & \cdot & 0 \\ 0 & x_{2i}' & 0 & \cdot & 0 \\ 0 & 0 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & \cdot & 0 & x_{Ji}' \end{bmatrix} \times \begin{bmatrix} \theta_1 \\ \theta_2 \\ \cdot \\ \cdot \\ \theta_J \end{bmatrix} + \begin{bmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \\ \cdot \\ \cdot \\ \varepsilon_{Ji} \end{bmatrix}, \quad (4.15)$$

or

$$y_i^* = X_i \theta + \varepsilon_i, \quad i = 1, 2, \dots, N, \quad (4.16)$$

where  $\varepsilon_i = (\varepsilon_{1i}, \varepsilon_{2i}, \dots, \varepsilon_{ji})' \sim iid \phi(0, \Sigma)$ , and  $\phi$  is the normal density function. The dimension

of  $\theta_j$  is  $k_j \times 1$ ,  $\Sigma$  is the  $J \times J$  covariance matrix,  $y_i^* = (y_{1i}^*, y_{2i}^*, \dots, y_{ji}^*)'$ ,

$X_i = diag(x'_{1i}, x'_{2i}, \dots, x'_{ji})$ , and  $\theta = (\theta'_1, \theta'_2, \dots, \theta'_j)$  is a  $k \times 1$  vector with  $k = \sum_{j=1}^J k_j$ .

With corner solutions present there are a total of  $L$  possible combinations of censored and uncensored outcomes; i.e., there are  $L$  vectors of  $S_k, k = 1, 2, \dots, L$ , where  $L = 2^J$  and each  $S_k$  represents a different demand regime. Let  $S = \{S_1, S_2, \dots, S_L\}$  and

$$S_1 = (\underbrace{0, 0, \dots, 0, 0}_J)', \quad \dots, \quad S_h = (\underbrace{0, \dots, 0}_r, \underbrace{+, \dots, +}_{J-r})', \quad \dots, \quad S_L = (\underbrace{+, +, \dots, +, +}_J)',$$

where '+' positive amount and '0' censored at 0.

If the individual visits all the alternative sites, i.e. no corner solutions are present, the likelihood function is the multivariate normal distribution function  $\phi$  :

$$f_i^L(y_i^*; \theta, \Sigma) = (2\pi)^{-n/2} |\Sigma|^{-1/2} \exp(-.5(y_i^* - X_i \theta)' \times \Sigma^{-1} \times (y_i^* - X_i \theta)), \quad (4.17)$$

where  $y_i^*$  and  $y_i$  are identical in this case.

The likelihood function of the  $i^{\text{th}}$  individual in the demand regime  $S_h$  can be expressed as

$$f_i^{S_h}(y_i^*; \theta, \Sigma) = \int_{-\infty}^{-x_{1i}\theta_1} \dots \int_{-\infty}^{-x_{ri}\theta_r} f(y_i^*; \theta, \Sigma) d\varepsilon_{r1} \dots d\varepsilon_{1i}, \quad (4.18)$$

where  $f(y_i^*; \theta, \Sigma) = (2\pi)^{-J/2} \Sigma^{-1/2} \exp(-.5(y_i^* - X_i \theta)' \times \Sigma^{-1} \times (y_i^* - X_i \theta))$ .

The likelihood function for an independent sample with N observations is

$$L(Y; \theta, \Sigma) = \prod_{i=1}^N \prod_{S_h} [f_i^{S_h}(y_i^*; \theta, \Sigma)]^{I_i(S_h)} \quad (4.19)$$

where  $I_i(c) = 1$  if demand regime  $S_h$  is observed for individual  $i$ , zero otherwise,

$f_i^{S_h}(y_i^*; \theta, \Sigma)$  is the contribution of individual  $i$  with demand regime  $S_h$  to the likelihood function, and  $Y = (y_1', y_2', \dots, y_N')$ . Parameter estimates can be obtained through maximizing the above likelihood function.

### 4.3 Estimation via Gibbs Sampling with Data Augmentation

Although the usefulness of the AT model is widely recognized, few applications exist due to the difficulties of computing multiple dimension integrals to obtain the maximum likelihood estimates. Currently available computer packages are not able to evaluate multivariate normal distribution integrals higher than a handful of dimensions, and even three dimensions becomes difficult.

A new and promising method to avoid evaluation of multivariate normal integrals is the Gibbs sampler with data augmentation. The Gibbs sampler is a popular technique for generating random variables from a distribution indirectly by a sequence of easier calculations, without having to calculate the original density. Though most existing applications focus on Bayesian models (e.g., Albert and Chib [1993] and Chib [1995]), the Gibbs sampler also shows promise for classical likelihood calculations (Hajivassiliou and Macfadden [1998]). As we will see next, through the use of a Gibbs sampler, we are able to recover the parameters of the AT model without evaluating multiple dimension integrals.

### 4.3.1 Overview of the Gibbs Sampler

The Gibbs sampler is an algorithm for generating random variables from a multivariate distribution by using its set of full conditional distributions. Geman and Geman [1984] consider simulation from Gibbs distribution using sequential simulations from full conditional distributions and Gelfand and Smith [1990] argue that the Gibbs sampler has general potential for more conventional statistical problems. The sampling-based methods (in particular the Gibbs sampler) are attractive since these methods are conceptually simple and easy to implement even without numerical analytic expertise.

I illustrate the general picture of the Gibbs sampler using an example of three random variables,  $U_1$ ,  $U_2$  and  $U_3$ . Densities are denoted generically by brackets, so joint, conditional, and marginal forms appear as  $[U_1, U_2]$ ,  $[U_1 | U_2]$ , and  $[U_1]$ , respectively. The process of marginalization (i.e., integration) is denoted as  $[U_1] = \int [U_1 | U_2] \times [U_2]$ , where all variables appearing in the integrand but not in the resulting density have been integrated out. Thus the integration is with respect to  $U_2$ . In the case of three random variables, although we want to simulate from  $[U_1, U_2, U_3]$ , drawing variables directly from the joint distribution may be either impossible or expensive which prohibits us from using direct drawing. In general, we are interested in the marginal distributions from which we can make inference about parameters of interest. Using the above notation the marginal distributions are written as:

$$[U_1] = \int [U_1 | U_2, U_3] \times [U_3 | U_2] \times [U_2]$$

$$[U_2] = \int [U_2 | U_1, U_3] \times [U_1 | U_3] \times [U_3]$$

$$[U_3] = \int [U_3 | U_2, U_1] \times [U_2 | U_1] \times [U_1].$$

The full conditional distributions alone, i.e.,  $[U_1 | U_2, U_3]$ ,  $[U_2 | U_1, U_3]$  and  $[U_3 | U_2, U_1]$ , convey all the information needed to uniquely determine the joint distributions and the marginal distributions (Gelfand and Smith [1990]). Gibbs sampling is a Markovian updating scheme that sequentially draws random variables from the full conditional distributions. The three-variable case proceeds as follows. Given a set of starting values  $U_1^{(0)}$ ,  $U_2^{(0)}$  and  $U_3^{(0)}$ , one draws  $U_1^{(1)} \sim [U_1 | U_2^{(0)}, U_3^{(0)}]$ ,  $U_2^{(1)} \sim [U_2 | U_1^{(1)}, U_3^{(0)}]$ ,  $U_3^{(1)} \sim [U_3 | U_1^{(1)}, U_2^{(1)}]$ , which concludes one sequence for the three-variable case. The process is repeated a large number of times until convergence is achieved. After  $t$  such iterations we would arrive at  $(U_1^{(t)}, U_2^{(t)}, U_3^{(t)})$ . Under mild conditions,

$(U_1^{(t)}, U_2^{(t)}, U_3^{(t)}) \xrightarrow{d} [U_1, U_2, U_3]$  and hence for each  $s$ ,  $U_s^{(t)} \xrightarrow{d} U_s \sim [U_s]$  as  $t \rightarrow \infty$  (Geman and Geman [1984] and Gelfand and Smith [1990]).

The above algorithm indicates that the Gibbs sampler requires only the full conditional distributions to determine the joint distribution, while other sampling approach, such as substitution sampling, requires all six conditional distributions including the full conditional distributions to determine the joint distribution. Since the Gibbs sampler can extract the marginal distributions which jointly converge to the true joint distribution we can use the properties of the marginal distributions, such as the mean and variance, to draw inference about the parameter estimates.

The term data augmentation refers to methods for constructing iterative algorithms via the introduction of unobserved data or latent variables (van Dyk and Meng [2001]). The method was popularized in the statistical literature by Tanner and Wong's [1987] data augmentation algorithm for posterior sampling for stochastic algorithms. The introduction of the unobserved data through data augmentation makes the observed data easier to analyze. Data augmentation approaches (Chib[1992] and Albert and Chib [1993]) have been shown useful for analyzing limited dependent variables models, such as Probit and Tobit models. The recreation demand data are censored at zero, i.e., the nonconsumption might mean a negative demand or zero. Through data augmentation the unobserved negative latent demands are constructed, making the Gibbs sampler feasible and simple. The AT model assumes the latent recreation demand data have a multivariate normal distribution. Values for the censored observations can be drawn from suitable truncated normal distributions (see equation (4.21) for details). Given the latent data, the values of the parameters are updated by drawing from the proper underlying distribution (see equations (4.25) and (4.29) for details). Finally, the parameters are used to sample new latent data, and the process is iterated within the Gibbs sampler.

#### *4.3.2 Details on Implementation of Gibbs Sampler with Data Augmentation*

In the current application, the desired density function to be sampled is  $f(\theta, \Sigma | y_i)$ , corresponding to the above likelihood function, where  $y_i$  is the observed demand. After data augmentation, the actual density is sampled from  $f(\theta, \Sigma | y_i^*)$ , where  $y_i^*$  is the latent demand. Because sampling from the conditional density is much more convenient than

from the full density function, the following sequential steps are taken at iteration  $t$  (Pitt and Millimet [2001]):

1. Augment Demand  $y_i^*$  given  $\theta, \Sigma$  and  $y$ , i.e., draw  $y_i^{*t} | \theta^{t-1}, \Sigma^{t-1}$ ;
2. Update  $\Sigma$  given  $y_i^*$  and  $\theta$ , i.e., draw  $\Sigma^t | \theta^{t-1}, y^{*t}$ ;
3. Update  $\theta$  given  $y_i^*$  and  $\Sigma$ , i.e., draw  $\theta^t | \Sigma^t, y^{*t}$ ;

These three steps are repeated until certain convergence conditions are met (Hajivassiliou and Mcfadden [1998]). In practice I assume convergence is achieved after  $\ln N$  iterations. In the following subsections I describe each step in detail.

#### *Obtaining Starting Values of Gibbs Sampler*

Because the covariance matrix is not known the starting values for the Gibbs sampler are obtained using Seemingly Unrelated Regression Estimation (SURE hereafter) through the Feasible Generalized Least Squares (FGLS hereafter) procedure. To ensure weak integrability of the incomplete demand system we must impose the relevant linear restrictions on the FGLS procedure. Estimation of the linear model with exact linear restrictions was implemented as follows (Hirschberg and Slottje [1999]). Suppose that the matrix of parameters,  $\theta$ , have the following structure

$$\begin{bmatrix} \alpha_1 & \gamma_1 & \beta_{11} & \beta_{12} & \beta_{13} \\ \alpha_2 & \gamma_2 & \beta_{21} & \beta_{22} & \beta_{23} \\ \alpha_3 & \gamma_3 & \beta_{31} & \beta_{32} & \beta_{33} \end{bmatrix},$$

and that the linear restrictions that we want to impose are

- $\beta$  matrix is symmetric;

- $\sum_{i=1}^3 \alpha_i = 1$ ;
- $\gamma_1 = \gamma_2 = \gamma_3$ .

We define that  $R\theta - r = 0$  as

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} \alpha_1 \\ \gamma_1 \\ \beta_{11} \\ \beta_{12} \\ \beta_{13} \\ \alpha_2 \\ \gamma_2 \\ \beta_{21} \\ \beta_{22} \\ \beta_{23} \\ \alpha_3 \\ \gamma_3 \\ \beta_{31} \\ \beta_{32} \\ \beta_{33} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Corresponding to equation (4.16), FGLS estimator (Chipman [1989]) of  $\theta$  subject to  $l$  independent linear restrictions

$$R\theta - r = 0$$

is

$$\hat{\theta} = (I - \mathfrak{R}R)\tilde{\theta} + \mathfrak{R}r \tag{4.20}$$

where  $\mathfrak{R} = (X\hat{\Sigma}^{-1}X)^{-1}R'[R(X\hat{\Sigma}^{-1}X)^{-1}R']^{-1}$ ,  $\tilde{\theta} = (X\hat{\Sigma}^{-1}X)^{-1}X\hat{\Sigma}^{-1}Y$ ,  $Y = (y'_1, y'_2, \dots, y'_N)$ ,  $X = (X'_1, X'_2, \dots, X'_N)$ ,  $\hat{\Sigma}$  is estimated variance covariance matrix from OLS,  $l$  is the rank of  $\mathfrak{R}$ ,  $R$  is  $l \times k$  with rank  $l$  (recall  $k$  is the total number of parameters of interest).

This example shows the general techniques on how to impose linear restrictions on the linear model and I apply this technique to a five site model in this dissertation.

### *Augmenting Demand Step*

As mentioned earlier, the major theoretical difference between AT approach and virtual price approach is whether the virtual price plays a role in the individual decision making process. As a consequence of this theoretical difference the data augmentation process is also quite different: in the AT approach only nonconsumed goods are augmented, i.e., if regime  $S_h$  is chosen, only the first  $r$  nonconsumed alternatives are augmented, the remaining positively consumed demands are not augmented. As we will see later, in the virtual price approach both the nonconsumed goods and consumed goods are augmented due to the role of the virtual price.

If all goods are consumed the observed demands are the latent demands. In this case there is no need to augment the demands. If at least one good is not consumed the latent demands for the non-consumed goods are augmented by simulating the latent data from a suitable truncated multivariate normal distribution conditional on the observed demands (Huang [2001]). Formally, assume the demand regime  $S_h$  is observed in which the first  $r$  sites are not visited, i.e.,  $S_h = (0_1, \dots, 0_r, y_{r+1}, \dots, y_j)$ . The latent demands,  $(y_{1i}^*, y_{2i}^*, \dots, y_{ri}^*)$ ,

conditional on the market prices of the all available goods, can be simulated by the following process:

$$y_{ji}^* | y_{1i}^*, y_{2i}^*, \dots, y_{j-1,i}^*, y_{j+1,i}^*, \dots, y_{ri}^*, y_{r+1,i}^*, \dots, y_{ji}, \theta, \Sigma \sim TN_{(-\infty, 0)}(\mu_{j|-j}, \varpi_{j|-j}^2), \quad (4.21)$$

where  $j = 1, 2, \dots, r$ ,  $TN_{(-\infty, 0)}(\mu_{j|-j}, \varpi_{j|-j}^2)$  is the truncated univariate normal distribution with mean  $\mu_{j|-j}$  and variance  $\varpi_{j|-j}$ . Letting  $y_{-j}^* = (y_{1,i}^*, \dots, y_{j-1,i}^*, y_{j+1,i}^*, y_{r+1,i}^*, \dots, y_{j,i})'$ , the mean and variance are,

$$\mu_{j|-j} = \mu_j + \Sigma'_{j-j} \Sigma_{-j-j}^{-1} (y_{-j}^* - \mu_{-j}) \quad (4.22)$$

$$\varpi_{j|-j}^2 = \varpi_{jj}^2 - \Sigma'_{j-j} \Sigma_{-j-j}^{-1} \Sigma_{j-j}, \quad (4.23)$$

where  $\mu = X_i \theta$ ,  $\mu_j$  is the  $j^{\text{th}}$  row element of  $\mu$  and  $\mu_{-j}$  is obtained by deleting the  $j^{\text{th}}$  row element of  $\mu$ . The matrix  $\Sigma_{-j-j}$  is the  $(J-1) \times (J-1)$  matrix derived from  $\Sigma$  by eliminating its  $j^{\text{th}}$  row and  $j^{\text{th}}$  column, and  $\Sigma_{j-j}$  is the  $((J-1) \times 1)$  vector derived from the  $j^{\text{th}}$  column of  $\Sigma$  by removing the  $j^{\text{th}}$  row term.

With this simulation strategy the generation of each univariate truncated normal distribution proceeds as follows (Devroye [1986], Griffiths [2001]). Suppose that  $\zeta$  is a truncated normal random variable with location parameter,  $\mu$ , and scale parameter,  $\sigma$ , and truncation  $a < \zeta < b$ . To draw  $\zeta$ :

1. Draw a uniform(0,1) random variable U.
2. Calculate

$$\zeta = \mu + \sigma \times \Phi^{-1}[\Phi(\frac{a-\mu}{\sigma}) + U \times (\Phi(\frac{b-\mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma}))] \quad (4.24)$$

where  $\Phi$  is the standard normal cumulative distribution function.

Although the above data augmentation process employs the same idea as Pitt and Millimet [2001], the details of implementation are different in two aspects. The first difference is whether the unobserved data are augmented in one step or in separate steps. While Pitt and Millimet simultaneously generate *multivariate* random variables to augment demands for all nonconsumptions together, the current application generates a series of univariate random variables to augment each nonconsumption at each step. For example, if two goods are censored a bivariate random variable is generated to augment the two corners simultaneously in Pitt and Millimet's procedure. In contrast, the current application first generates a univariate random variable to augment the first corner, then this procedure is repeated to augment the second corner. The second difference is the way of generating truncated normal random variables. While Pitt and Millimet use rejection sampling to generate the multivariate random variables, the current application employs the inversion method. Rejection sampling generates the random variables from the underlying untruncated distribution first and keeps only the variables falling within the truncated zone, while the inversion method draws the random variables from the truncated zone directly as illustrated above. Although these two methods are theoretically equivalent, the inversion method is more efficient than the rejection sampling (Chib[1992]). If the probability of random draws from the underlying distribution falling within the truncated zone is significantly smaller than one, the inversion method is much faster than rejection sampling.

### Updating the Covariance Matrix

The second step of the Gibbs sampler is to update the estimate of the covariance  $\Sigma$ . Given the augmented demands and current estimates of the parameters, the residuals are calculated from equation (4.12), and then the sample covariance matrix is computed. An updated  $\bar{\Sigma}$  is drawn as follows

$$\bar{\Sigma} \sim IW[(N-1)\tilde{\Sigma}, N-1], \quad (4.25)$$

where  $\tilde{\Sigma}$  is the sample covariance matrix, IW denotes the inverse Wishart distribution, and N is the number of observations ( Schafer [1997]).

The Wishart distribution is the counterpart of the Chi-Square distribution in the multivariate setting. Let  $X_i, i = 1, \dots, m$  be a multivariate normal distribution with mean vector  $\mu = 0$  and covariance  $\Sigma$ , and X denote the  $m \times p$  matrix consisting of the row vectors of  $X_i$ . Define  $W = X'X$ . W has a Wishart distribution with scale matrix  $\Sigma$  and degrees of freedom m, i.e.,  $W \sim W_p(\Sigma, m)$ . If  $W \sim W_p(\Sigma, m)$  then

$$W^{-1} \sim IW_p(\Sigma^{-1}, m) \text{ (Schafer[1997])}.$$

The algorithm to sample from  $W_p(\Sigma, m)$  is as follows (Schafer[1997]):

- i. Compute the Choleski decomposition of  $\Sigma : \Sigma = CC'$ .
- ii. Generate a p by p matrix U as

$$U(i, j) \begin{cases} = 0, & \forall i, j, j > i, \\ \sim \phi(0,1), & \forall i, j, j < i, \end{cases}$$

$$U(i, i) \sim \sqrt{\chi_{m-i+1}^2}, i = 1, \dots, p.$$

iii Define  $Q = UU'$  and  $W = CQC'$ .

Since  $W \sim W_p(\Sigma, m)$ , we can invert  $W$  to obtain  $W^{-1} \sim IW_p(\Sigma^{-1}, m)$ , then we can draw  $\bar{\Sigma}$  from  $W^{-1}$ .

### *Updating the Parameters*

The third step of Gibbs Sampler is to update the estimates of parameters. Once the updated covariance matrix has been obtained, the SURE model is utilized to estimate  $\theta$  conditional on  $\Sigma$  and the augmented demands. Although the same SURE model is also used in obtaining the starting values of Gibbs sampler, the Generalized Least Squares estimators (GLS thereafter) instead of FGLS should be employed here because the current step is conditional on the updated covariance matrix rather than under the assumption that both the parameters and covariance matrix are unknown as was the case in obtaining the starting values of estimates.

Corresponding to equation (4.16), the GLS ( Chipman [1999]) estimator of  $\theta$  subject to  $l$  independent linear restrictions

$$R\theta - r = 0$$

is

$$\hat{\theta} = (I - \Gamma R)\tilde{\theta} + \Gamma r \quad (4.26)$$

and the covariance matrix of  $\hat{\theta}$  is given as follows

$$\Sigma_{\theta} = (X\bar{\Sigma}^{-1}X)^{-1} - \Gamma R(X\bar{\Sigma}^{-1}X)R'\Gamma' \quad (4.27)$$

where  $\Gamma = (X\bar{\Sigma}^{-1}X)^{-1}R'[R(X\bar{\Sigma}^{-1}X)^{-1}R']^{-1}$ ,  $\tilde{\theta} = (X\bar{\Sigma}^{-1}X)^{-1}X\bar{\Sigma}^{-1}Y$ ,  $Y = (y'_1, y'_2, \dots, y'_N)$ ,  $X = (X'_1, X'_2, \dots, X'_N)$ ,  $\bar{\Sigma}$  is updated covariance matrix in the second step of the Gibbs sampler,  $l$  is the rank of  $r$ , and  $R$  is  $l \times k$  with rank  $l$  (recall  $k$  is the total number of parameters of interest).

Given the linearity of  $\hat{\theta}$  with respect to  $Y$ , and  $Y$  being normally distributed, the normality of  $\hat{\theta}$  is warranted, i.e.,

$$\hat{\theta} \sim \phi(\theta_0 - \Lambda(R\theta_0 - r), \Sigma_{\theta_0}),$$

where  $\theta_0$  is the true values of parameter,  $\Lambda = (X\Sigma^{-1}X)^{-1}R'[R(X\Sigma^{-1}X)^{-1}R']^{-1}$ , and

$$\Sigma_{\theta_0} = (X\Sigma^{-1}X)^{-1} - \Gamma R(X\Sigma^{-1}X)R'\Gamma'. \quad (4.28)$$

With this, the values of  $\theta$  are updated by drawing

$$\bar{\theta} \sim \phi(\hat{\theta}, \Sigma_{\theta}), \quad (4.29)$$

where  $\hat{\theta}$  are the SURE estimates and  $\Sigma_{\theta}$  is the covariance matrix of the estimated parameters. After the new estimates are obtained, the weakly integrable condition is checked. For the specification of equation (4.1) the weakly integrable condition is that the  $\beta$  matrix is symmetric, negative semidefinite. Recall that this condition is necessary to recover the quasi-indirect utility function from which the welfare measures are constructed. If any of these conditions are violated, new values  $\bar{\theta}$  are redrawn using equation (4.29). This whole process must be repeated at least  $\ln N$  times to assure convergence (Hajivassiliou and Mcfadden [1998]), where  $N$  is the number of observations.

#### 4.4 Welfare Analysis Methodology

After we obtain the parameter estimates welfare analysis is the next task. Von Haefen and Phaneuf [2003] discuss different welfare measurement methods that depend on the assumptions used to generate structural functions in count variable and continuous demand systems. In general, there are two types of structural models. The first is the representative consumer case, which estimates the statistical expectations of the consumer demand and ignores the role of unobserved heterogeneity. The second is the actual consumer case in which the errors are the unobserved components of preferences.

The AT approach fits the first interpretation in that censoring is treated in a statistical manner, i.e., there is no extra information on preferences obtained from corners and the system characterizes the expectations of the individual demands while accounting for censoring. In contrast, the virtual price approach fits the second interpretation in that censoring is treated in an economic manner, i.e., the virtual prices are recovered from corners and the system characterizes the actual individual's preference. The next chapter discusses the implications of this for welfare measurement.

There are two potential ways to construct welfare measures in the AT model. First, welfare measures can be constructed using the quasi-indirect utility function (Hausman [1981]),

$$\tilde{V}(p, q, m) = m - \sum_{k=1}^J \alpha_k(z) p_k - \frac{1}{2} \sum_{j=1}^J \sum_{k=1}^J \beta_{jk} p_j p_k. \quad (4.30)$$

Von Haefen and Phaneuf [2003] point out that the quasi-indirect utility function in equation (4.30) does not represent the consumer's preference ordering, but rather is the

structure of preferences that generate the corresponding incomplete demand system equations (4.1). The representative consumer's compensating variation associated with a change in the price and/or quality vectors from  $(p^0, q^0)$  to  $(p^1, q^1)$  is implicitly defined by

$$V(p^0, q^0, m; \alpha, \beta) = V(p^1, q^1, m + CV(p^0, q^0, p^1, q^1, m; \alpha, \beta); \alpha, \beta). \quad (4.31)$$

Although in general there is no closed form solution for the compensating variation due to nonlinearity in price, the linear demand system (i.e., equation (4.4)) does lead to a closed form solution.

Second, welfare measures can be constructed by calculating the change in consumer surplus equation by equation (Just et.al [1982]). With income effects assumed to be zero, the change in consumer surplus is equal to the Hicksian measure and is path independent.

There are potential problems with both welfare measures that arise from the lack of attention given to non-negativity constraints in the economic model. The expected demands given in equation (4.3) may be negative for observed price levels. Thus, calculation of the welfare effects via both methods will include in some cases surplus measures extending into the negative domain for demand. This represents a significant weakness in the AT model and suggests the importance of addressing corner solutions structurally as well as statistically.

To calculate meaningful welfare measures from this model we rely on a modified version of equation (4.33). The welfare impact can be calculated by sequentially evaluating the change in consumer surplus for each good resulting from a change from  $(p^0, q^0)$  to  $(p^1, q^1)$ . The formula is given by

$$CS = \sum_{j=1}^J \int_{p_0^j}^{p_1^j} \int_{q_0^j}^{q_1^j} y_i(p, q) dq dp . \quad (4.33)$$

Since  $y_j$  is not constrained to be positive, this formula calculates surplus measures for negative values of  $y_j$ , resulting in a meaningless welfare measure.

To address this we modify the welfare calculation formula to depend on the choke price for the good(s) whose price/quality changes. Without loss of generality assume the welfare calculation is being constructed only for a change in price/quality for good 1. Define  $p_1^c$  as the choke price for good 1. For a price increase the change in consumer surplus for good  $j$  is calculated according to the following conditions:

- If  $p_1^c < p_1^0$ ,  $CS_j = 0$
- If  $p_1^0 < p_1^c < p_1^1$ ,  $CS_j = CS_j(p_1^0, p_1^c)$
- If  $p_1^0 < p_1^1 < p_1^c$ ,  $CS_j = CS_j(p_1^0, p_1^1)$ .

For a price decrease the conditions are modified as follows:

- If  $p_1^c < p_1^1 < p_1^0$ ,  $CS_j = 0$
- If  $p_1^1 < p_1^c < p_1^0$ ,  $CS_j = CS_j(p_1^1, p_1^c)$
- If  $p_1^1 < p_1^0 < p_1^c$ ,  $CS_j = CS_j(p_1^0, p_1^1)$ .

The logic for a quality change is slightly different and requires recognition of the fact that the choke price is dependent on the level of quality. Define  $p_1^c(q_1^0)$  as the choke price for good 1 at the original quality level and  $p_1^c(q_1^1)$  as the choke price for good 1 at the new quality level. Furthermore note that, for the linear specification, a quality change at site  $j$

impacts only the demand for trips to site  $j$ . Thus, for a quality increase at site 1 the following conditions apply:

- If  $p_1^c(q_1^0) < p_1^c(q_1^1) < p_1^0$ ,  $CS_1 = 0$
- If  $p_1^c(q_1^0) < p_1^0 < p_1^c(q_1^1)$ ,  $CS_1 = CS(p_1^0, p_1^c(q_1^1))$
- If  $p_1^0 < p_1^c(q_1^0) < p_1^c(q_1^1)$ ,  $CS_1 = CS(p_1^0, p_1^c(q_1^1)) - CS(p_1^0, p_1^c(q_1^0))$ .

Intuitively these conditions enforce the non-negativity constraints that are absent in the original specification of the problem by computing the consumer surplus triangle that lies strictly in the positive orthant.

In the current application the latter is preferable since we can enforce censoring on latent variables when necessary. The Gibbs sampler with data augmentation introduces the latent (i.e., negative) demands to obtain the parameter estimates; welfare measures calculated without censoring will include the “surplus” resulted from the negative orthant which, by definition, should not be considered as a part of consumer surplus.

#### 4.5 Estimation Results

In the following sections estimation results of the main AT model and its variations are presented. Then welfare measures for several hypothetical scenarios of price and/or quality changes are calculated using the estimated results.

##### 4.5.1 Main Model Estimation

In estimating the main model, the econometric specification is defined as follows:

$$y_{ij} = \alpha_j + \varphi q_j + \sum_{k=1}^J \beta_{jk} p_{ik} \quad j = 1, 2, \dots, J, \quad i = 1, 2, \dots, N. \quad (4.34)$$

Recall the beta matrix has to be symmetric and negative semidefinite to assure the weak integrable conditions. As mentioned in chapter 3, because the pheasant counts at each site do not vary across individuals, the quality variable  $q_j$  in equation (4.34) is defined as product of hunting permit (dummy variable) and site pheasant count in the above specification.

The main AT model was estimated for the 1967 users in the Iowa Wetland data by running 2,400 loops of Gibbs sampler as detailed above. The initial 800 sets of estimates were discarded to ensure the estimates converge. To reduce serial correlation across parameter draws only every eighth loop was kept (Chib and Greenberg [1996]) to calculate the means of the parameters reported.

The mean and standard deviation of the remaining 200 sets of parameter estimates are reported in the Table 4.1. The estimates of interest have the expected signs and are statistically significant at the 5 percent level. The coefficient on the quality variable is positive which means that higher pheasant counts add attractiveness to the corresponding site. The coefficients for all own prices are negative, which is what the travel cost model predicts: the higher the travel cost the fewer visits.

The mean and standard deviation of the remaining 200 sets of covariance matrix are provided in Table 4.2. The off diagonal elements of the matrix are mixed with negative and positive elements which implies that there are no uniform relationships among the errors across sites.

The results show the current method has several advantages over many previous applications. For example, the popular multinomial logit RUM model has zero income and cross price effects. The assumption that the random terms have independent extreme value distributions implies the RUM model has a restrictive error structure. In contrast, although the AT model also has no income effect, it does have rich cross price effects which allows more fully characterized cross price impacts. The error terms in the AT model are assumed to have a full normal distribution and thus allow more general correlation among recreation sites. Although the KT model has the advantage of treating corner solutions in a theoretically consistent manner, its reliance on explicitly specified first order conditions restricts the utility function used in its application. In practice, additive separable utility functions are employed in the KT model, which have restrictive cross price effects. The KT model has limited stochastic substitution if the generalized extreme distribution is assumed to be the underlying distribution of the error terms. Again the AT model has richer cross price effects and stochastic substitution patterns than the KT model.

#### 4.5.2 Variations on the main model

The estimation results of two variations on the main model are reported in this section. The first variation estimates the AT approach without quality variable in the specification. The econometric specification is

$$y_{ij} = \alpha_j + \sum_{k=1}^J \beta_{jk} p_{ik} \quad j = 1, 2, \dots, J, \quad i = 1, 2, \dots, N. \quad (4.35)$$

The estimates and the covariance matrix are reported in Table 4.5 and Table 4.6. The estimates follow a similar pattern as the main model. All the own price coefficients are

negative and significant at 5 the percent level. The only negative cross price coefficient is the one between site 2 and site 5, but it is not significant. Again the off diagonal elements of the covariance matrix are mixed with positive and negative terms indicating no uniform correlation of error terms among sites.

The second variation estimates the AT model with demographic variables, such as age , gender and education, in the intercept along with the quality variable:

$$y_{ij} = \alpha_j + \varphi q_j + \rho_1 male + \rho_2 age + \rho_3 college + \sum_{k=1}^J \beta_{jk} p_{ik}, j = 1, 2, \dots, J, i = 1, 2, \dots, N, \quad (4.36)$$

where male is a dummy variable with value 1 if the individual is male and zero otherwise, age is the actual age of the individual, and college is dummy variable with value 1 if the individual holds a four-year college degree.

The estimates and the covariance matrix for this specification are reported in Table 4.7 and Table 4.8. All the own price coefficients are negative and significant at 5 percent level. All the cross price coefficients are positive as expected. The coefficient on the quality variable is positive and significant at 5 percent level. The coefficients on male dummy and college dummy are positive but none is significant at 5 percent level. Age has negative impacts on the number of trips made in general since the coefficient is negative and significant at 5 percent level. Again the covariance matrix has similar structure as the other models.

#### 4.5.3 Inclusion of nonusers in the estimation

In estimation of the main model and its two variations, only a part of sample, i.e, users of Iowa wetlands, was included. The AT approach provides a natural technique to

incorporate nonusers into the estimation through data augmentation. The econometric specification is the same as the main model. Table 4.3 and Table 4.4 provide the estimates for the full sample. All the own price coefficients are negative and significant at 5 percent level. All the cross price coefficients are positive as expected. The most notable change is the coefficient on the quality variable, which is much larger than the other models including only users of the Iowa wetlands and is significant at 5 percent level. This change indicates that the quality variable has a larger impact on whether to take the recreation trip than how many trips to take. Another notable change is the larger variance than that of other models, which is correlated with the bigger negative coefficients on of the intercept brought by the nonusers of the wetlands.

#### *4.5.4 Welfare measures results*

To make comparisons across models possible the same hypothetical policy scenarios are estimated for the AT models:

- Scenario A: Increase in the access fee \$50.00 for site 2.
- Scenario B: Increase in the pheasant counts 20% for site 2.

The welfare estimates and related standard deviations are reported for each scenario of the above four models in Table 4.9. The welfare measures reported here are the average consumer surplus calculated using the methods describe in section 4.4. Each set of the 200 sets of estimates from the Gibbs sampler are used to calculate the consumer surplus and the corresponding means and standard deviations are computed. For the access fee scenario, the welfare measures for the three models including only users are very close ( between

\$14 and \$15 ), but the welfare measures for the variation including both users and nonusers are much lower (about \$5 ), which is expected since nonusers would not lose or gain much from the access fee scenario. For the pheasant counts scenario, the welfare measures are similar for all the models (about \$3), which is also expected. The reason why the variation including both users and nonusers has this comparable welfare measures to other models is that the coefficient on the quality variable in this variation is much larger than that of all other models, which implies the 20% increase in pheasant counts not only could make the users take more trips but also could convert some nonusers into users under this scenario.

## Chapter 5: Virtual Price Approach

In the last section, the incomplete demand system was estimated using the AT approach. In this section the incomplete demand system is estimated using the virtual price (VP hereafter) approach (Lee and Pitt [1986]). As mentioned in the last chapter, the major difference between the AT approach and the VP approach is whether the virtual price has a role in the individual's decision making process. The virtual price is the reservation price at which the individual is indifferent between zero consumption and positive consumption. If the market price is higher than the virtual price for a good, the individual will consume zero; if the market price is lower than the virtual price for a good, the individual will end up with positive consumption. Through the construction of the virtual price zero consumption is no longer a corner solution, it is an interior solution, since the virtual price exactly supports zero consumption. The virtual price is the economically meaningful price in that there is no behavior response for price increases above the virtual price. The Lee and Pitt approach follows this reasoning and allows the virtual price, rather than the market price, to play the key role in the individual's decision making process.

The section starts with a brief review of the incomplete demand system and the parameter restrictions that are necessary to recover the quasi-indirect utility function. Then this section follows with derivation of the estimable equations and the likelihood functions under the VP approach. The details on the strategy for estimation and welfare measures are presented, and the techniques for implementation are also provided. The chapter ends with the results of the main model and its variations, as well as welfare measures.

## 5.1 Brief Review of the Incomplete Demand System

To effectively compare the AT approach and the dual approach the same specification of the incomplete demand system is employed. Suppose that consumer demand for a set of  $J$  alternative choices with  $N$  individuals in the demand system is expressed by the following system of Marshallian demand functions:

$$y_j = \alpha_j(z) + \sum_{k=1}^J \beta_{jk} p_k + \gamma_j m + \varepsilon_j, \quad j \in \{1, \dots, J\}, \quad (5.1)$$

where  $y_j$  is the number of trips to site  $j$ ,  $p_k$  is a vector of travel costs to site  $k$ ,  $z$  is a vector of prices of all other goods,  $\varepsilon \sim \phi(0, \Sigma)$  and  $\Sigma$  is the covariance matrix. Again in the current application the intercept contains the quality variables in the above demand equations. Note here the subscripts denoting the individuals are omitted to make the notation simple. We add the unobserved terms  $\varepsilon_j$ 's to the demand functions, and note they have different interpretation from that of the AT approach. In the AT approach the error terms are the measurement errors, while the error terms under the VP approach are unobserved heterogeneity and assumed to be an important component of preferences.

Similar to the AT approach, we have to impose the following restrictions on the parameters to make the incomplete demand system weakly integrable (Lafrance [1985,1986,1990] ):

- $\beta_{jk} = \beta_{kj}, k, j \in \{1, \dots, J\}, k \neq j,$
- $\gamma_j = 0, j \in \{1, \dots, J\}.$

That is, the incomplete demand system becomes

$$y_{ij} = \alpha_j(z_i) + \sum_{k=1}^J \beta_{jk} p_{ik} + \varepsilon_j, \quad j \in \{1, \dots, J\}. \quad (5.2)$$

The quasi-indirect utility function (Hausman [1981]) can be recovered by solving a series of partial differential equations. The quasi-indirect utility functions for the above incomplete demand system is given by von Haefen [2002]

$$\Phi(p, q, m) = m_i - \sum_{k=1}^J \alpha_k(z_i) p_{ik} - \frac{1}{2} \sum_{j=1}^J \sum_{k=1}^J \beta_{jk} p_{ij} p_{ik}, \quad (5.3)$$

with this quasi-indirect utility function exact welfare measures can be constructed.

## 5.2 General Structures on Virtual Price Approach

With corner solutions present there are a total of  $L$  possible combinations of consumed goods, i.e., there are  $L$  vectors of  $S_k$ ,  $k = 1, 2, \dots, L$ , where  $L = 2^J - 1$ , and each  $S_k$  represents a different demand regime. Let  $S = \{S_1, S_2, \dots, S_L\}$  and

$$S_1 = (\underbrace{0, 0, \dots, 0}_J, +)', \quad \dots, \quad S_h = (\underbrace{0, \dots, 0}_r, \underbrace{+, \dots, +}_{J-r}), \quad \dots, \quad S_L = (\underbrace{+, +, \dots, +}_J)',$$

where '+' indicates a positive amount and '0' indicates censored at 0. This definition of the demand regime is similar to that of the AT model, but here  $S_1 = (0, 0, \dots, 0, 0)'$  is not included in the demand regime list, because the virtual prices of the nonconsumed goods have to be a function of prices of the positively consumed goods by the definition of the virtual price. This difference between the AT approach and the VP approach makes it difficult to include nonusers in estimation under the VP approach. Recall that nonusers in the Iowa application were incorporated into estimation using the actual market prices as the right hand variables, but the same method could not be used under the VP approach, since

if no goods are consumed, we cannot construct virtual prices. We return to this in the following chapter.

Assume the demand regime  $S_h$  is chosen, i.e., the first  $r$  alternatives are not consumed.

The virtual prices can be solved by setting the demands of non-consumption to zero:

$$\begin{bmatrix} \pi_1 \\ \cdot \\ \cdot \\ \cdot \\ \pi_r \end{bmatrix} = -B_r^{-1} \begin{bmatrix} \alpha_1 + \sum_{j=r+1}^J \beta_{1j} p_j + \varepsilon_1 \\ \vdots \\ \alpha_r + \sum_{j=r+1}^J \beta_{rj} p_j + \varepsilon_r \end{bmatrix}, \quad (5.4)$$

where  $\pi$  denotes the virtual price and

$$B_r = \begin{bmatrix} \beta_{11} & \cdots & \beta_{1r} \\ \vdots & & \vdots \\ \beta_{r1} & \cdots & \beta_{rr} \end{bmatrix}.$$

By subtracting  $[p_1, \dots, p_r]^T$  from both sides of equation (5.4), we get

$$\begin{bmatrix} \pi_1 \\ \cdot \\ \cdot \\ \cdot \\ \pi_r \end{bmatrix} - \begin{bmatrix} p_1 \\ \cdot \\ \cdot \\ \cdot \\ p_r \end{bmatrix} = -B_r^{-1} \begin{bmatrix} \alpha_1 + \sum_{j=r+1}^J \beta_{1j} p_j + \varepsilon_1 \\ \vdots \\ \alpha_r + \sum_{j=r+1}^J \beta_{rj} p_j + \varepsilon_r \end{bmatrix} - \begin{bmatrix} p_1 \\ \cdot \\ \cdot \\ \cdot \\ p_r \end{bmatrix}. \quad (5.5)$$

Since the virtual prices are less than the market prices for the non-consumed goods, each element of the left hand side of equation (5.5) is less than zero. That is

$$\begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \geq -B_r^{-1} \begin{bmatrix} \alpha_1 + \sum_{j=r+1}^J \beta_{1j} p_j + \varepsilon_1 \\ \vdots \\ \alpha_r + \sum_{j=r+1}^J \beta_{rj} p_j + \varepsilon_r \end{bmatrix} - \begin{bmatrix} p_1 \\ \cdot \\ \cdot \\ \cdot \\ p_r \end{bmatrix}. \quad (5.6)$$

Multiplying both sides of equation (5.6) by  $B_r$  and moving  $[\varepsilon_1, \dots, \varepsilon_r]^T$  to the left hand side of equation (5.6), we get the conditions for nonconsumption for the first  $r$  goods defined as:

$$\begin{bmatrix} \varepsilon_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_r \end{bmatrix} \geq \begin{bmatrix} t_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ t_r \end{bmatrix}, \quad (5.7)$$

where

$$\begin{bmatrix} t_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ t_r \end{bmatrix} = - \begin{bmatrix} \alpha_1 + \sum_{j=r+1}^J \beta_{1j} p_j \\ \vdots \\ \alpha_r + \sum_{j=r+1}^J \beta_{rj} p_j \end{bmatrix} - B_r \begin{bmatrix} p_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ p_r \end{bmatrix}. \quad (5.8)$$

The demand for the remaining  $J-r$  positively consumed goods is obtained by substituting the virtual prices into the remaining demand equations equation (5.2):

$$y_j = \alpha_j + \sum_{k=1}^r \beta_{kj} \bar{\pi}_k + \sum_{k=r+1}^J \beta_{jk} p_k + \varepsilon_j + \eta_j, \quad j = r+1, \dots, J, \quad (5.9)$$

where

$$\eta_j = B_r^j B_r^{-1} \begin{bmatrix} \varepsilon_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_r \end{bmatrix},$$

$B_r^j = (\beta_{j1}, \dots, \beta_{jr})$ ,  $y_j$  is the observed demand for the  $j^{\text{th}}$  good, and  $\bar{\pi}_k$  is the deterministic component of the virtual price in equation (5.9).

Rearranging equation (5.9) we get

$$\varepsilon_j = t_j, \quad j = r+1, \dots, J, \quad (5.10)$$

where

$$t_j = y_j - \left( \alpha_j + \sum_{k=1}^r \beta_{jk} \ln \bar{\pi}_k + \sum_{k=r+1}^J \beta_{jk} p_k + \eta_j \right). \quad (5.11)$$

Equation (5.7) and equation (5.10), along with the joint density function  $f_\varepsilon(\varepsilon)$  for  $\varepsilon$ , provide the necessary information to construct the likelihood function.

The contribution to the likelihood function for an individual who positively consumes  $J-r$  goods is

$$f_i^{S_h}(y_i; \alpha, \beta, \Sigma) = \int_{t_1}^{\infty} \dots \int_{t_r}^{\infty} g(\varepsilon_1, \dots, \varepsilon_r) h(\varepsilon_{r+1}, \dots, \varepsilon_J | \varepsilon_1, \dots, \varepsilon_r) d\varepsilon_1 \dots d\varepsilon_r, \quad (5.12)$$

where the  $g(\cdot)$  is a marginal distribution and  $h(\cdot)$  is the conditional distribution. There are  $2^J - 1$  possible demand regimes from which equation (5.12) can be constructed. The likelihood function for an independent sample with  $N$  observations is

$$L(Y; \alpha, \beta, \Sigma) = \prod_{i=1}^N \prod_{S_h} [f_i^{S_h}(y_i; \alpha, \beta, \Sigma)]^{I_i(S_h)} \quad (5.13)$$

where  $I_i(S_h) = 1$  if demand regime  $S_h$  is observed for individual  $i$ , zero otherwise, and

$f_i^{S_h}(y_i; \alpha, \beta, \Sigma)$  is the contribution of individual  $i$  with demand regime  $S_h$  to likelihood function and where  $Y = (y'_1, y'_2, \dots, y'_N)$ . Parameter estimates can be obtained through maximizing the above likelihood function.

### 5.3 Estimation via Gibbs Sampling with Data Augmentation

Similar to the problem we encounter with the AT model, this estimation involves evaluation of a product of multivariate normal integrals. Numerical techniques, such as the crude frequency simulator and the normal importance simulator, handle only a handful of dimensions. The new and fast normal simulator, i.e., the GHK simulator, is not appropriate for equation (5.12) which involves a product of two multivariate normal density functions. A new and promising method to avoid evaluation of the multivariate normal distribution is the Gibbs sampler with data augmentation (Pitt and Millimet [2002]). As we already discussed in chapter 4, the Gibbs sampler is a technique for generating random variables from a distribution indirectly by a sequence of easier calculations, without having to calculate the original density. By the introduction of latent variables, data augmentation make Gibbs sampler feasible and simple.

While the three steps of the estimation process in VP approach are the same as that in the AT approach, the differences come from the fact that the virtual prices rather than the market prices play a key role under the VP approach. This conceptual difference has important implications on the data augmentation step, and we will return to this point later.

Corresponding to the demand system specified in equation (5.2), the desired density function to be sampled from is  $f(\alpha, \beta, \Sigma | y)$ , or  $f(\alpha, \beta, \Sigma | y^*)$  after augmenting demands, where  $y$  and  $y^*$  are the observed and augmented demands, respectively. Because sampling from the conditional density is much more convenient than the full density function, the following sequential steps are taken (Pitt and Millimet [2001]):

1. Augment Demand  $y_i^*$  given  $\theta, \Sigma$  and  $y$ , i.e. draw  $y_i^{*t} | \theta^{t-1}, \Sigma^{t-1}$ ;
2. Update  $\Sigma$  given  $y_i^*$  and  $\theta$ , i.e. draw  $\Sigma^t | \theta^{t-1}, y^{*t}$ ;
3. Update  $\theta$  given  $y_i^*$  and  $\Sigma$ , i.e. draw  $\theta^t | \Sigma^t, y^{*t}$ ;

These three steps are repeated until certain convergence conditions are met (Hajivassiliou and Macfadden [1998]). In practice assume convergence is achieved after  $\ln N$  iterations. In the following subsections I describe each step in detail.

*Augmenting Demand Step:*

Through the data augmentation step the full system of latent demands is constructed. In the VP approach, even for the positively consumed goods the latent demands are in general not same as the observed demands due to the use of the virtual price, so we need to augment the demands for both consumed and nonconsumed goods. Similar methods to the AT approach are taken to obtain the starting values for Gibbs sampler under the VP approach. Given the starting values for the parameters and the observed data, the Gibbs sampler first augments the demands for the nonvisited sites and obtains the unique virtual prices. Conditional on these virtual prices, the latent demands for the visited sites are derived uniquely. The current application focuses on a five site model using the linear incomplete demand system, and the corresponding processes are derived for this specification.

Two cases are considered in estimation. The first is that all sites are visited. In this case the non-negativity constraints are not binding, the actual prices are the virtual prices and the observed demands are the latent demands. No augmentation is needed.

The second case is that at least one site is not visited. In this case at least one of the non-negativity constraints is binding. Without loss of generality, assume the first  $r$  goods are not consumed and set the notional demands for the first  $r$  goods to zero and solve the virtual prices as,

$$\begin{bmatrix} \pi_1 \\ \cdot \\ \cdot \\ \cdot \\ \pi_r \end{bmatrix} = -B_r^{-1} \begin{bmatrix} \alpha_1 + \sum_{j=r+1}^J \beta_{1j} p_j + \varepsilon_1 \\ \vdots \\ \alpha_r + \sum_{j=r+1}^J \beta_{rj} p_j + \varepsilon_r \end{bmatrix}, \quad (5.14)$$

where  $\pi$  denotes the virtual price and

$$B_r = \begin{bmatrix} \beta_{11} & \cdots & \beta_{1r} \\ \vdots & & \vdots \\ \beta_{r1} & \cdots & \beta_{rr} \end{bmatrix}.$$

As we mentioned earlier economic theory indicates that it is the virtual price that determines whether to consume, and if so, how much to consume. As we can see from equation (5.14) the virtual price is a function of the prices of the positively consumed goods, which means the market prices of the nonconsumed goods are irrelevant in the individuals' decision making process. Another point about equation (5.14) is that the virtual prices are random variables since random terms,  $\varepsilon_{nc} = [\varepsilon_1, \dots, \varepsilon_r]^T$ , are the random components of the virtual prices.

After the virtual prices are solved from equation (5.14), the demands for remaining  $J-r$  positively consumed goods are obtained by substituting the virtual prices into equation (5.2), that is,

$$y_j = \alpha_j + \sum_{k=1}^r \beta_{kj} \bar{p}_k + \sum_{k=r+1}^J \beta_{jk} p_k + \tilde{\varepsilon}_j, \quad j \in c, \quad (5.15)$$

where  $\tilde{\varepsilon}_j = \varepsilon_j - B_r^j B_r^{-1} \varepsilon_{nc}$ ,  $nc = (1, \dots, r)'$ ,  $c = (r+1, \dots, J)$ , and  $B_r^j = (\beta_{j1}, \dots, \beta_{jk})$ .

There are several points about Equation (5.15) needed to be emphasized. First, the demands for the positively consumed goods are functions of the prices of the positively consumed goods only because the introduction of the concept of the virtual price. Second, in contrast to equation (4.16) in chapter 4, the latent demand for the positively consumed goods

$$y_j^* = \alpha_j + \sum_{k=1}^J \beta_{jk} p_k + \varepsilon_j, \quad j \in c$$

does not equal the actual demand in equation (5.15). Equation (5.15) indicates that the demands for the positively consumed goods depend on the error terms for the nonconsumed goods through the virtual prices of the nonconsumed goods. This explains why the latent demands for the positively consumed goods have to be augmented too. Thus, even for the goods not at a corner, the latent and observed demands are not identical. Finally, equation (5.15) shows that the random component of the demands for positively consumed goods combines error terms from both goods at corners and goods not at corners and because of this we have to construct a new covariance matrix for  $\varepsilon$  (representing  $\varepsilon_{nc}$  and  $\tilde{\varepsilon}_j$ ,  $\forall j \neq nc$ ),  $\Omega$ .

Data augmentation is implemented in the following way. First, given the initial estimates of parameters for  $\alpha$  and  $\beta$  and the observed data, the  $\tilde{\varepsilon}'_j$ s are computed from

equation (5.15). Second, we draw errors for the nonconsumed goods conditional on the  $\tilde{\varepsilon}'_j$ s from equation (5.16). Third, we compute the latent demands and virtual prices for the nonconsumed goods from equations (5.2) and (5.4). Forth, we compute errors for the positively consumed goods from equation (5.15). Finally, we compute the latent demands for the positively consumed goods from equation (5.2). In what follows I describe the technical steps needed to accomplish these steps. To properly augment the data, first derive the components of,  $\Omega$ , the covariance matrix of  $\varepsilon_{nc}$  and  $\tilde{\varepsilon}_j, \forall j \neq nc$ , given the initial estimates of the covariance matrix,  $\Sigma$ , as follows:

$$\Omega = COV \begin{bmatrix} \varepsilon_1 \varepsilon_1 & \cdot & \cdot & \cdot & \varepsilon_1 \varepsilon_r & \varepsilon_1 \tilde{\varepsilon}_{r+1} & \cdot & \cdot & \cdot & \varepsilon_1 \tilde{\varepsilon}_J \\ \cdot & & & & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & & & & \cdot & \cdot & & & & \cdot \\ \varepsilon_r \varepsilon_1 & \cdot & \cdot & \cdot & \varepsilon_r \varepsilon_r & \varepsilon_r \tilde{\varepsilon}_{r+1} & \cdot & \cdot & \cdot & \varepsilon_r \tilde{\varepsilon}_J \\ \tilde{\varepsilon}_{r+1} \varepsilon_1 & \cdot & \cdot & \cdot & \tilde{\varepsilon}_{r+1} \varepsilon_k & \tilde{\varepsilon}_{r+1} \tilde{\varepsilon}_{r+1} & \cdot & \cdot & \cdot & \tilde{\varepsilon}_{r+1} \tilde{\varepsilon}_J \\ \cdot & & & & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & & & & \cdot & \cdot & & & & \cdot \\ \tilde{\varepsilon}_J \varepsilon_1 & \cdot & \cdot & \cdot & \tilde{\varepsilon}_J \varepsilon_r & \tilde{\varepsilon}_J \tilde{\varepsilon}_{r+1} & \cdot & \cdot & \cdot & \tilde{\varepsilon}_J \tilde{\varepsilon}_J \end{bmatrix}.$$

After  $\Omega$  is constructed we draw  $\varepsilon_{nc}$  conditional on  $\tilde{\varepsilon}'_j$ s using  $\Omega$ . Assume  $\varepsilon$  (representing  $\varepsilon_{nc}$  and  $\tilde{\varepsilon}_j, \forall j \neq nc$ ) is normally distributed with mean  $\mu$  and covariance  $\Omega$ , i.e.,  $\varepsilon \sim \phi(\mu, \Omega)$ .

Each element of  $\varepsilon_{nc}$  can be drawn by the following process:

$$\varepsilon_j \mid \varepsilon_1, \varepsilon_2, \dots, \varepsilon_{j-1}, \varepsilon_{j+1}, \varepsilon_r, \tilde{\varepsilon}_{r+1}, \dots, \tilde{\varepsilon}_J, \alpha, \beta, \Sigma \sim TN_{(-\infty, \delta_j)}(\mu_{j|j}, \sigma_{j|j}^2), \quad j \in nc, \quad (5.16)$$

where  $TN_{(-\infty, \delta_j)}(\mu_{j|-j}, \varpi_{j|-j}^2)$  is the truncated univariate normal distribution with mean  $\mu_{j|-j}$  and variance  $\varpi_{j|-j}$  and is truncated above from  $\delta_j$  to assure the augmented demands for the non-consumed goods to be negative, where  $\delta_j = -(\alpha_j + \sum_{k=1}^J \beta_{jk} p_k)$ . Let

$\varepsilon_{-j} = (\varepsilon_1, \varepsilon_2, \dots, \varepsilon_{j-1}, \varepsilon_{j+1}, \varepsilon_r, \tilde{\varepsilon}_{r+1}, \dots, \tilde{\varepsilon}_J)'$ . The conditional mean and variance terms are,

$$\mu_{j|-j} = \mu_j + \Omega'_{j-j} \Omega^{-1}_{-j-j} (\varepsilon_{-j} - \mu_{-j}), \quad (5.17)$$

$$\varpi_{j|-j}^2 = \varpi_{jj}^2 - \Omega'_{j-j} \Omega^{-1}_{-j-j} \Omega_{j-j}, \quad (5.18)$$

where  $\mu$  is a  $J \times 1$  vector of zeros,  $\mu_j$  is the  $j^{\text{th}}$  row element of  $\mu$  and  $\mu_{-j}$  is obtained by deleting the  $j^{\text{th}}$  row element of  $\mu$ . The term  $\Omega_{-j-j}$  is the  $(J-1) \times (J-1)$  matrix derived from  $\Omega$  by eliminating its  $j^{\text{th}}$  row and  $j^{\text{th}}$  column and  $\Omega_{j-j}$  is the  $((J-1) \times 1)$  vector derived from the  $j^{\text{th}}$  column of  $\Omega$  by removing the  $j^{\text{th}}$  row term.

There are two points about the above process. First, equation (5.16) draws each element of  $\varepsilon_{\text{nc}}$  in a separate step from a univariate truncated normal distribution, rather than drawing all elements of  $\varepsilon_{\text{nc}}$  together from multivariate truncated normal distribution. Second, the inversion method instead of the rejection sampling is used to generate elements of  $\varepsilon_{\text{nc}}$ . See detailed discussion of the above two points for the AT approach in chapter 4.

After the  $\varepsilon_{\text{nc}}$ 's are drawn, the  $\varepsilon_j$ 's,  $j = r+1, \dots, J$ , are calculated from equation (5.15).

These two steps provide a draw for the full vector of unobserved heterogeneity in the model. To complete the augmentation, substitute the parameter values and the error terms for the positively consumed goods into equation (5.2). With this the augmented latent

demands for the positively consumed goods are uniquely determined. Although the  $\varepsilon_{nc}$ 's are drawn such that the latent demands for nonconsumed goods  $y_{nc}^*$  are nonpositive, there is no assurance that the latent demands for positively consumed goods  $y_c^*$  are positive. If any element of  $y_c^*$  is nonpositive, the entire data augmentation step must be repeated. The regime switching conditions require that the virtual prices are less than the actual prices. If any of these regime switching conditions are not satisfied, again the above process must be repeated.

#### *Updating the Covariance Matrix*

The second step of the Gibbs sampler is to update estimate of the covariance matrix,  $\Sigma$ . Given the augmented demands and current values of parameters, the residuals are calculated from equation (5.2), and the sample covariance matrix is calculated. An updated  $\bar{\Sigma}$  is drawn as follows

$$\bar{\Sigma} \sim IW[(N-1)\check{\Sigma}, N-1] \quad (5.19)$$

where  $\check{\Sigma}$  is the sample covariance matrix, IW denotes the inverse Wishart distribution, and N is the number of observations ( Schafer[1997]). The details on how to generate random variables from the Wishart distribution are given in chapter 4.

#### *Updating the Parameters*

The third step of Gibbs Sampler is to update the estimates of parameters. Once the updated covariance matrix has been obtained, the SURE model is utilized to estimate  $\alpha$  and  $\beta$  conditional on  $\Sigma$  and the augmented demands. Because the covariance matrix is

given from the previous step, GLS instead of FGLS is used in the estimation. The same procedures used in the AT approach in Chapter 4 are implemented in the VP approach; the details are given in the previous chapter.

After the new parameters are obtained, the values of  $\alpha$  and  $\beta$  are updated by drawing

$$\bar{\alpha}, \bar{\beta} \sim N\left(\begin{bmatrix} \hat{\alpha} \\ \hat{\beta} \end{bmatrix}, \Sigma_{\alpha\beta}\right), \quad (5.20)$$

where  $\hat{\alpha}, \hat{\beta}$  are the SURE estimates and  $\Sigma_{\alpha\beta}$  is the covariance matrix of the estimated parameters. The properties of the SURE estimates (i.e.,  $\hat{\alpha}, \hat{\beta}$ ) are given in chapter 4. Again we need to check the  $\beta$  matrix to assure the incomplete demand is weakly integrable. Recall that for equation (5.2) the weakly integrable conditions are that the  $\beta$  matrix is symmetric and negative semidefinite, which requires that the eigenvalues are nonpositive. If any of these conditions are not satisfied,  $\alpha$  and  $\beta$  must be redrawn according to equation (5.20).

This whole process must be repeated at least  $\ln N$  times to assure convergence (Hajivassiliou and Macfadden 1998), where  $N$  is the number of observations.

#### **5.4 Welfare Analysis Methodology**

Similar to the comparison of the continuous and count data modeling frameworks (von Haefen and Phaneuf [2002]), the AT approach and the VP approach differ substantially in the interpretation of the structural estimates. The VP approach specifies the demand system up to the unobserved heterogeneity matrix, and the AT approach specifies expectations of consumer demand. As I have discussed in other sections the error terms in the AT approach and the VP approach have different interpretations. In the AT approach we assume that

individuals know exactly what their utility function looks like, and the error term represents the mistakes made by the individuals when choosing among the available alternatives or the measurement error in estimation process. In contrast, the error term in the VP approach represents the individual's unobserved heterogeneity, a random component of the utility function known by the individual and unobserved by the analyst. These differences have implications for the interpretation of constructed welfare measures for the AT approach and the VP approach. The VP approach recovers an estimate of *each individual's* compensating variation, while the AT approach recovers the *representative consumer's* welfare measure.

Denote the quasi-indirect utility function equation (5.3) recovered from incomplete demand system equation (5.2) by  $V(p, m, q; \alpha, \beta, \varepsilon)$  to include the income and the quality variables  $q$ . The compensating variation associated with a change in the price and/or quality vectors from  $(p^0, q^0)$  to  $(p^1, q^1)$  is implicitly defined by

$$V(p^0, m, q^0; \alpha, \beta, \varepsilon) = V(p^1, m + CV(p^0, q^0, p^1, q^1, m; \alpha, \beta, \varepsilon), q^1, m; \alpha, \beta, \varepsilon). \quad (5.21)$$

Since the error term  $\varepsilon$  is part of compensating variation, the compensating variation is a random variable from analyst's perspective. The expectation for the individual and population will be calculated. In general, the nonlinear nature of the utility function and maximization process will prevent a closed form solution, but for CV in the current application (due to linear demand function chosen) a closed form solution exists. Phaneuf, Kling, and Herriges [2000] ( PKH [2000] hereafter ) suggest a methodology for computing the CV in this context, which is employed in the current application.

Because of the nonnegativity constraints, the quasi-indirect utility function of interest for welfare analysis is the maximum of the set of conditional indirect utility functions.

Once again let

$$S = \{S_k, k = 1, \dots, L\}, \quad L = 2^J - 1,$$

denote the collection of all possible subsets of the index  $M = \{1, \dots, J\}$ , each representing a possible demand regime. A conditional indirect utility function can then be defined for each  $\varpi \in S$  as

$$V_{\varpi}(p_{\varpi}, m, q; \alpha, \beta, \varepsilon) = V(p_{\varpi}, \pi(p_{\varpi}), m, q; \alpha, \beta, \varepsilon), \quad (5.22)$$

where the commodities indexed by  $\varpi$  are consumed,  $p_{\varpi} = \{p_j : j \in \varpi\}$ , and  $\pi(p_{\varpi})$  is the vector of the virtual price of non-consumed goods.

The quasi-indirect utility function equation (5.3) is the *notional* indirect utility function since the non-negativity constraints are not imposed on the function. But as we have already discussed economically meaningful demands should be obtained using virtual prices, hence the corresponding indirect utility function should be a function of the virtual prices. The actual utility function replaces the market price with the virtual price to construct conditional indirect utility functions for each possible demand regime.

Constraining a subset of the commodities to zero via virtual prices provides no assurance that the demands for the remaining goods will be positive. Let

$$\tilde{D} = \{\varpi \in S : y_{\varpi_j}(p_{\varpi}, m, q; \alpha, \beta, \varepsilon) > 0, \forall j \in \varpi\} \quad (5.23)$$

denote the set of  $\varpi$ 's for which the corresponding conditional quasi-indirect utility function yields non-negative demands. The non-notional quasi-indirect utility function of interest for welfare analysis is then the maximum of the feasible conditional quasi-indirect utility functions. That is,

$$V(p, m, q; \alpha, \beta, \varepsilon) = \text{Max}_{\varpi \in \bar{D}} \{V_{\varpi}(p_{\varpi}, m, q; \alpha, \beta, \varepsilon)\}. \quad (5.24)$$

The computation of compensating variation in equation (5.21) corresponds to implicitly solving for  $CV(p^0, q^0, p^1, q^1, m; \alpha, \beta, \varepsilon)$  in

$$\begin{aligned} \text{Max}_{\varpi \in \bar{D}} \{V_{\varpi}(p_{\varpi}^0, m, q^0; \alpha, \beta, \varepsilon)\} = \\ \text{Max}_{\varpi \in \bar{D}} \{V_{\varpi}(p_{\varpi}^1, m + CV(p^0, q^0, p^1, q^1, m; \alpha, \beta, \varepsilon), q^1; \alpha, \beta, \varepsilon)\}. \end{aligned} \quad (5.25)$$

### *The Challenge of Quality Change in Welfare Calculation*

Recreation demand models are typically specified to allow estimation of “use value” of the recreation sites. Weak complementarity (WC hereafter) has been the typical vehicle for environmental quality change based on revealed preference models. In the recreation demand context, WC (Maler [1974]) implies that if an individual does not visit a recreation site, changes in the quality attributes of that site have no value to the individual. This in turn implies the welfare measures under the weak complementarity assumption only include “use value” of the recreation site. The two models estimated in the current application have different positions regarding the implementation of weak complementarity. For the AT approach, WC is implicitly incorporated in that the AT approach treats the corners in a statistical manner and assumes an “interior solution” in

derivation; for the VP approach, this is more of an issue since we endogenously determine the positively consumed goods.

One way to impose WC assumption in the VP approach is to construct quality adjusted prices. For example, rewrite the incomplete demand system for the current application as follows:

$$y_j = \alpha_j(z) + \sum_{k=1}^J \beta_{jk} \bar{p}_k + \varepsilon_j, \quad j \in \{1, \dots, J\}$$

where  $y_j$  is the number of trips to site  $j$ ,  $z$  is a vector of prices of all other goods, other demographic variables, such as age or school, could also be part of  $\alpha$ . Define  $\bar{p}_k$  as the quality-adjusted travel cost to site  $k$ , specifically,

$$\bar{p}_k = p_k + \gamma q_k,$$

where  $p_k$  is the travel cost to site  $k$  and  $q_k$  is the level of quality of site  $k$ .

The virtual prices are calculated based on the quality-adjusted prices  $\bar{p}$  rather than actual money price in this suggestion. Recall that the actual demands and the conditional indirect utility function contain only the quality-adjusted prices of the positively consumed goods, so the quality variables associated with the non-consumed goods drop out of the conditional indirect utility function. This specification satisfies the weak complementarity criteria.

Though this specification is appealing for the above reasons, it poses new estimation challenges. Since the parameter space becomes nonlinear we cannot use the FGLS or GLS estimators to recover the parameters for this specification, and maximum likelihood

methods are needed to update the parameter values. Several attempts failed to estimate this model.

Since both the conditional indirect utility  $V_{\omega}$  and the latent demand  $y_{\omega}$  are functions of  $q$  and not  $q_{\omega} = \{q_j : j \in \omega\}$  in equation (5.22) weak complementarity assumption is not imposed in calculating welfare measures. This implies that without adjustment the compensating variation will contain both use and non-use value components of the recreation sites. I will return to this below.

## 5.5 Estimation Results

In the following sections estimation results of the main VP model and its variations are presented. Then welfare measures for several hypothetical scenarios of price and/or quality changes are calculated using the estimated results.

### 5.5.1 Main Model Estimation

In estimating main model, the econometric specification is defined as follows:

$$y_{ij} = \alpha_j + \varphi q_j + \sum_{k=1}^J \beta_{jk} p_{ik} + \varepsilon_{ij} \quad j = 1, 2, \dots, J, \quad i = 1, 2, \dots, N. \quad (5.26)$$

where  $q_j$  is the quality variable at site  $j$ . Because the pheasant counts at each site do not vary across individuals, the quality variable in equation (5.26),  $q_j$ , is defined as product of hunting permit (dummy variable) and site pheasant counts in the above specification.

The VP model was estimated for the 1967 users in the Iowa Wetland data by running 2,400 loops of Gibbs sampler as detailed above. The initial 800 sets of estimates were discarded to ensure the estimates convergence. To reduce serial correlation across

parameter draws only every eighth loop was kept (Chib and Greenberg [1996]) to calculate the mean of parameters presented.

The mean and standard deviation of the remaining 200 sets of parameter estimates are reported in the Table 5.1. The estimates of interest have the expected signs and are statistically significant at 5 percent level. The coefficient on the quality variable is positive, which means that higher pheasant counts add attractiveness to the corresponding site. The coefficients of own prices are all negative, which is what the travel cost model predicts: the higher travel cost the fewer visits.

The estimates for the covariance matrix are provided in Table 5.2. The off diagonal elements of matrix are negative, which mean the error terms are negatively correlated across sites. Note that this result is different from the AT model, where there is no uniform relationship on error terms across sites.

As we have described in the chapter 4, the AT model and the VP approach have notable advantages over previous applications in that both models provide richer parametric and stochastic substitutions among sites. The VP model not only possesses all the advantages of the AT model but also gives the corner solutions an economic treatment, so in this sense the VP approach is superior to the AT model. Another advantages of this VP model is that it has the ability to estimate a full covariance matrix. Previous applications (e.g. Winer et. al[1994] and Phaneuf [1999]) assume exchangeability among the error terms to make the estimation feasible and simpler.

### 5.5.2 Variations on the main model

The estimation results for two variations on the main model are reported in this section. The first variation estimates the VP approach without the quality variable in the specification. The econometric specification is

$$y_{ij} = \alpha_j + \sum_{k=1}^J \beta_{jk} p_{ik} + \varepsilon_{ij} \quad j = 1, 2, \dots, J, \quad i = 1, 2, \dots, N. \quad (5.27)$$

The estimated results are reported in Table 5.3. All the own price coefficients are negative and significant at 5 percent level. The cross price coefficients are mixed with positive and negative elements. Table 5.4 provides the estimated covariance matrix. The results are similar to the main model with the off diagonal elements are all negative, which means that the error terms of recreation sites are negatively correlated.

The second variation estimates the VP model with demographic variables, such as age, gender and education, in the intercept along with the quality variable:

$$y_{ij} = \alpha_j + \varphi q_j + \rho_1 \text{male} + \rho_2 \text{age} + \rho_3 \text{college} + \sum_{k=1}^J \beta_{jk} p_{ik} + \varepsilon_{ij}, \quad (5.28)$$

where  $j = 1, 2, \dots, J, i = 1, 2, \dots, N$ , male is a dummy variable with value 1 if the individual is male and zero otherwise, age is the actual age of the individual, and college is dummy variable with value 1 if the individual holds a four-year college degree.

The estimates and the covariance matrix of this specification are reported in the Table 5.5 and Table 5.6. All the own price coefficients are negative and significant at 5 percent level. The coefficient on the quality variable is positive and significant at 5 percent level. The coefficients on male dummy and college dummy are positive and negative,

respectively, but none is significant at 5 percent level. The age has negative impacts on the number of trips made in general since the coefficient is negative and significant at 5 percent level. Again off diagonal of the covariance matrix are all negative.

There are two notable differences between the AT models and the VP models in the estimated results. The first is the structure of the covariance matrix. For the VP models the off diagonal elements are all negative, which clearly indicates that the error terms of the sites are negatively correlated, while the covariance matrix of the AT models indicates no such pattern. The second is the estimated intercept. For the VP models all intercepts are positive except the one of site 2 of the main model. But the intercepts of the AT models are all negative and much larger than those of the VP models in absolute value, which indicates that the latent demands augmented using the market prices in the AT models are larger than the latent demands augmented using the virtual prices in absolute values. This is exactly what we expect since the virtual price is less than the market price.

### *5.5.3 Welfare Measures Results*

To illustrate the usefulness of the VP approach in welfare analysis, we consider three hypothetical policy scenarios:

- Scenario A: Increase in the access fee \$50.00 for site 1 and 5.
- Scenario B: Increase in the access fee \$50.00 for site 2.
- Scenario C: Increase in the pheasant counts 20% for site 2.

For these three scenarios, average compensating variation for the sample was estimated using the following modified procedure outlined in PKH [2000]:

1. Select  $N_\theta = 200$  parameter vectors (i.e.  $\theta^{(n)}$ ,  $n = 1, \dots, N_\theta$ ) from the estimated  $\theta$  matrix during 1,600 loops.
2. Draw  $N_\varepsilon = 50$  vectors of random disturbances terms (i.e.  $\varepsilon^{(ink)}$ ,  $k = 1, \dots, N_\varepsilon$ ) for each  $\theta^{(n)}$  and each observation in the sample (i.e.  $i = 1, \dots, N$ ) with the corresponding estimated covariance matrix during 1,600 loops.
3. Substitute  $\theta^{(n)}$  and  $\varepsilon^{(ink)}$  for  $\theta$  and  $\varepsilon$  in equation (5.25), and solve the compensating variation,  $CV^{(ink)}$ .
4. Obtain the average compensating variation,  $\overline{CV}^{(n)}$ , for the  $n^{\text{th}}$  draw by averaging the  $CV^{(ink)}$ 's over the  $N_\varepsilon$  and  $I$  observations in the sample.

The distribution of the mean compensating variation of interest,  $\overline{CV}$ , results from the distribution of  $\overline{CV}^{(n)}$ . The welfare estimates and related standard deviations are reported for each scenario in Table 5.7.

As we mentioned earlier, weak complementarity is not imposed in current application. To obtain welfare measures including only use value for the quality change scenario, we assume the pheasant counts are zero for the nonvisited sites when computing the compensating variations in step 3 of the above procedure. For the \$50 access fee increase scenario A, the mean compensating variations for the three models are very close, about -\$64, while for the \$50 access fee increase scenario B, the mean compensating variations for the three models are of reasonable magnitudes, ranging from -\$15.24 to -\$17.75. For the

20% pheasant counts increase in site 2, the mean compensating variation is \$17.62 for the main model and \$14.55 for the model with demographic variables. These results show that the three models are similar for the welfare analysis.

## **Chapter 6: Summary and Conclusion**

In this chapter I summarize the major findings of this dissertation. I first discuss the findings for the Amemiya-Tobin model and the virtual price approach conceptually, and then I discuss the estimation results and interpretation of welfare measures. The chapter ends with limitations and suggestions for future research.

This dissertation has investigated dual approaches to corner solution estimation that are tractable and flexible in recreation demand. The following are desirable characteristics of such models. First, the models should be utility consistent so that welfare analysis can be applied. The purpose of recreation demand estimation is to address valuation related issues, which are based on the recovered household preferences, therefore utility consistent models are required to calculate the welfare measures. Second, since single or isolated site rarely exists and recreators usually only visit a subset of the available sites, appropriate models should allow estimation using seasonal and multiple site data and address the associated corner solution issue. Further, the models should allow rich parametric and stochastic substitution effects. Finally, the models should incorporate both discrete and continuous aspects of recreation demand decision describing whether to participate, and if so, how many trips to make. The general purpose of this dissertation is to investigate dual approaches to accomplish these tasks.

This research has centered on examining the ability of the virtual price and Amemiya-Tobin models to meet these criteria discussed. The general contributions from my examination are threefold. First, this dissertation advances knowledge on estimating utility

consistent recreation models allowing corner solutions. The AT model and the dual approach estimated in this dissertation are utility consistent with functional forms that allow comparatively flexible parametric and stochastic substitution effects. Second, this dissertation presents feasible computational techniques for the use in recreation demand estimation. Although the attractiveness of dual approaches is well recognized, the difficulty associated with estimation has prevented wider applications. This research takes advantage of modern computational techniques and effectively addresses the problem of evaluating multidimensional integrals, which is crucial in addressing corner solutions using dual approaches. Third, this dissertation combines the incomplete demand system framework with corner solution estimation. The incomplete demand system is attractive in the recreation demand context since welfare measures are conditional on the household's total income rather than the predetermined recreation expenditure.

The two approaches have been applied to a common data on wetland use in Iowa. The two models provide intuitive parameter estimates with similar patterns in term of parametric substitution effects. The site quality measures have positive and significant impacts on demand, which is consistent with prior expectations. However, the two models predict different patterns in terms of stochastic substitution effects. For the virtual price models the off diagonal elements of the covariance matrix are all negative, which clearly indicates that the error terms of the alternative sites are negatively correlated, while the Amemiya-Tobin models indicate no such a pattern.

The Amemiya-Tobin model and the virtual price approach are designed as structural models to deal with corner solutions in recreation demand estimation, but the two models are quite different conceptually. The major difference separating them is whether the reservation price plays a role in the individual's decision making process. Economic theory tells us that it is the comparison between the reservation price and the market price that determines whether an individual consumes a good or not. The AT approach ignores the behavioral explanation for corner solutions and addresses corners statistically in the estimation process. The virtual price approach, on the other hand, treats corner solutions in an economic manner, explaining behaviorally and statistically why an individual has zero consumption. The conceptual difference between the two models has implications for the structural estimates. The VP approach specifies the demand system up to the unobserved heterogeneity matrix and recovers an estimate of *each individual's* compensating variation, while the AT approach specifies expectations of consumer demand and recovers the *representative consumer's* welfare measure.

#### *Limitations and Suggestions for Future Research*

As shown in the previous chapters the dual approaches are appealing in many respects. However, there are several limitations associated with the two models. First, even though the simulation method is conceptually simple, this process requires the ability to code the Gibbs sampler via data augmentation, and the simulation processes are still computationally intensive. Second, while the incomplete demand system allows us to calculate the welfare measures conditional on the individual's total income, researchers

must choose zero income effects or restrictive substitution effects to satisfy the weakly integrable conditions that are required for recovering the quasi-indirect utility function. Third, while the latent demands introduced by the data augmentation facilitate the estimation process on one hand, the negative demands also make the welfare calculation more complicated. Since the “surplus” under the demand curve in the negative orthant are not the welfare measures of interests, adjustments to ensure the positive demands are required to obtain meaningful welfare measures, particularly for the AT approach. For the AT model the welfare measures are constructed by calculating the changes in consumer surplus equation by equation and restricted to the positive orthant. For the VP approach the welfare measurement calculation is conceptually more pleasing but computationally intensive. Since the compensating variation is a function of the virtual price (and therefore the random terms) the compensating variation is a random variable. In general, due to the nonlinear nature of the utility function, the compensating variation has no closed form solution. Hence numerical techniques are required for calculations.

There are several ways to improve the current research for future applications. The first is to incorporate quality variables into the model consistent with weak complementarity. Recreation demand models are typically specified to allow estimation of “use value” of the recreation sites through the weak complementarity. The two models estimated in this dissertation have different characteristics on this issue. The AT approach assumes *interior solutions* in derivation and thus WC is implicitly incorporated, while the VP approach endogenously determines the positively consumed goods. Imposition of WC is more of

issue in the VP approach. Although the quality-adjusted price method is an attractive alternative vehicle to address the issue of use values, the techniques to implement this idea are needed to be further addressed. It would also be worthwhile to investigate other functional forms for the demand equations. Although this may make the estimation more complicated, it may provide a better way to address the issue of the nonuse value.

The second potential area for future research is to find a better way to impose the restrictions associated with the curvature conditions. Gibbs sampling allows us to effectively address the dimension challenges, but imposing restrictions on the parameters when estimating the structural models poses additional challenges. To demonstrate the ability of the dual approaches to handle larger dimensions and to examine the importance of finding more effective ways to impose the restrictions required by curvature conditions, I have tried to estimate a 15-site model and a 9-site model using the AT approach.

Corresponding to the above map, the 9 aggregate sites are grouped as {1,2,3}, {4,5,8}, {9}, {10}, {11}, {13,14,15}, {6}, {7}, and {12}. I was unsuccessful in estimating the 15-site model since the curvature conditions cannot be satisfied. However, the estimation of 9-site model was successful and the results are reported in Table 6.1 and Table 6.2. The results show that the own-price coefficients are negative as expected, the cross-price coefficients are mixed, and the beta matrix are symmetric and negative semidefinite.

Including the nonusers into the estimation under the virtual price approach is another issue to be further addressed. As has already been shown in previous chapters, the nonusers can be easily incorporated under the AT model since the latent demands are augmented

using the market price. In contrast, the latent demands are augmented using virtual price in the VP approach. The implicit assumption underlying the VP approach is that every individual is a potential user, and the only reason the individual does not consume one or more goods is because the actual market price of the good is higher than the individual's reservation price of that good. This assumption is not entirely true. For example, a nonsmoker may still choose not to smoke even if cigarettes are provided free of charge. For this individual, the virtual price is meaningless. Accordingly, this poses a challenge in recreation demand estimation when using general population survey data. To incorporate potential recreators and nonrecreators into estimation, other estimation strategies, such as hurdle models (Haab and McConnell [1996] and Shonkwiler and Shaw [1996]), are needed to augment the dual approaches.

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Table 3.1  
 Descriptive Statistics :The Iowa Wetlands Data (1997)  
 Site Specific Data -5 Sites

Site	Trips	Travel Cost	Pheasant Counts
Site 1	0.81 (3.53)	155.94 (71.28)	17.94
Site 2	1.18 (4.14)	119.37 (58.08)	50.70
Site 3	3.60 (7.06)	75.53 (48.34)	55.68
Site 4	1.44 (4.68)	118.32 (69.31)	27.58
Site 5	1.47 (4.51)	105.67 (50.47)	40.41
All Sites	8.51 (10.83)	114.97 (65.56)	38.46 (14.06)

Note: Standard deviation in parentheses.

Table 3.2  
 Descriptive Statistics :The Iowa Wetlands Data (1997)  
 Site Specific Data -15 Sites

Site	Trips		Travel Cost		Pheasant Counts
	Mean	S. D.	Mean	S.D.	
Site 1	0.38	2.35	166.40	78.51	25.22
Site 2	0.22	1.55	156.68	70.49	9.77
Site 3	0.21	1.58	150.63	65.65	18.83
Site 4	0.36	2.06	150.94	70.31	53.16
Site 5	0.49	2.56	109.92	53.62	50.54
Site 6	0.28	1.74	114.05	54.01	27.81
Site 7	0.41	2.33	118.05	53.33	47.18
Site 8	0.33	2.12	105.45	48.29	48.42
Site 9	1.02	3.44	80.36	43.95	61.55
Site 10	0.94	3.68	92.34	50.90	51.44
Site 11	1.65	4.84	80.87	54.43	54.04
Site 12	0.79	3.25	102.15	56.06	46.22
Site 13	0.63	2.97	129.31	66.15	37.00
Site 14	0.36	2.08	119.92	70.27	19.50
Site 15	0.46	2.65	119.98	71.73	26.25
All Sites	8.51	10.81	119.80	66.55	38.46

Note: The standard deviation of pheasant counts is 15.92 for all sites.

Table 3.3  
 Number of Observations by Regime: The Iowa Wetlands Data (1997)  
 5 Sites

Regime	# of observations	Cumulative Percentage
Non-users	924	0.32
1 Site Visited	1076	0.69
2 Site Visited	613	0.90
3 Site Visited	210	0.98
4 Site Visited	58	1.00
5 Site Visited	10	1.00

Note: There are 2891 observations in this data.

Table 3.4  
 Number of Observations by Regime: The Iowa Wetlands Data (1997)  
 15 Sites

<u>Regime</u>	<u># of observations</u>	<u>Cumulative Percentage</u>
Non-users	924	0.32
1 Site Visited	894	0.63
2 Site Visited	534	0.81
3 Site Visited	303	0.92
4 Site Visited	118	0.96
5 Site Visited	48	0.98
6 Site Visited	27	0.99
7 Site Visited	20	0.99
8 Site Visited	8	0.99
9 Site Visited	10	1.00
10 Site Visited	5	1.00
11 Site Visited	0	1.00
12 Site Visited	0	1.00
13 Site Visited	0	1.00
14 Site Visited	0	1.00
15 Site Visited	0	1.00

Table 3.5  
 Descriptive Statistics :The Iowa Wetlands Data (1997)  
 Individual Specific Data

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Income		\$43,266 (\$28,038)
Age		49.09 (15.64)
Male		2129 out of 2891
With Hunt/Fish Permit		1976 out of 2891
With 4-Year College Degree		810 out of 2891

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Note: Standard deviation in parentheses.

Table 4.1  
 Estimates of Own and Cross Price Marginal Effects on Notional Demands  
 Amemiya-Tobin Model (Main Model)

Mean	Intercept	Quality Variable	Site1	Site 2	site 3	site 4	Site 5
	$\alpha$	$\varphi$	$\beta$				
Site 1	-7.1188 (1.1312)	0.0682 (0.0081)	-0.1281 (0.0077)	0.0633 (0.0060)	0.0025 (0.0062)	0.0085 (0.0048)	0.0552 (0.0053)
Site 2	-7.7728 (1.1465)	0.0682 (0.0081)	0.0633 (0.0060)	-0.1775 (0.0083)	0.0349 (0.0060)	0.0394 (0.0046)	-0.0015 (0.0055)
Site 3	1.1394 (0.9514)	0.0682 (0.0081)	0.0025 (0.0062)	0.0349 (0.0060)	-0.2614 (0.0094)	0.0391 (0.0042)	0.0538 (0.0054)
Site 4	-8.9013 (1.0190)	0.0682 (0.0081)	0.0085 (0.0048)	0.0394 (0.0046)	0.0391 (0.0042)	-0.1068 (0.0059)	0.0334 (0.0042)
Site 5	-2.7279 (0.9635)	0.0682 (0.0081)	0.0552 (0.0053)	-0.0015 (0.0055)	0.0538 (0.0054)	0.0334 (0.0042)	-0.2164 (0.0076)

Note: standard error in parentheses

Table 4.2  
 Estimates of Variance-Covariance Matrix ( $\Sigma$ ) on Notional Demands  
 Amemiya-Tobin Model (Main Model)

Mean	Site1	Site 2	site 3	site 4	Site 5
Site 1	111.6605 (9.4995)	10.5058 (4.4842)	-7.2964 (4.3784)	-6.4455 (6.1751)	-0.8015 (4.1695)
Site 2	10.5058 (4.4842)	91.8321 (6.7512)	6.2035 (3.3910)	7.2197 (4.3582)	6.4199 (3.4969)
Site 3	-7.2964 (4.3784)	6.2035 (3.3910)	90.3146 (3.9268)	9.5667 (3.0221)	11.1753 (2.8158)
Site 4	-6.4455 (6.1751)	7.2197 (4.3582)	9.5667 (3.0221)	97.3278 (6.0391)	4.6364 (3.6368)
Site 5	-0.8015 (4.1695)	6.4199 (3.4969)	11.1753 (2.8158)	4.6364 (3.6368)	73.7106 (4.1398)

Note: standard error in parentheses.

Table 4.3  
 Estimates of Own and Cross Price Marginal Effects on Notional Demands  
 Amemiya-Tobin Model (User and Non-User)

Mean	Intercept	Quality Variable	Site1	Site 2	site 3	site 4	Site 5
	$\alpha$	$\phi$	$\beta$				
Site 1	-12.8729 (1.2535)	0.1461 (0.0087)	-0.1366 (0.0080)	0.0696 (0.0067)	0.0012 (0.0067)	0.0048 (0.0051)	0.0628 (0.0057)
Site 2	-14.4545 (1.2229)	0.1461 (0.0087)	0.0696 (0.0067)	-0.1897 (0.0080)	0.0419 (0.0064)	0.0372 (0.0050)	-0.0055 (0.0058)
Site 3	-5.0647 (0.9713)	0.1461 (0.0087)	0.0012 (0.0067)	0.0419 (0.0064)	-0.3037 (0.0097)	0.0440 (0.0045)	0.0569 (0.0060)
Site 4	-12.5141 (1.1238)	0.1461 (0.0087)	0.0048 (0.0051)	0.0372 (0.0050)	0.0440 (0.0045)	-0.1211 (0.0062)	0.0337 (0.0045)
Site 5	-7.6584 (1.0076)	0.1461 (0.0087)	0.0628 (0.0057)	-0.0055 (0.0058)	0.0569 (0.0060)	0.0337 (0.0045)	-0.2343 (0.0087)

Note: standard error in parentheses

Table 4.4  
 Estimates of Variance-Covariance Matrix ( $\Sigma$ ) on Notional Demands  
 Amemiya-Tobin Model (User and Non-User)

Mean	Site1	Site 2	site 3	site 4	Site 5
Site 1	150.1813 (11.7009)	36.6595 (5.9691)	14.1509 (5.3213)	15.0862 (7.4681)	17.8882 (4.9773)
Site 2	36.6595 (5.9691)	120.4833 (8.0926)	30.0840 (4.0730)	28.2067 (4.8253)	23.8170 (4.1070)
Site 3	14.1509 (5.3213)	30.0840 (4.0730)	115.9045 (5.4111)	34.9614 (3.7111)	32.1224 (3.4425)
Site 4	15.0862 (7.4681)	28.2067 (4.8253)	34.9614 (3.7111)	121.8566 (7.5911)	22.5733 (4.2707)
Site 5	17.8882 (4.9773)	23.8170 (4.1070)	32.1224 (3.4425)	22.5733 (4.2707)	88.5862 (5.3659)

Note: standard error in parentheses.

Table 4.5  
 Estimates of Own and Cross Price Marginal Effects on Notional Demands  
 Amemiya-Tobin Model (without Quality Variables, User Only)

Mean	Intercept	Site1	Site 2	site 3	site 4	Site 5
	$\alpha$			$\beta$		
Site 1	-6.1845 (1.2100)	-0.1296 (0.0072)	0.0640 (0.0062)	0.0031 (0.0062)	0.0083 (0.0050)	0.0555 (0.0055)
Site 2	-5.0852 (1.0848)	0.0640 (0.0062)	-0.1770 (0.0080)	0.0345 (0.0060)	0.0396 (0.0045)	-0.0008 (0.0053)
Site 3	3.9299 (0.9100)	0.0031 (0.0062)	0.0345 (0.0060)	-0.2627 (0.0089)	0.0393 (0.0044)	0.0553 (0.0054)
Site 4	-7.4659 (1.0761)	0.0083 (0.0050)	0.0396 (0.0045)	0.0393 (0.0044)	-0.1072 (0.0060)	0.0337 (0.0040)
Site 5	-0.7864 (0.9429)	0.0555 (0.0055)	-0.0008 (0.0053)	0.0553 (0.0054)	0.0337 (0.0040)	-0.2177 (0.0084)

Note: standard error in parentheses

Table 4.6  
 Estimates of Variance-Covariance Matrix ( $\Sigma$ ) on Notional Demands  
 Amemiya-Tobin Model (without Quality Variables, User Only)

Mean	Site1	Site 2	site 3	site 4	Site 5
Site 1	113.5864 (9.5050)	10.0170 (4.8942)	-8.4042 (4.8570)	-6.3575 (6.0008)	-0.3974 (4.1852)
Site 2	10.0170 (4.8942)	90.5839 (6.5539)	7.9258 (3.3859)	7.5227 (4.4250)	7.7893 (3.5359)
Site 3	-8.4042 (4.8570)	7.9258 (3.3859)	93.5065 (4.0464)	11.2748 (3.1346)	13.2698 (3.0696)
Site 4	-6.3575 (6.0008)	7.5227 (4.4250)	11.2748 (3.1346)	99.1691 (6.5195)	5.9470 (3.6898)
Site 5	-0.3974 (4.1852)	7.7893 (3.5359)	13.2698 (3.0696)	5.9470 (3.6898)	76.4356 (4.5878)

Note: standard error in parentheses.

Table 4.7  
 Estimates of Own and Cross Price Marginal Effects on Notional Demands  
 Amemiya-Tobin Model (with Demographic Variables, User Only)

Mean	Intercept	Quality Variable	Male Dummy	Age	College Dummy	Site1	Site 2	site 3	site 4	Site 5
	$\alpha$	$\varphi$		$\rho$				$\beta$		
Site 1	-5.0503 (1.2485)	0.0628 (0.0086)	0.5884 (0.3415)	-0.0466 (0.0094)	0.3318 (0.3161)	-0.1285 (0.0078)	0.0624 (0.0059)	0.0014 (0.0058)	0.0081 (0.0048)	0.0547 (0.0055)
Site 2	-5.5782 (1.2069)	0.0628 (0.0086)	0.5884 (0.3415)	-0.0466 (0.0094)	0.3318 (0.3161)	0.0624 (0.0059)	-0.1774 (0.0074)	0.0347 (0.0057)	0.0383 (0.0044)	-0.0015 (0.0054)
Site 3	3.2516 (1.0642)	0.0628 (0.0086)	0.5884 (0.3415)	-0.0466 (0.0094)	0.3318 (0.3161)	0.0014 (0.0058)	0.0347 (0.0057)	-0.2608 (0.0080)	0.0384 (0.0041)	0.0537 (0.0051)
Site 4	-6.7443 (1.1452)	0.0628 (0.0086)	0.5884 (0.3415)	-0.0466 (0.0094)	0.3318 (0.3161)	0.0081 (0.0048)	0.0383 (0.0044)	0.0384 (0.0041)	-0.1081 (0.0055)	0.0331 (0.0041)
Site 5	-0.7826 (1.0985)	0.0628 (0.0086)	0.5884 (0.3415)	-0.0466 (0.0094)	0.3318 (0.3161)	0.0547 (0.0055)	-0.0015 (0.0054)	0.0537 (0.0051)	0.0331 (0.0041)	-0.2177 (0.0086)

Note: standard error in parentheses

Table 4.8  
 Estimates of Variance-Covariance Matrix ( $\Sigma$ ) on Notional Demands  
 Amemiya-Tobin Model (with Demographic Variables, User Only)

Mean	Site1	Site 2	site 3	site 4	Site 5
Site 1	112.2446 (9.2404)	10.9198 (4.1725)	-6.9833 (4.5485)	-5.2151 (5.6920)	0.5855 (4.2150)
Site 2	10.9198 (4.1725)	90.6712 (5.9693)	5.3056 (3.4312)	7.0402 (4.3331)	5.7738 (3.5154)
Site 3	-6.9833 (4.5485)	5.3056 (3.4312)	88.6198 (3.8359)	8.4709 (3.0120)	10.5689 (2.8122)
Site 4	-5.2151 (5.6920)	7.0402 (4.3331)	8.4709 (3.0120)	96.5790 (5.6932)	4.5708 (3.5095)
Site 5	0.5855 (4.2150)	5.7738 (3.5154)	10.5689 (2.8122)	4.5708 (3.5095)	74.3623 (4.8298)

Note: standard error in parentheses.

Table 4.9  
Welfare Estimates for Amemiya-Tobin Model  
Average Consumer Surplus

Model Number	Policy Scenario	
	\$50 Access Fees at Site 2	20% Increase in Pheasant Counts at Site 2
Main Model (User Only)	-14.99 (1.47)	3.00 (0.49)
Without Quality Variables (User Only)	-14.38 (1.56)	
With Demographic Variables (User Only)	-14.88 (1.44)	2.82 (0.47)
Main Model (User and Nonuser)	-4.61 (0.53)	2.82 (0.41)

Note: standard error in parentheses

Table 5.1  
 Estimates of Own and Cross Price Marginal Effects on Notional Demands  
 Virtual Price Model (Main Model)

Mean	Intercept	Quality Variable	Site1	Site 2	site 3	site 4	Site 5
	$\alpha$	$\phi$	$\beta$				
Site 1	0.5274 (0.7096)	0.0319 (0.0031)	-0.2124 (0.0098)	0.1278 (0.0082)	0.0021 (0.0082)	-0.0031 (0.0042)	0.1267 (0.0081)
Site 2	-1.1100 (0.8274)	0.0319 (0.0031)	0.1278 (0.0082)	-0.3073 (0.0125)	0.1758 (0.0108)	0.0182 (0.0052)	-0.0123 (0.0072)
Site 3	4.5600 (1.1602)	0.0319 (0.0031)	0.0021 (0.0082)	0.1758 (0.0108)	-0.7426 (0.0268)	0.2108 (0.0109)	0.2428 (0.0128)
Site 4	1.3505 (0.6466)	0.0319 (0.0031)	-0.0031 (0.0042)	0.0182 (0.0052)	0.2108 (0.0109)	-0.2060 (0.0083)	0.0382 (0.0058)
Site 5	2.8891 (0.8575)	0.0319 (0.0031)	0.1267 (0.0081)	-0.0123 (0.0072)	0.2428 (0.0128)	0.0382 (0.0058)	-0.4212 (0.0159)

Note: standard error in parentheses

Table 5.2  
 Estimates of Variance-Covariance Matrix (  $\Sigma$  ) on Notional Demands  
 Virtual Price Model (Main Model)

Mean	Site1	Site 2	site 3	site 4	Site 5
Site 1	104.2688 (7.8408)	-13.0776 (4.0741)	-68.3495 (9.0038)	-5.1537 (3.7251)	-28.5073 (5.0161)
Site 2	-13.0776 (4.0741)	151.2993 (10.3144)	-93.2218 (11.3726)	-5.1233 (4.6998)	-31.3840 (5.8777)
Site 3	-68.3495 (9.0038)	-93.2218 (11.3726)	465.8730 (33.9086)	-114.2434 (11.4588)	-83.9030 (12.3329)
Site 4	-5.1537 (3.7251)	-5.1233 (4.6998)	-114.2434 (11.4588)	136.6578 (9.1271)	-18.1011 (5.4832)
Site 5	-28.5073 (5.0161)	-31.3840 (5.8777)	-83.9030 (12.3329)	-18.1011 (5.4832)	194.9628 (13.9310)

Note: standard error in parentheses.

Table 5.3  
 Estimates of Own and Cross Price Marginal Effects on Notional Demands  
 Virtual Price Model (without Quality Variables, User Only)

Mean	Intercept	Site1	Site 2	site 3	site 4	Site 5
	$\alpha$	$\beta$				
Site 1	0.7968 (0.7097)	-0.2112 (0.0091)	0.1268 (0.0079)	0.0018 (0.0081)	-0.0025 (0.0044)	0.1273 (0.0078)
Site 2	0.6187 (0.8004)	0.1268 (0.0079)	-0.3079 (0.0118)	0.1730 (0.0103)	0.0171 (0.0051)	-0.0112 (0.0075)
Site 3	5.7740 (1.1667)	0.0018 (0.0081)	0.1730 (0.0103)	-0.7349 (0.0266)	0.2065 (0.0107)	0.2446 (0.0124)
Site 4	1.9256 (0.6483)	-0.0025 (0.0044)	0.0171 (0.0051)	0.2065 (0.0107)	-0.2019 (0.0080)	0.0389 (0.0056)
Site 5	3.8884 (0.9296)	0.1273 (0.0078)	-0.0112 (0.0075)	0.2446 (0.0124)	0.0389 (0.0056)	-0.4238 (0.0155)

Note: standard error in parentheses

Table 5.4  
 Estimates of Variance-Covariance Matrix (  $\Sigma$  ) on Notional Demands  
 Virtual Price Model (without Quality Variables, User Only)

Mean	Site1	Site 2	site 3	site 4	Site 5
Site 1	103.8834 (6.9725)	-12.9400 (4.0383)	-67.1977 (8.4286)	-5.1191 (3.7367)	-29.1295 (5.1432)
Site 2	-12.9400 (4.0383)	151.5538 (9.6812)	-91.5075 (10.5161)	-4.6416 (4.7333)	-31.7501 (6.0376)
Site 3	-67.1977 (8.4286)	-91.5075 (10.5161)	458.7090 (31.7955)	-110.2043 (10.9277)	-83.9459 (11.9575)
Site 4	-5.1191 (3.7367)	-4.6416 (4.7333)	-110.2043 (10.9277)	133.1760 (8.4349)	-17.9549 (5.1110)
Site 5	-29.1295 (5.1432)	-31.7501 (6.0376)	-83.9459 (11.9575)	-17.9549 (5.1110)	198.4191 (13.3367)

Note: standard error in parentheses.

Table 5.5  
 Estimates of Own and Cross Price Marginal Effects on Notional Demands  
 Virtual Price Model (with Demographic Variables, User Only)

Mean	Intercept	Quality Variable	Male Dummy	Age	College Dummy	Site1	Site 2	site 3	site 4	Site 5
	$\alpha$	$\varphi$	$\rho$			$\beta$				
Site 1	1.5587 (0.7546)	0.0280 (0.0031)	0.2087 (0.1172)	-0.0195 (0.0032)	-0.1960 (0.1060)	-0.2124 (0.0093)	0.1271 (0.0078)	0.0004 (0.0081)	-0.0029 (0.0043)	0.1268 (0.0079)
Site 2	0.0376 (0.8255)	0.0280 (0.0031)	0.2087 (0.1172)	-0.0195 (0.0032)	-0.1960 (0.1060)	0.1271 (0.0078)	-0.3068 (0.0119)	0.1738 (0.0108)	0.0180 (0.0052)	-0.0117 (0.0070)
Site 3	5.4686 (1.1910)	0.0280 (0.0031)	0.2087 (0.1172)	-0.0195 (0.0032)	-0.1960 (0.1060)	0.0004 (0.0081)	0.1738 (0.0108)	-0.7314 (0.0260)	0.2071 (0.0106)	0.2406 (0.0122)
Site 4	2.1274 (0.6552)	0.0280 (0.0031)	0.2087 (0.1172)	-0.0195 (0.0032)	-0.1960 (0.1060)	-0.0029 (0.0043)	0.0180 (0.0052)	0.2071 (0.0106)	-0.2029 (0.0081)	0.0391 (0.0058)
Site 5	3.5910 (0.8603)	0.0280 (0.0031)	0.2087 (0.1172)	-0.0195 (0.0032)	-0.1960 (0.1060)	0.1268 (0.0079)	-0.0117 (0.0070)	0.2406 (0.0122)	0.0391 (0.0058)	-0.4187 (0.0167)

Note: standard error in parentheses

Table 5.6  
 Estimates of Variance-Covariance Matrix (  $\Sigma$  ) on Notional Demands  
 Virtual Price Model (with Demographic Variables, User Only)

Mean	Site1	Site 2	site 3	site 4	Site 5
Site 1	104.7655 (7.3207)	-13.2569 (3.9103)	-66.6420 (8.4045)	-5.1730 (3.6893)	-28.5517 (5.0381)
Site 2	-13.2569 (3.9103)	149.8352 (10.1241)	-90.7228 (10.9674)	-4.3966 (4.4591)	-31.8988 (5.9767)
Site 3	-66.6420 (8.4045)	-90.7228 (10.9674)	451.9309 (31.8548)	-111.3941 (11.1009)	-81.9919 (11.8490)
Site 4	-5.1730 (3.6893)	-4.3966 (4.4591)	-111.3941 (11.1009)	133.0796 (9.1700)	-18.3290 (5.1739)
Site 5	-28.5517 (5.0381)	-31.8988 (5.9767)	-81.9919 (11.8490)	-18.3290 (5.1739)	192.3686 (13.8382)

Note: standard error in parentheses.

Table 5.7  
Welfare Estimates for Virtual Price Model  
Average Compensating Variation

Model Number	Policy Scenario		
	\$50 Access Fees at Site 1 and 5	\$50 Access Fees at Site 2	20% Increase in Pheasant Counts at Site 2
Main Model (User Only)	-63.95 (5.54)	-15.24 (1.81)	17.62 (2.38)
Without Quality Variables (User Only)	-64.88 (6.13)	-17.75 (2.06)	
With Demographic Variables (User Only)	-64.14 (4.97)	-15.82 (1.83)	14.55 (2.43)

Note: standard error in parentheses

Table 6.1  
 Estimates of Own and Cross Price Marginal Effects on Notional Demands  
 Amemiya-Tobin Model (without Quality Variables, User Only, 9 Sites)

Mean	Intercept	Site1	Site 2	site 3	site 4	Site 5	Site 6	Site 7	site 8	site 9
	$\alpha$	$\beta$								
Site 1	-7.6049 (1.2917)	-0.1517 (0.0104)	0.0476 (0.0082)	-0.0109 (0.0141)	0.0417 (0.0147)	-0.0552 (0.0161)	0.0277 (0.0104)	0.0703 (0.0132)	0.0181 (0.0121)	0.0152 (0.0152)
Site 2	-4.9536 (1.1358)	0.0476 (0.0082)	-0.1869 (0.0112)	0.0023 (0.0107)	0.0355 (0.0145)	0.0269 (0.0156)	0.0241 (0.0079)	0.0147 (0.0149)	0.0224 (0.0142)	-0.0220 (0.0144)
Site 3	-0.7654 (1.0479)	-0.0109 (0.0141)	0.0023 (0.0107)	-0.3958 (0.0256)	0.0850 (0.0185)	0.0729 (0.0214)	-0.0607 (0.0109)	0.0860 (0.0227)	0.0566 (0.0215)	0.0245 (0.0204)
Site 4	-7.2531 (1.3620)	0.0417 (0.0147)	0.0355 (0.0145)	0.0850 (0.0185)	-0.3502 (0.0245)	0.1174 (0.0238)	0.0520 (0.0096)	0.0269 (0.0291)	-0.0777 (0.0259)	0.0006 (0.0215)
Site 5	-0.1908 (1.0442)	-0.0552 (0.0161)	0.0269 (0.0156)	0.0729 (0.0214)	0.1174 (0.0238)	-0.5303 (0.0340)	0.0805 (0.0096)	-0.0103 (0.0320)	0.0296 (0.0309)	0.1174 (0.0254)
Site 6	-6.9819 (1.1239)	0.0277 (0.0104)	0.0241 (0.0079)	-0.0607 (0.0109)	0.0520 (0.0096)	0.0805 (0.0096)	-0.1721 (0.0080)	-0.0099 (0.0171)	0.0211 (0.0134)	0.0455 (0.0100)
Site 7	-6.2629 (1.3851)	0.0703 (0.0132)	0.0147 (0.0149)	0.0860 (0.0227)	0.0269 (0.0291)	-0.0103 (0.0320)	-0.0099 (0.0171)	-0.3695 (0.0357)	0.1161 (0.0263)	-0.0211 (0.0287)
Site 8	-6.7791 (1.3897)	0.0181 (0.0121)	0.0224 (0.0142)	0.0566 (0.0215)	-0.0777 (0.0259)	0.0296 (0.0309)	0.0211 (0.0134)	0.1161 (0.0263)	-0.3634 (0.0306)	0.1398 (0.0257)
Site 9	-4.2666 (1.2255)	0.0152 (0.0152)	-0.0220 (0.0144)	0.0245 (0.0204)	0.0006 (0.0215)	0.1174 (0.0254)	0.0455 (0.0100)	-0.0211 (0.0287)	0.1398 (0.0257)	-0.3673 (0.3673)

Note: standard error in parentheses

Table 6.2  
 Estimates of Variance-Covariance Matrix ( $\Sigma$ ) on Notional Demands  
 Amemiya-Tobin Model (without Quality Variables, User Only, 9 Sites)

Mean	Site1	Site 2	site 3	site 4	Site 5	Site 6	Site 7	site 8	site 9
Site 1	119.6154 (9.0098)	10.6543 (4.5900)	-5.0423 (4.8043)	1.7313 (7.0489)	-12.1493 (5.8120)	-6.0936 (6.2983)	8.6733 (5.8435)	4.4507 (5.3794)	-12.1486 (5.7518)
Site 2	10.6543 (4.5900)	91.7828 (6.2858)	15.0975 (3.4932)	0.9503 (5.8168)	3.1169 (4.6258)	4.7438 (4.4106)	17.8298 (5.2618)	12.3163 (4.9540)	0.2630 (5.2347)
Site 3	-5.0423 (4.8043)	15.0975 (3.4932)	72.5784 (5.5004)	9.1034 (4.5225)	8.7035 (3.3738)	10.2182 (3.5087)	18.9782 (4.3346)	21.5532 (4.4243)	14.7718 (3.8337)
Site 4	1.7313 (7.0489)	0.9503 (5.8168)	9.1034 (4.5225)	121.4573 (9.7709)	1.0285 (4.7437)	19.4120 (4.4880)	19.9617 (8.3870)	8.0092 (7.4882)	15.1295 (5.9215)
Site 5	-12.1493 (5.8120)	3.1169 (4.6258)	8.7035 (3.3738)	1.0285 (4.7437)	89.2709 (5.5276)	9.5576 (3.6565)	9.3273 (6.5601)	11.8396 (6.2481)	7.8815 (4.0502)
Site 6	-6.0936 (6.2983)	4.7438 (4.4106)	10.2182 (3.5087)	19.4120 (4.4880)	9.5576 (3.6565)	95.8008 (5.6483)	7.8283 (7.2763)	-1.4184 (6.0323)	11.2135 (3.9472)
Site 7	8.6733 (5.8435)	17.8298 (5.2618)	18.9782 (4.3346)	19.9617 (8.3870)	9.3273 (6.5601)	7.8283 (7.2763)	79.3206 (9.7191)	27.7735 (6.0141)	1.8132 (6.8292)
Site 8	4.4507 (5.3794)	12.3163 (4.9540)	21.5532 (4.4243)	8.0092 (7.4882)	11.8396 (6.2481)	-1.4184 (6.0323)	27.7735 (6.0141)	94.5471 (10.7347)	21.1011 (5.1834)
Site 9	-12.1486 (5.7518)	0.2630 (5.2347)	14.7718 (3.8337)	15.1295 (5.9215)	7.8815 (4.0502)	11.2135 (3.9472)	1.8132 (6.8292)	21.1011 (5.1834)	84.8150 (6.7997)

Note: standard error in parentheses.