Dynamic Behaviour of Deformable and Rigid Articulated Systems

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INTRODUCTION

The calculation of structures sometimes involves articulations and it is essential that these be modelled in order to analyse the mechanical behaviour of the ensemble. The increasing complexity of these connections shows the desirability of having a program for the calculation of mechanisms. The problems are, then, to determine the applied stresses on structures during their operation and to evaluate the maximum admissible stress in the structural members so as to choose the most suitable configurations. The program should also allow simulations and, through a better knowledge of their behaviour, it should help find an optimal solution for the design and the use of mechanically linked systems.

PLEXUS

These developments appear to be an extension of the calculation of structures by the method of finite elements. We have, therefore, started with PLEXUS, a program of the CASTEM system, which was developped by the C.E.A.-D.E.M.T. (Lepareux and al. 1988).This allowed us to benefit from all of the advantages attached to CASTEM and, in particular, from the pre-processor GIBI for the meshing program and the post-processor ALICE for the visualisations.

PLEXUS is an explicit program which makes it possible to readily introduce non-linear behaviour laws. It has already been well verified for dynamic calculations (Lepareux and al. 1982 & 1983). Moreover, there is a general algorithm for treating the couplings between degrees of freedom (dof) which facilitates the introduction of kinetic links associated with the mechanisms. However, as for all explicit programs, there is a stability time step which should not be exceeded in order to ensure the convergence of the calculations.

RIGID SOLIDS

In the case where the mechânical system is made up of an assembly of sub-structures with very different degrees of stiffness, the stability step is set by the component with the highest degree of stiffness. In the extreme case the calculation may become impractical.

If the user is not especially interested in this sub-structure it is reasonable to designate it as rigid. Thus, it does not limit the time step but its inertia and its kinetic properties are taken into account. In addition, it is necessary to be able to combine deformable and rigid structures in the same calculation.

It seemed interesting to us to be able to use a program that meshes these rigid solids together as finely as possible while keeping their complex form and their distribution of mass. Once the solid is perfectly defined, PLEXUS automatically calculates its tensor of inertia and then keeps the minimum of the necessary dof. Of course, if the solid is an assembly of simple shapes, the tensor of inertia can be given directly.

Reduction of a solid

Six dof are sufficient to describe a solid. However, we will replace the solid by 4 point masses. In fact, the inconvenience of introducing 6 additionnal dof is largely compensated for by the advantage of having a diagonal mass matrix.

Let M be the mass of the solid and J its principal tensor of inertia. The coordinates in the principal inertial reference frame of the 4 physical points are the following (Fig. 1) :

$$\begin{aligned} m1 &= (\ 0,\ -b,\ -c) \\ m2 &= (\ 0,\ b,\ -c) \\ m3 &= (-a,\ 0,\ c) \\ m4 &= (\ a,\ 0,\ c) \end{aligned}$$

with : $a = \sqrt{\dfrac{-jx + jy + jz}{M}}$ $\quad b = \sqrt{\dfrac{jx - jy + jz}{M}}$

$c = \sqrt{\dfrac{jx + jy - jz}{2M}}$

and : $J = \begin{bmatrix} jx & 0 & 0 \\ 0 & jy & 0 \\ 0 & 0 & jz \end{bmatrix}$

In addition, a quarter of the total mass is attributed to each point.

Complementary equations

We still have to write 6 kinetic equations representing the conservation of the distances between the 4 points :

$$\overrightarrow{M_1 M_2}^2 = \text{constant} \quad \Rightarrow \quad \overrightarrow{M_1 M_2} \cdot (\vec{V}_1 - \vec{V}_2) = 0$$

This is put in the form : C v = 0, where C is a matrix of the coefficients and v the vector of the velocities.

Let us suppose that this condition is true for the cycle n, it is necessary to determine the acceleration γ^{n+1} so that it remains true for the cycle n+1.

The geometry for the cycle n+1 is known by starting from the state of cycle n :

$$x^{n+1} = x^n + tv^n + \frac{t^2}{2}\gamma^n \qquad (t = \text{time step})$$

This leads to knowing the matrix C^{n+1}. On the other hand, the velocity at the end of the step is not known :

$$v^{n+1} = v^n + \frac{t}{2}(\gamma^n + \gamma^{n+1})$$

or $\quad v^{n+1} = v^{n+\frac{1}{2}} + \frac{t}{2}\gamma^{n+1} \qquad$ with $v^{n+\frac{1}{2}} = v^n + \frac{t}{2}\gamma^n$

The condition to be respected is, therefore :

$$C^{n+1} v^{n+1} = 0 \quad \Rightarrow C^{n+1} v^{n+\frac{1}{2}} + \frac{t}{2} C^{n+1} \gamma^{n+1} = 0$$

or

$$C^{n+1} \gamma^{n+1} = -\frac{2}{t} C^{n+1} v^{n+\frac{1}{2}}$$

The condition is of the form $C\gamma = S$, where C and S are known.

The complementary forces, F_C, that ensure that the liaisons are respected, still have to be determined. For this let us use the Lagrange multipliers :

$M\gamma = F_{ei} + F_C \qquad$ (F_{ei} = external and internal applied forces)

$F_C = C^T \lambda \qquad$ (λ = multipliers)

$C\gamma = S \quad \Rightarrow \quad S = CM^{-1}(F_{ei} + C^T\lambda)$

$S = CM^{-1} C^T\lambda + CM^{-1} F_{ei}$

The matrix $H = CM^{-1} C^T$ is symmetric. It can be inverted if the equations are compatible. From this, the multipliers can be deduced and then the complementary forces which give us the desired accelerations :

$$\lambda = H^{-1} (S - CM^{-1} F_{ei})$$

$$\gamma^{n+1} = M^{-1} (F_{ei} + C^T\lambda)$$

This is a general treatment of couplings and can be applied to all kinetic equations. It reduces to the inversion of a symmetric matrix which has a rank depending only on the number of coupled degrees of freedom.

MECHANISMS

We shall designate by the term mechanism any link between deformable or rigid sub-structures. A mechanism will be modelled by an "element" with two points with 6 dof, each point belonging to different sub-structures (Bung and al, 1986). The meshing program will thus be able to localise the mechanism readily and it will be easy to introduce the behaviour laws in the case of non linear articulations.

The equations relating the two points to each other will depend on the kinetic linking. If this is placed in a local frame of reference connected to the mechanism their form will be especially simple.

In the case where the points making up the sub-structure have only 3 dof (the case of a solid), it will be necessary to designate 2 additionnal points to be able to write the equations describing the rotations (Figure 2). These two points then define a proximity of the mechanism which will assumed to be rigid. If we are dealing with a rigid solid then it will suffice to take 2 points resulting from the reduction.

Motor torque

The choice of an element to describe a mechanism immediatly allows us to transform a hinge joint into a motor. All that is necessary is to situate it in the local frame of reference and to apply the torque about the hinge axis. The same method can be use for a guide rod, which can thus be very simply transformed into an actuator.

Moreover, the torque or the force can be servo-controlled by a command chosen by the user. At first, we used a form of the type P.I.D. (proportionnal, integral, derivative) for the servo-control :

$$C_m = K_o - K_p (\Theta - \Theta_c) - K_d (\dot{\Theta} - \dot{\Theta}_c) - K_i (I - I_c)$$

where Θ_c, $\dot{\Theta}_c$, I_c are the commands for the position, the velocity and for the integral of the position and Θ, $\dot{\Theta}$, I are the corresponding calculated values. The constants K_o, K_p, K_i, K_d are left to choice of the user.

Direct current motor

The model of the servo-control moter described above neglects the electromechanical torques that are brought into play in a direct current motor. In practise, it is essential to take the laws governing electrical behaviour into account (Barraco and al, 1987). These depend on the torque produced by the motor and the velocity. In addition, the servo-controlled physical quantity is not the torque but the input voltage, as this latter is always limited by the saturation voltage.

The behaviour of such a motor can be modified according to the following equations :

R, L	resistance and self-inductance of the coils
I, V	intensity and voltage of the input current
N	reduction ratio
E	counter electromotive force
T_m, Ω_m	torque and angular velocity at the motor axis

$$V = RI + L\frac{dI}{dt} + E \qquad E = K\Omega_m \qquad T_m = KI$$

The torque T applied at the axis at the position of the arm is then :

$$T = N\,T_m - f\dot{\Theta} - T_d \qquad \text{with } \dot{\Theta} = N\,\Omega_m$$

The constant K depends on the motor and f and T_d are, respectively, the coefficients of viscous and dry friction.

To complete these equations we need only those related to the servo-control :

$$V = V_o - K_p (\Theta - \Theta_c) - K_d (\dot{\Theta} - \dot{\Theta}_c)\,)\,Ki\,(I - I_c) \qquad |V| \leq V_{max}$$

EXAMPLES

The majority of the examples are taken from robotics.

Flexible arm with servo-controlled rotation (Fig. 3).

The extremity of an elastic beam is fixed to a motor, the other end carries a point mass. The angle of rotation and the angular velocity are servo-controlled. This test shows the additionnal flexibility provided by the servo-control.

Manipulator with 3 axes of rotation (Fig. 4).

The manipulator consists of 5 rigid solids connected together. There are three motors for actuating the ensemble. This test was the object of a benchmark study carried out in liaison with the E.N.S.A.M. at Paris and the University of Karlsruhe (F.R.G.) (Wittenburg and al, 1985).

Simulation of an industrial robot (Fig. 5)

This test shows the advantage of a meshing program to define the complicated shapes that are sometimes found in robotics. This simulation made it possible to optimise the motor torques and the servo-control parameters for efficient use of the robot.

CONCLUSIONS

These developments have made PLEXUS a reference program for the study of complex structures, for simulating their behaviour and optimising their performance. It has been verified using simple tests and the latest studies show that it is now operational for use in industrial applications.

REFERENCES

Barraco A., Cuny B., Lepareux M., Bung H., (1987) Computer simulations of manipulator dynamic using different control laws. 2nd Int. Conf. on Robotics and Factories of the Future - San Diego, USA July 28-31.

Bung H., Lepareux M., (1986) Dynamique des robots - Mécanismes spatiaux - Articulations - Rapport DEMT 86/201.

Lepareux et al. (1982) Calcul et analyse de structures en mécanique et thermique - Fiches de validation - Rapport DEMT/82-075.

Lepareux M., Bung H., (1983) CASTEM System - PLEXUS Program - Summarised presentation of its feasibilities - DEMT/83-084 Report.

Lepareux et al (1988) CASTEM System - PLEXUS - User's manuel.

Wittenburg J., Wolz U., (1985) The program MESA VERDE for robot dynamic simulations - 3rd International Symposium of Robotics Research - Gouvieux.

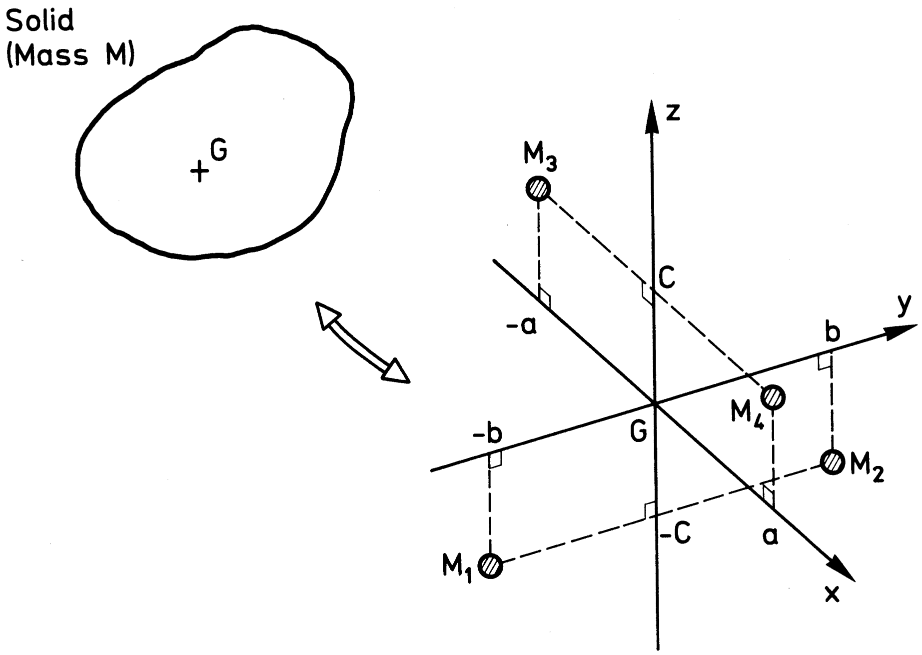

FIGURE 1 : REDUCTION OF A SOLID INTO 4 NODAL MASSES

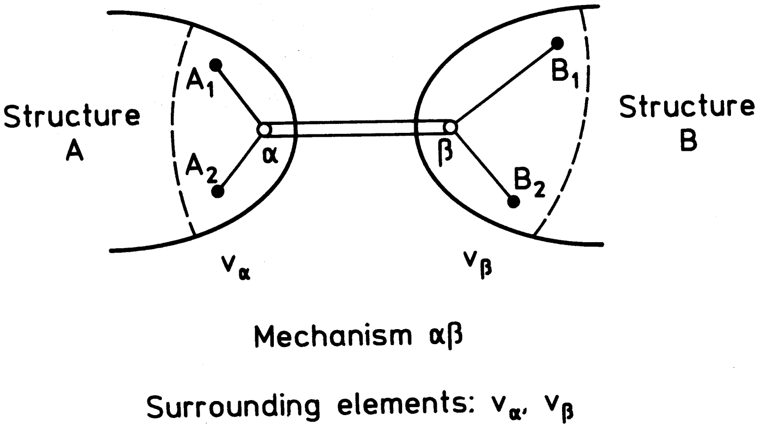

FIGURE 2 : DEFINITION OF A MECHANISM AND OF ITS SURROUNDING ELEMENTS

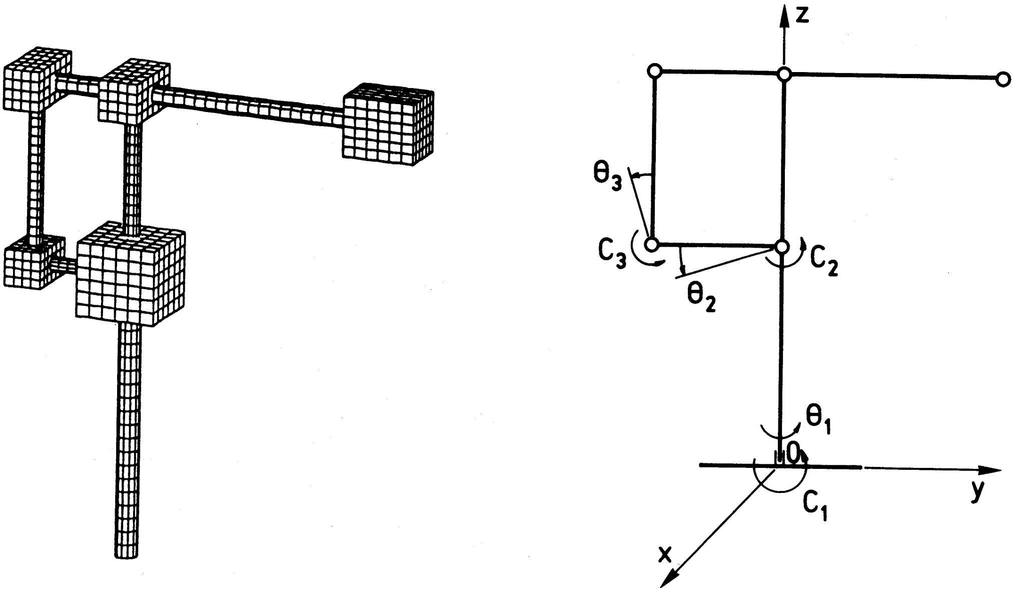

FIGURE 3 : THREE ROTATIONS MANIPULATOR

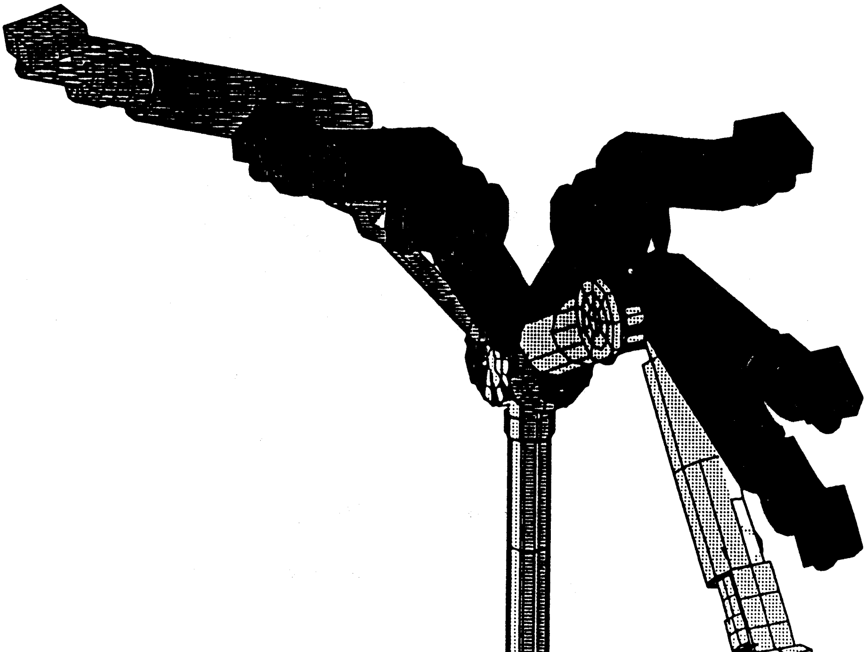

FIGURE 4 : SIMULATION OF AN INDUSTRIAL ROBOT