

## Approximate Formulas to Evaluate Non-Linear Soil Amplifications

R. Carpaneto, A. Del Grosso, S. Lagomarsino, G. Solari  
*Università degli Studi di Genova, Istituto di Scienza delle Costruzioni,  
 Via Montallegro 1, I-16145 Genova, Italy*

### Abstract

In this paper the results are presented of a parametric analysis concerning the evaluation of the amplification characteristics of surficial deposits of homogeneous cohesive or granular soils with nonlinear behaviour. The analysis has led to the definition of empirical formulas, able to determine an estimate of the period for which the amplification function of the stratum exhibits the first peak. This estimated value, called "equivalent fundamental period", can replace in the applications the fundamental period usually calculated under linear hypotheses.

### 1. Introduction

In local amplification and soil-structure interaction studies, the consideration of non-linear soil behaviours is taking increasing relevance, especially when strong seismic motions are of concern. Recently, several methods have been developed, for the evaluation of the dynamic nonlinear response of soils, based upon numerical time integration techniques or upon the definition of equivalent linear problems, either in time or frequency domain. Comparative analyses and critical evaluations of these methods are presented, for example, in [1] and [2].

Although this problem can be considered relatively well established from the theoretical and computational point of view, the assessment of simple techniques, eventually applicable in codes and standards, is still an open aspect. As a matter of fact, in present regulations, the consideration of the effects due to local dynamic characteristics of sites is substantially reducible to the two following approaches:

- a) through the definition of site dependent spectra from statistical analyses of recorded accelerograms pertaining to sites of known characteristics, representing typical geological conditions (New Zealand, Mexico, Costarica, Japan [3] );
- b) through the modification of design spectra referred to standard geological conditions, based upon the fundamental period of the surficial soil deposit (USA, Turkey, Chile [3]).

The first approach presupposes the availability of a sufficient number of records and, on the other hand, includes the effects of nonlinearities through the real soil behaviour. The second method is generally applicable and gives approximate evaluations of particular local

need of a proper evaluation of nonlinear soil behaviours

- 2) it depends on the frequency content of the input motion; this dependency is becoming more evident as the soil flexibility increases and, therefore, as the nonlinearity increases as well.

The second remark is quite important since it leads to the definition of an equivalent fundamental period  $T_0$  also as a function of the frequency, and not only as a function of soil properties and earthquake intensity, as would be desirable in view of applications. This consideration suggests the opportunity of defining  $T_0$  through the following expression:

$$T_0 = \bar{T}_0 + \eta \Delta T_0 \quad (-1 \leq \eta \leq +1) \quad (2)$$

where  $\bar{T}_0$  is the mean value of  $T_0$  with respect to the variation of the frequency content of the input motions, and  $\Delta T_0$  is the halfwidth of the range of  $T_0$  values. It should be pointed out that, independently of the difficulty of defining appropriate parameters characterizing the frequency content of an earthquake, the earthquake is essentially an unpredictable event and this justifies the definition of a range  $2\Delta T_0$  of the possible, and a priori equiprobable,  $T_0$  values.

The results of the parametric analysis suggest the choice of the following expression for  $T_0$ :

$$\bar{T}_0 = \alpha T_0^* \quad (3)$$

in which  $T_0^*$  is the (linear) fundamental period of the deposit and  $\alpha$  is a dimensionless coefficient.  $T_0^*$  is given by the well known expression:

$$T_0^* = \frac{4H}{v_s} \quad (4)$$

$v_s$  being the shear wave velocity. This latter quantity is practically constant for a homogeneous cohesive soil deposit and varies with depth in granular soils. In this case an average  $v_s$  value can be defined, or, according to [5], the following formula can be used:

$$T_0^* = c F_{Dr} F_\phi F_\rho F_M H^{3/4} \quad (5)$$

where  $c = 3.5 \cdot 10^{-2} \text{ s} \cdot \text{m}^{-3/4}$ , and:

$$F_{Dr} = \frac{1.8}{\sqrt{3.75Dr + 1}} \quad ; \quad F_\phi = \frac{1}{(3 - 2 \sin \phi)^{1/4}}$$

$$F_\rho = \left[ \frac{\rho}{\rho_M} \right]^{1/4} \quad F_M = \frac{1}{(1-f)^{3/4} (1-\lambda) + \lambda \left[ 1-f \left( 1 - \frac{1}{\lambda} \right) \right]^{3/4}}$$

$$\lambda = \frac{\rho}{\rho_s - \rho_M} \left[ \frac{\rho}{\rho_s} \right]^{1/2} \quad \lambda_\rho = \frac{\rho}{\rho_s - \rho_M}$$

in which  $\rho$ ,  $\rho_s$  and  $\rho_M$  are the dry or moist soil density, the saturated soil density and the water density, respectively.

The parametric studies carried out for granular soil deposits have shown that the coefficient

$\alpha$  in Eq.(3) depends on the peak acceleration, the relative density and the water table elevation with respect to the free surface. An expression fitting the results of the numerical analyses was found to be:

$$\alpha = \left( 1 + 3.4i + \frac{2.6f^2 i^{1/2}(1+3.4i)}{10 + 2f^2} \right) \left( \frac{3.75Dr + 1}{3.25} \right)^{0.44i^{3/4}} \quad (6)$$

where  $f$  and  $D$  have the meaning already defined, and  $i$  is the peak acceleration in g's. The scattering of the results was also fitted with a function  $\Delta T_0$  given by:

$$\Delta T_0 = \begin{cases} 0.025 \sqrt{I} H \bar{T}_0 & \text{for } 0 \leq H \leq 20\text{m} \\ 0.5 \sqrt{I} \bar{T}_0 & \text{for } H > 20\text{m} \end{cases} \quad (7)$$

The fitting of the numerical results presented some more difficulties in cohesive soil models, because of the more relevant effect of nonlinearity, and the  $\alpha$  coefficient was found to be dependent, besides on the undrained cohesion and on the peak acceleration, also on the equivalent fundamental period itself. The best fitting expression of  $\alpha$  was found to be:

$$\alpha = \frac{205}{K (0.02 C_u)^{\beta}} \quad (8)$$

being:

$$K = \frac{17.5 + 33i^{2/3}}{0.085 + i^{2/3}} + \frac{1.6 i^{2/3}}{0.003 + \bar{T}_0 i^{4/3}}$$

$$\beta = \frac{4.75 i}{1 + 22.5 i}$$

In the formulas above,  $C_u$  is expressed in  $\text{kN/m}^2$  and  $\bar{T}_0$  in seconds.

Due to the substitution of Eq.(9) into Eq.(3),  $\bar{T}_0$  is the root of a second degree equation. However, without significant loss of accuracy in most practical cases,  $\bar{T}_0$  can also be given by the linear equation:

$$\bar{T}_0 = Aw + B \quad (9)$$

where:

$$w = \frac{173H\sqrt{\rho}}{C_u^{2/3}}$$

$$A = (2.6 \sqrt{I} + 0.33) \cdot 10^{-2}$$

$$B = -(17.5 \sqrt{I} - 1.2) \cdot 10^{-2}$$

In the above,  $H$  is expressed in meters and  $\rho$  in  $\text{Kg/m}^3$ . A good fitting of the numerical results

situations although it utilizes behavioural parameters derived from linear assumptions. This paper, which synthesizes and completes two preceding researches [4], [5], represents an attempt to extend the second approach to include nonlinear soil effects, by means of a parameter obtainable with simple formulas. This parameter, herein defined as the "equivalent fundamental period"  $T_0$ , is given by semi-empirical expressions derived from a numerical parametric analysis performed on homogeneous cohesive and granular soil deposits, for varying geometrical and mechanical properties of the soil and expected peak acceleration.

## 2. Parametric Analyses

With reference to fig.1, the amplification function of the deposit is defined as the ratio between the Fourier spectra of the earthquake in B and in A. Parametric analyses have been carried out by means of the computer program SHAKE [6], in order to determine, from the amplification function, the value of the period corresponding to the first peak, to be interpreted as the equivalent fundamental period of the deposit. In the analyses the relevant soil parameters have been varied according to the values shown in Table I. It should be noted that preliminary analyses have shown negligible influence of the friction angle of granular soils onto the nonlinear response of the deposit. The shear moduli  $G$ , and the damping ratios  $D$  have been determined, as functions of the shear strain  $\gamma$ , according to the laws given by SEED and IDRIS [7]. The models of the deposits have been subjected to various horizontal acceleration time histories, taken from recently recorded Italian earthquakes. The time histories have been scaled to several values of the peak acceleration, ranging from 0.005 g to 0.35 g. The accelerograms have been applied to the outcropping of rock (point A in fig.1) in the SHAKE models. For each time history the characteristics of the rock have been defined accordingly to the geologic conditions of the corresponding seismic stations. Among the available recorded earthquakes, the ones considered in this study were selected on the basis of their energy content and in such a way to obtain a broad range of frequency contents. The time histories used belong to the set of records shown in Table II, in which the real peak ground acceleration and the mean frequency

$$\bar{n}_A = \frac{\int_0^{\infty} n S^2(n) dn}{\int_0^{\infty} S^2(n) dn} \quad (1)$$

are also shown. In eq. (1) above,  $n$  indicates the frequency and  $S(n)$  is the Fourier spectrum.

## 3. Results of the analyses

The parametric analyses have shown that the period corresponding to the first peak of the amplification function is obviously dependent on the geometrical and mechanical soil properties and, moreover, presents the following characteristics:

- 1) it is strictly related to the acceleration level of the input motion; this confirms the

was observed for  $w > 10$  and  $0.05 \leq i \leq 0.35$ .

The parametric analyses for cohesive soils have finally suggested the following expression for the scattering:

$$\Delta T_{\alpha} = \begin{cases} \frac{2.45 \bar{T}_o^2 i}{4+100 i} & \text{for } \bar{T}_o < 1.5 \text{ s} \\ \frac{36.75 \bar{T}_o i}{4+100 i} & \text{for } \bar{T}_o \geq 1.5 \text{ s} \end{cases} \quad (10)$$

It should be noted that in all the above formulas  $\Delta T_{\alpha} \rightarrow 0$  and  $\alpha \rightarrow 1$  when  $i \rightarrow 0$ . The same behaviour is obviously not experienced by Eq. (9).

#### 4. Concluding remarks

This paper has presented a methodology for the evaluation of the "equivalent fundamental period" of homogeneous surficial soil deposits, composed by cohesive or granular soils. The methodology is based onto empirical formulas derived from parametric numerical analyses. These formulas could be used in applications usually requiring the knowledge of the fundamental period of soil deposits.

The analysis has put into evidence that nonlinear effects are greater in cohesive soils. Generally, nonlinearities in soils give increases in the value of the period corresponding to the peak of the amplification function. This increase can be very significant in some instances.

From the point of view of the structural seismic response, the study has also suggested the following considerations:

- a) the evaluation of the equivalent fundamental period through Eq.(2) leads to the definition of a range of possible value for  $T_o$ ; the fundamental period of a structure founded on such a deposit should be compared with all the values within this range;
- b) due to the above consideration it is not a priori possible to exclude that a structure would experience seismic forces greater than the ones due to the design earthquake defined for the site, because of seismic events characterized by peak accelerations less than the ones of the design earthquake.

#### Acknowledgements

This research has been supported by CNR-GNDT (Italian National Research Council) under Grant No. 83.01328.54.

#### 4. References

- [1] CONSTANTOPOULOS I.V., ROESSET J.M., CHRISTIAN J.T.: "A Comparison of Linear and Exact Nonlinear Analyses of Soil Amplification", Proceedings, 5th World Conference on Earthquake Engineering, Rome, Vol.2, (1973).
- [2] CORSANEGO A., SOLARI G., STURA D.: "A Comparison of Approximate Techniques for Nonlinear Seismic Soil Response", Earthquake Engineering and Structural Dynamics, Vol.12, (1984).

- [3] INTERNATIONAL ASSOCIATION OF EARTHQUAKE ENGINEERING: "Earthquake Resistant Regulations. A World List", (1973); supplement (1980).
- [4] CARPANETO R., DEI GROSSO A., LAGOMARSINO S., SOLARI G.: "Un procedimento per la valutazione dell'amplificazione di terreni in regime nonlineare", Atti del Secondo Convegno Nazionale "L'Ingegneria Sismica in Italia", Rapallo (Genova), (1984).
- [5] LAGOMARSINO S.: "Sull'amplificazione di depositi di terreno incoerente in regime nonlineare", Pubblicazione dell'Istituto di Scienza delle Costruzioni, University of Genoa (in press).
- [6] SCHNABEL P.B., LYSMER J., SEED H.B.: "Shake: A Computer Program for Earthquake Response Analysis of Horizontally Layered Sites", Earthquake Engineering Research Center, Report N.EERC 72-12, University of California, Berkeley, (1972).
- [7] SEED H.B., IDRIS I.M.: "Soil Moduli and Damping Factors for Dynamic Response Analyses", Earthquake Engineering Research Center, Report N. EERC 70-10, University of California, Berkeley, (1970).

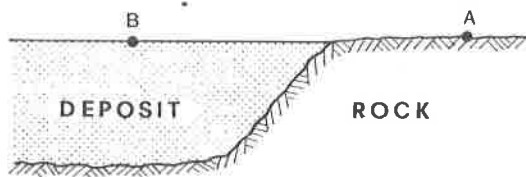


Fig.1

TABLE I - Ranges of Parametric Analyses

	GRANULAR SOILS	COHESIVE SOILS
	$D_r = 30+90\%$ $\phi = 33 \text{ degrees}$	$C_u = 15+220k \text{ N/m}^2$
$\rho$	1700+2000 Kg/m <sup>3</sup> dry or moist 1900+2150 Kg/m <sup>3</sup> saturated	1600+2000 Kg/m <sup>3</sup>
$f$	$0+1$	-
$H$	6 + 70 m	

$D_r$  = relative density  
 $\phi$  = friction angle  
 $f$  =  $H_w/H$  = ratio between the water table elevation and height of the stratum  
 $H$  = height of the stratum  
 $C_u$  = undrained cohesion

TABLE II - Selected Input Motions

EARTHQUAKE	STATION	DIR.	$n(\text{Hz})$	$a_{max}(g)$
Irpinia 1980	Calitri	E-W	2.04	0.13897
	Mercato S.S.	E-W	2.96	0.13127
	Sturmo	E-W	3.01	0.31144
	Bagnoli Ir.	N-S	3.70	0.13697
	Brienza	N-S	5.25	0.21347
Friuli 1976	Buia	N-S	1.82	0.23256
	Tolmezzo	N-S	3.50	0.34455
	S.Rocco	E-W	3.73	0.23751
	Folgaria-C.	E-W	4.35	0.30737
	Torcento	N-S	7.23	0.13150