Transactions of the **13th International Conference on Structural Mechanics in Reactor Technology** (SMiRT 13), Escola de Engenharia - Universidade Federal do Rio Grande do Sul, Porto Alegre, Brazil, August 13-18, 1995

Some numerical results for nonlinear bending of tubes

Selke, C.A.C.[1], Luersen, M.A.[1], Pereira, J.T.[2], Imaeda, F.T.[3]

1) UFSC, Mechanical Engineering Department, GRATE, Florianópolis, SC, Brazil
2) CEFET/PR, Mechanical Department, Curitiba, PR, Brazil
3) COSIPA - Companhia Siderúrgica Paulista, São Paulo, SP, Brazil

ABSTRACT: This work presents a study on numerical simulations of the problem of nonlinear bending of tubes, using a thin shell finite element. Possible secondary bifurcations in finite-length tubes, with circular and elliptic cross sections, are investigated.

1. INTRODUCTION

The finite element method (FEM) has been used successfully to solve a great number of nonlinear problems in the field of solid and structural mechanics. Despite the immense success of the FEM in performing numerical simulations of several problems of the area of elastic stability of structures (Gallagher, 1987), the nonlinear bending and buckling of shells are areas that still have some problems, which require additional studies (Ramm and Stegmüller, 1982). Among these problems and considering the great use of pipes in the industry, the nonlinear bending of tubes is a topic of major research interest. Lately, special attention has been devoted to the study of certain secondary bifurcation problems, that occur in bent tubes, with circular and elliptic cross sections.

The problem of a curved tube, being bent by the action of end couples, was first investigated by T. van Kármán in 1911, who studied the flattening of the cross sections, known as the van Kármán effect (Emmerling, 1984). A special case of this problem, which is a straight long tube, bent by end couples, has its failure occurring by a limit-point instability, and this nonlinear problem was first studied by L.G. Brazier in 1927. This limit-point instability case is characterized by the excessive ovalization of the cross section, the so called Brazier effect, and it is associated with the maximum value of the bending moment (Brush and Almroth, 1976).

Despite some experimental evidences showing that long bent tubes would fail by a bifurcation-point instability, at levels of loading lower than the maximum value of the correspondent bending moment to the limit-point instability, it was not until the 60's that E.L. Axelrad obtained a set of equations that allow for the simulation of the failure of a long bent tube as a secondary bifurcation phenomenon (Emmerling, 1984).

Among the more elaborated theories, proposed lately for the study of bending problems in shells, it is important to make a reference, again, to the works of E. L. Axelrad, on the failure of bent tubes. To study this problem, he introduced the concept of Flexible Shells, a shell theory where the bending theory aspects are kept only for the circunferencial direction of the bent cylinders, while, in the meridional direction, a simplified membrane stress state is assumed (Axelrad, 1984 and 1992; Axelrad and Emmerling, 1986).

A study on the numerical simulation of the nonlinear problem of bending of tubes is presented in this work, using a nine node thin shell element, obtained through an updated lagrangian formulation, based on the principle of virtual work, with the use of the discrete Kirchhoff hypothesis. The results presented are related to the determination of the nonlinear deformation and of the collapse load of elastic straight finite-length tubes, with circular and elliptic cross sections, subjected to bending loads.

2. FORMULATION OF THE THIN SHELL ELEMENT

The numerical simulations of the phenomenon of nonlinear bending of tubes are performed using a thin shell finite element, whose formulation is now presented, in a summarized way. This formulation was originally utilized for obtaining a finite element, which was used in numerical simulations of elastic-plastic problems in plates and shells (Imaeda, 1992).

The incremental version of the principle of virtual work for elastic-plastic problems in a tridimensional body Ω, can be written as (McMeeking and Rice, 1975)

(1) $$\int_{\Omega}\left\{\tau^{*}_{ij}\,\delta D_{ij}-\tfrac{1}{2}\sigma_{ij}\,\delta\left(2\,D_{ik}D_{kj}-v_{k,i}v_{k,j}\right)\right\}d\Omega=\int_{\Gamma}\dot{t}_{i}\,\delta v_{i}\,d\Gamma+\int_{\Omega}\dot{b}_{i}\,\delta v_{i}\,d\Omega$$

where $\overset{*}{\tau}_{ij}$ are the components of the Jaumann stress rate tensor, D_{ij} are the components of the strain rate tensor, σ_{ij} are the components of the Cauchy stress tensor, v_i are the components of the velocity vector, and $\dot{b}_i$ and $\dot{t}_i$ are, respectively, the components of the body force rate vector and the surface traction vector on the boundary Γ.

The principle of virtual work, given by (1) in its incremental version, is submitted to a discretization procedure, in which the body Ω, in this case a thin shell, is assumed to be divided into N_E nine nodes thin shell finite elements. Inside of each element, the velocities are approximated by

(2) $$\{v\}=[N]\{\dot{\psi}\}$$

where $\{\dot{\psi}\}$ are the nodal displacements and rotations rate vector, and $[N]$ is the shape function matrix. As a consequence, the strain rate tensor D_{ij} is interpolated as

(3) $$\{D\}=[B]\{\dot{\psi}\}$$

where the components of $[B]$ are given by

(4) $$B_{ij}=\frac{1}{2}\left(N_{i,j}+N_{j,i}\right).$$

This element is similar to the "SemiLoof" shell element (Irons, 1976), and it contains nine nodes, with the nodal displacements and rotations being its degrees of freedom. For the corner nodes, the degrees of freedom are the displacements and for the nodes at the sides and at the centroid, the rotations are the degrees of freedom. The formulation, which includes the discrete Kirchhoff hypothesis, is such that, in the end, all these rotations are eliminated, being represented by the rotations at the mid-side nodes of the element. The detailed formulation of this finite element is given in the literature (Nagtegaal and Slater, 1981; Imaeda, Selke and Blass, 1993). Finally, the discretization procedure leads to the incremental finite element equations of the body Ω (Imaeda, 1992)

$$(5) \qquad \sum_{E=1}^{N_E} \{\delta\dot{\psi}\}^T \left(\left[{}^E K^o \right] + \left[{}^E K^\sigma \right] \right) \{\dot{\psi}\} = \sum_{E=1}^{N_E} \{\delta\dot{\psi}\}^T \left\{ {}^E \Delta F \right\}$$

where

$$(6) \qquad \left[{}^E K^o \right] = \int_{\Omega_E} [B]^T [C] [B] d\Omega$$

is the incremental stiffness matrix, similar to the matrix for the small strain and displacements analysis, where [C] is the constitutive matrix, given by the adopted constitutive law $\overset{*}{\tau}_{ij} = C_{ijkl} D_{kl}$,

$$(7) \qquad \left[{}^E K^\sigma \right] = \int_{\Omega_E} \left([N]_{,i} [\sigma] [N]_{,j} - 2 \, [B]^T [\sigma] [B] \right) d\Omega$$

is the initial stresses stiffness matrix, where $[\sigma]$ is the stress matrix, and

$$(8) \qquad \left\{ {}^E F \right\} = \int_{\Omega_E} [N]^T \left\{ \dot{b} \right\} d\Omega - \int_{\Gamma_E} [N]^T \left\{ \dot{t} \right\} d\Gamma$$

is the incremental load vector. The system of equations (5) is solved, at each increment, using the BFGS method (Bathe, 1982).

3. NUMERICAL RESULTS

The finite element, briefly described above, was used to make a numerical study of the nonlinear bending of tubes, with a special interest on possible secondary bifurcations that may occur in finite-length tubes, with circular and elliptic cross sections.

The first problem to be studied is the bending, by end couples M, of a finite-length tube, with a circular cross section. The results obtained in this analysis, using the thin shell finite element, are presented in the Figure 1, where they are compared with the analytical solution, given in literature, for very long tubes (Emmerling, 1984; Axelrad and Emmerling, 1986). All the physical characteristics of the problem are shown in a nondimensional form. Emmerling had predicted that the secondary bifurcation should occur at the nondimensional value of the bending moment, M = 1.016, while the limit-point instability would occur at the value of M = 1.057, a little smaller than the analytical value given by Brazier, which is M = 1.089. As can be seen from the results shown, the finite element model, where one quarter of the tube is discretized using 1440 thin shell elements, was able to catch the secondary bifurcation, obtaining it at the value M = 1.01404 which, in comparison with the solution given by Emmerling, has a relative error of $\in$ = -0.1927 %.

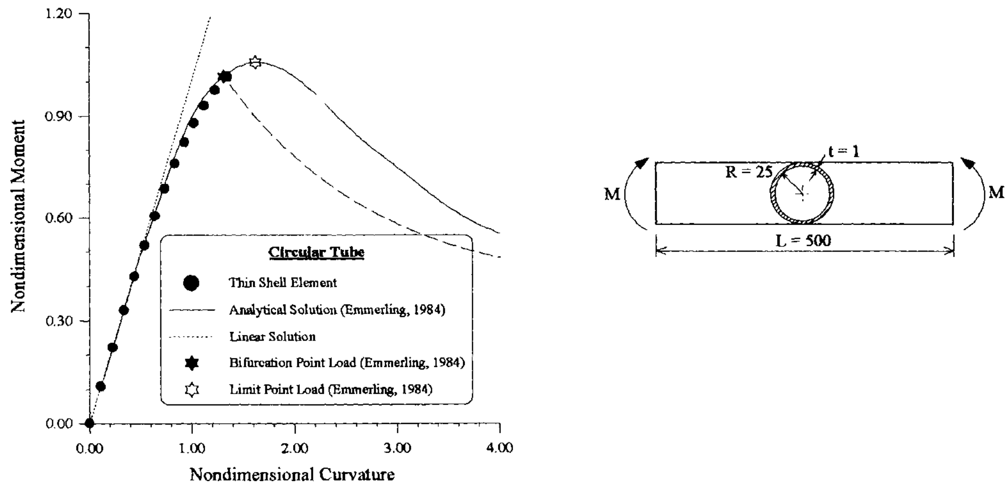

Figure 1. Nonlinear bending of a circular tube - Bending Moment X Curvature

The second problem studied was the bending, by end couples, of a finite-length tube, with an elliptic cross section. The results obtained in this analysis, using the thin shell element, are presented in Figure 2, where they are compared with the analytical solution, obtained for very long tubes, and given in the literature (Emmerling, 1984; Axelrad and Emmerling, 1986). Once again, the nondimensional form of the physical characteristics of the problem, is used. The graphical representation of the results show that the accuracy of the numerical solution for the case of an elliptic cross section, is not as good as the one obtained for the case of a circular cross section. The secondary bifurcation phenomenon was predicted by Emmerling to occur at the nondimensional value of bending moment M = 0.8851, while the limit-point instability was predicted to occur at the value M = 0.9896. Again, the model with one quarter of the tube being discretized by 1440 thin shell finite elements, was able to catch the secondary bifurcation, and it did at the value M = 0.7991, with a relative error of $\in$ = -9.7175%.

Finally, the bending, by end couples, of a shorter finite-length tube, with circular cross section, was numerically studied. In this case, the tube had its thickness/radius ratio with the value 0.1, which qualified it as a limit case, between a thin and a thick shell. The results obtained in this analysis are presented in the Figure 3. Once again, the nondimensional form of the physical characteristics of the problem is used. The model, one quarter of the tube discretized using 1200 thin shell finite elements, was able to catch the secondary bifurcation, obtaining it at the value M = 0.9983, with a relative error of $\in$ = -1.7438%, compared to the value predicted by Emmerling for the secondary bifurcation load of long tubes, which is M = 1.016. The bigger relative error, compared to the first case, can be explained by the fact that this tube is shorter, it is almost a thick shell, and that numerical solution is being compared with an analytical solution for thin shells and long tubes.

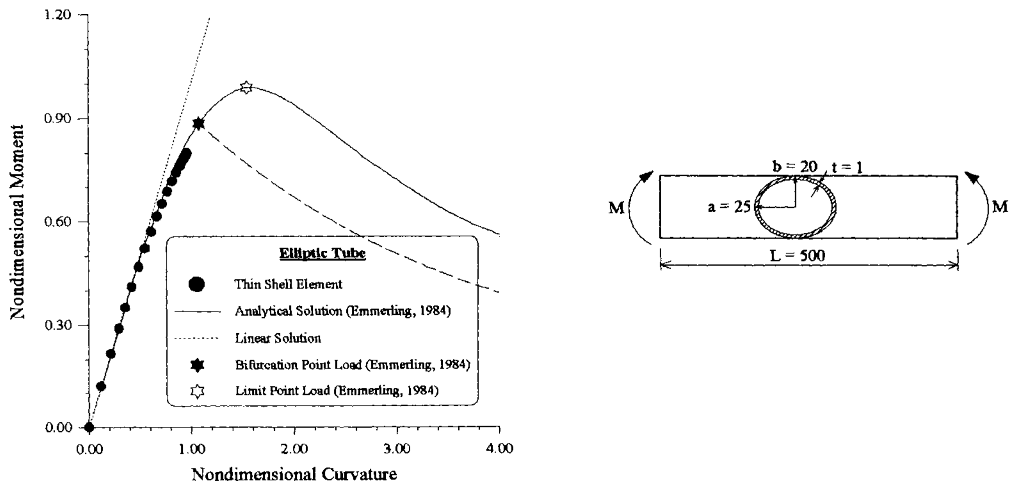

Figure 2. Nonlinear bending of an elliptic tube - Bending Moment X Curvature

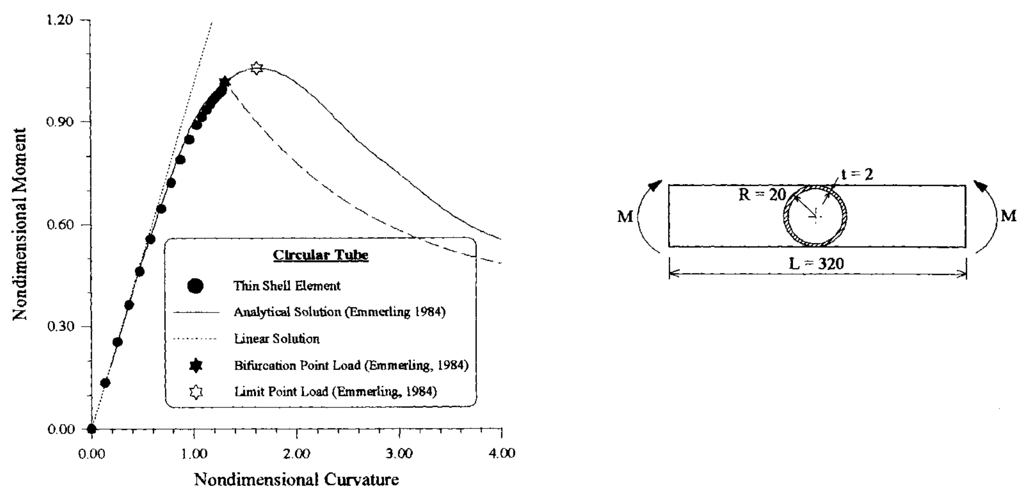

Figure 3. Nonlinear bending of a shorter circular tube - Bending Moment X Curvature

4. CONCLUSIONS

A numerical study of nonlinear bending problems of tubes was performed, using a thin shell finite element. The main objective of this study was the search for secondary bifurcation instabilities, which can cause the failure of these tubes. In the problems solved, one can see that the models with thin shell finite elements were able to catch the secondary bifurcations. The results presented for the nonlinear bending of tubes with circular cross section have shown a much better accuracy, compared to those obtained for a tube with elliptic cross section. Nevertheless, additional studies are recommended, especially with the use of other thin or thick shell finite elements, to perform a broader analysis for shells with various thickness/radius ratios and radius/length ratios.

5. ACKNOWLEDGMENTS

The authors would like to thank the Brazilian funding agencies CAPES and CNPq, with its program RHAE, for supporting this research.

REFERENCES

Axelrad, E.L. 1984. Flexible Shells, in Axelrad, E.L. and F.A. Emmerling (Editors), *Flexible Shells - Theory and Applications*, Springer Verlag, pp. 44-63, Berlin.

Axelrad, E.L. and Emmerling, F.A. 1986. Intrinsic Shell - Theory Formulation Effective for Large Rotations and an Application, in Pietraszkiewicz, W. (Editor), *Finite Rotations in Structural Mechanics*, Springer Verlag, pp. 01-18, Berlin.

Axelrad, E.L., 1992. Flexible - Shell Theory and Analysis of Tubes and Bellows, in the *Proceedings of the VII SIBRAT - 7th Brazilian Symposium on Piping and Pressure Vessels*, Vol. I, pp. 03-14, Florianópolis.

Bathe, K.-J. 1982. *Finite Element Procedures in Engineering Analysis*, Prentice-Hall, New Jersey.

Brush, D.O. and Almroth, B.O. 1976. *Buckling of Bars, Plates, and Shells*, McGraw-Hill, New York.

Emmerling, F.A. 1984. Nonlinear Bending of Curved Tubes, in Axelrad, E.L. and F.A. Emmerling (Editors), *Flexible Shells - Theory and Applications*, Springer Verlag, pp. 175-191, Berlin.

Fonseca, J.S.O. and Selke, C.A.C. 1990. Nonlinear Bending of Tubes Using a Tri-dimensional Modelling Technique, in the *Proceedings of the VI SIBRAT - 6th Brazilian Symposium on Piping and Pressure Vessels*, Vol. 2, pp. 449-459, Rio de Janeiro (in Portuguese).

Gallagher, R.H. 1987. Finite Element Method for Instability Analysis, in Kardestuncer, H. et alli. (Editors), *Handbook of Finite Elements*, McGraw-Hill, New York.

Imaeda, F.T. 1992. *A Thin Shell Finite Element for the Elastic-Plastic Analysis of Structures*, Master Dissertation, CPGEM/UFSC, Florianópolis (in Portuguese).

Imaeda, F.T., Selke, C.A.C. and Blass, A. 1993. Elastic-Plastic Analysis of Structures Using a Thin Shell Finite Element, in the *Proceedings of the XII COBEM - 12th Brazilian Congress of Mechanical Engineering*, Vol. I, pp. 169-172, Brasília (in Portuguese).

Irons, B.M. 1976. The SemiLoof Shell Element, in Ashwell, D.G. and R.H. Gallagher (Editors), *Finite Elements for Thin Shells and Curved Members*, John Wiley, London.

McMeeking, R.M. and Rice, J.R. 1975. Finite Element Formulations for Problems of Large Elastic-Plastic Deformation, *Int. J. Solids and Structures*, 11, pp. 601-616.

Nagtegaal, J.C. and Slater 1981. J.G., A Simple Noncompatible Thin Shell Element Based on Discrete Kirchhoff Theory, *ASME, AMD-48*, pp. 167-192.

Ramm, E. and Stegmüller, H. 1982. The Displacement Finite Element Method in Nonlinear Buckling Analysis of Shells, in Ramm, E. (Editor), *Buckling of Shells*, Springer-Verlag, pp. 201-235, Berlin.

Selke, C.A.C., Luersen, M.A. and Tomás Pereira, J. 1994. Numerical Simulations of the Nonlinear Bending of Tubes, in the *Proceedings of the VIII SIBRAT - 8th Brazilian Symposium on Piping and Pressure Vessels*, Vol. 2, pp. 752-763, Porto Alegre.