ASEISMIC DESIGN OF STRUCTURES WITH NUCLEAR REACTORS METHOD OF EARTHQUAKE RESPONSE ANALYSIS FOR REACTOR INTERNALS

Y. TSUSHIMA, J. JIDO, Y. ABE

Takenaka Komuten Co., Ltd.,
Takenaka Technical Research Laboratory, Tokyo, Japan

SUMMARY

Recently, new knowledge and analytical techniques have been developed in the field of structural response for nuclear power plants subjected to earthquake motions in Japan, the U.S.A. and some other countries. The authors have studied and developed analytical techniques for aseismic design of various structures for nuclear power plants and we have applied these techniques to structural analyses of existing structures. This paper shows the new method of the dynamic response analysis which is considered best with respect to light-weight structures such as fuel assemblages and shell structures, including, for example, drywells, which have dynamical membranous properties, and must be computed with multi-degrees of freedom. Further, this paper gives the general description concerning the theoretical and analytical investigations about our new method as follows:

1) Characteristics of composite structures
When we make dynamic response analysis of composite structures, such as nuclear power plants, we should use an appropriate method for analysis and set up suitable mathematical models for analytical purposes, because of the following two reasons:
One reason is that there are some differences in dynamical properties between soil-structure system and assemblages of mechanical equipment. The other reason is that the mathematical model would be large and more complicated with respect to degrees of freedom in case of idealizing the entire assemblage of structures, i.e. soil – structures – containments – fuel assemblages.

2) General description of response analysis techniques
According to the above-mentioned two reasons, we suggest separation of the mathematical model, including the entire assemblages of structures, that is, soil – structure – containment – fuel assemblages, into two models for carrying out response analysis for earthquake motions through the following two steps:

Step 1 (PHASE-1 of analyses):
We calculate the response values of heavy structures including soil-reactor building to investigate the aseismic safety thereof and at the same time to obtain the time history inputs for PHASE-2 of the analyses.

Step 2 (PHASE-2 of analyses):
Using the computed results (absolute accelerations and displacements) of PHASE-1 at the time of increment of computation, usually every 0.01 second, we calculate response values relating to the light-weight structures such as fuel assemblages and any other structures such as drywells, which should be analyzed by the multi-degrees of freedom system.

3) System program and numerical examples
We show an approximate description of the system program and an example of dynamic response analysis processes for the fuel assemblages of BWR type nuclear power plant under construction in Japan.

1. INTRODUCTION

When we want to analyze structures, we assume that it is better to improve aspects of techniques [1] already used. The reason why we suggest improvement of existing techniques is as follows: BWR type nuclear power plants can be said to be a sort of composite structures which consists of a variety of structures including the reactor building (R.B.), shield wall, reactor pressure vessel (R.P.V.), fuel assemblages, etc. Furthermore, each structure has dynamic properties different from those the others.

Weight of Each Mass Point of Lumped Mass System:

There are considerable differences in weight among these various types of structures. For an example, the weight at each mass point of R.B. is an average of 10,000 tons, and the total weight exceeds 160,000 tons. In respect of R.P.V. against R.B., the total weight amounts to less than 800 tons and each weight is an average of 50 tons, although other values are possible in case of another number of freedoms in the mathematical model. Further, concerning fuel assemblages in R.P.V., the total weight amounts to less than 400 tons and each weight is between 10 and 50 tons. In comparison of these weights of fuel assemblages with those of R.B., the ratio in each case averages between 1/1,000 and 1/200, and generally much less than 1/400. These very small weights of fuel assemblages would be considered almost erroneous in the process of computation.

Damping Mechanism in Various Components of Structures:

There are quantitative and qualitative differences in damping properties. While in the soil-structure model, it is a main requirement to evaluate the damping capacities of radiation phenomena through soil-structure interaction, in the R.P.V. - fuel assemblages model, it is more important to determine both the damping capacities of the material itself and the damping effects of the fluid surrounding the fuel assemblages, whose dynamical properties may be considered to be different from those in soil-structure interaction.

Objectives of This Study:

Because there are some differences in the dynamical properties mentioned above, and since the mathematical model would be larger in the number of degrees of freedom and more complicated in case of idealizing the entire assemblage of structures, that is, soil-structure-containment-fuel assemblages, it is considered better to use another method more suitable for the respective dynamic properties. The objectives of this report are as follows:

1) Describing our new response analysis method, which is the best with respect to lightweight structures such as fuel assemblages and shell structures such as a drywell with dynamic properties which must be computed by means of multidegrees of freedom.
2) Examining the validity of our method through studing a case of numerical models.
3) Applying this method to the dynamic response analysis of the fuel assemblages of an actual BWR type unclear power plant under construction in Japan.

2. GENERAL DESCRIPTION OF THIS METHOD

According to the objectives described in the preceeding chapter, we suggest to separate the mathematical model including the entire assemblages of structures, that is, soil-structure-containments-fuel assemblages, into two models and to make dynamic response analysis for earthquake motions through the following two steps:

Step 1 (PHASE-1 of Analysis):

Calculate the response values of the heavy structures including the soil and R.B., to investigate their aseismic safety and at the same time to make time hysteresis input motions for the following step (PHASE-2 of Analysis).

Step 2 (PHASE-2 of Analysis):

Using the computed results (absolute accelerations and displacements) of PHASE-1 at any time, usually 0.01 second, calculate the response values of the light weight structures such as fuel assemblages and any containments such as the drywell, which should be analyzed the multidegrees of freedom system.

Fig.-1 shows PHASE-1 mathematical model of an actual nuclear power plant. In the same figure, G-2, which means Group 2 of the entire assemblage of structures, shows the drywell. It is a typical thin-shell structure, which is assumed to move not only in bending and shearing displacements as a cantilever but also in membranous displacement as a shell. It is necessary to make analysis considering these properties if we want to study details of phenomena.

Drywell in Fig.-1 is the simplest lumped mass system consisting of the smallest number of masses from 9 to 16 for PHASE-1 analysis. On the other hand, the mathematical model shown in Fig.-2, as an example, can be assumed for PHASE-2 and is used for stress-strain analysis by means of the Finite Element Method considered better than others at present. Therefore, in order to calculate the response values of this structure, we must use the computed results of mass points 9 and 15 in the PHASE-1 model as the inputs to the mass points 1 and 22 in the PHASE-2 model, respectively.

Fig.-3 is a mathematical model of PHASE-2 for R.P.V. and fuel assemblages. When we want to calculate the detailed response values of fuel assemblages, we should follow the same procedures as those used for the drywell, i.e. the computed results of mass points 29, 30 and 42 in the PHASE-1 model as the input motions to the mass points 1, 2 and 21 in the PHASE-2 model.

Thus, we can say that by this method it is feasible to carry out response calculation for earthquake motions with consideration of the dynamical properties of each structure, this being beyond the limitation of computers or numerical techniques.

The above-mentioned response calculation can be done using the Step by Step Modal Analysis Method (mode superposing in the time domain) with the equations of motion expressed by the complex stiffness.

However, procedures of this type may be more complicated numerically and difficult than those of the usual method used hitherto because one must use accelerations and displacements of time hysteresis at two mass points in case of PHASE-2.

3. EQUATIONS OF MOTION

Using the coordinates and parameters shown in Fig.-4, the equations of motion of PHASE-1 and PHASE-2 will be developed in the next two sections to earthquake motions.

Coordinates and Parameters are presented as follows:

Coordinates:

Axis-I is the absolute coordinates. Axis-II is the relative coordinates located in the distance of horizontal ground motion from the Axis-I. Axis-III is the rotational coordinates rotated at the center of gravity of rigid foundation. And Axis-IV is the local coordinates defined as the straight line connecting the heigher mass point "P" and the lower mass point "Q".

Parameters:

x_i, Δx_i, and $\Delta \overline{x}_i$ are i-th mass point horizontal displacement from Axis-II, Axis-III and Axis-IV, respectively. θ_i and $\Delta\theta_i$ are i-th mass point rotational displacement from Axis-III and Axis-IV, respectively. "G" is center of gravity of rigid foundation. x_{BG} is horizontal displacement of "G" from Axis-II. θ_{R} is rotational displacement of rigid foundation. yo is horizontal ground motion from Axis-I. θ_{L} is rotational displacement of Axis-IV to Axis-III. "P" and "Q" are connection mass points between PHASE-1 model and PHASE-2 model.

3.1 Equations of Motion for Soil-Structure System (PHASE-1 System)

When a structure is subjected to ground horizontal accelerations, we obtain the following equations of motion, including horizontal and rotational displacements of rigid foundation, for the multidegrees of freedom system. The equations are expressed in matrix form as follows [2]:

$$[M]\{\ddot{x}\} + [K_{\mathrm{R}} + i\ sgn\ \omega K_I]\{x\} = -\ddot{y}_o\ [M]\{Co\} \tag{1}$$

where $[M]$, $[K_{\mathrm{R}} + i\ sgn\ \omega\ K_I]$ and $\{Co\}$ are mass matrix, complex stiffness matrix and constant vector, respectively. The term of $sgn\ \omega$ means as follows: $sgn\ \omega = 1$ for $\omega > 0$, $sgn\ \omega = 0$ for $\omega = 0$, $sgn\ \omega = -1$ for $\omega < 0$. $\{Co\}^T$ is $\{1, 1, \ldots\ldots, 1, 0\}$.

The stiffness matrix of aboveground structures can be formed by superposing matrix of each component in assemblages structures, and this is transfered into a global coordinate system and superposed, with consideration of the relationship among entire assemblages of structures.

It is possible to transform the second order differential equations, eq. (1), into those of the first order with twice the number of degrees of freedom, as follows:

$$[D]\{\dot{z}\} + [E]\{z\} = -\ddot{y}o\ [D]\{f\} \tag{2}$$

where

$$[D] = \begin{bmatrix} O & M \\ M & O \end{bmatrix},\quad [E] = \begin{bmatrix} -M & O \\ O & K_R + i\ sgn\ \omega\ K_I \end{bmatrix}$$

$$\{\dot{z}\} = \begin{Bmatrix} \ddot{x} \\ \dot{x} \end{Bmatrix},\quad \{z\} = \begin{Bmatrix} \dot{x} \\ x \end{Bmatrix},\quad \{f\} = \begin{Bmatrix} Co \\ O \end{Bmatrix}$$

3.2 Equations of Motion for R.P.V. - Fuel Assemblages System (PHASE-2 System)

When a structure is subjected to two input motions (accelerations, displacements) at the points "P" and "Q", we should introduce the equations of motion for PHASE-2 in the manner mentioned below. First, we assume the input motions of time hysteresis at the points "P" and "Q" in Fig.-4, i.e. $\ddot{x}_p + \ddot{y}o$, $\ddot{\theta}p$, Δx_p, θp and $\ddot{x}_Q + \ddot{y}o$, $\ddot{\theta}_Q$, Δx_Q, θ_Q. Then equilibrium equations are formed for the free-free structure by eq. (3).

$$\{F\} = [K]\{\delta\} \tag{3}$$

where $\{F\} = \{F_n, Fp, Mp, F_{n-1}, \ldots\ldots\ldots, F_{i+1}, Fi, Mi, F_{i-1}, \ldots\ldots, F_2, F_Q, M_Q\}^T$

$$\{\delta\} = \{\Delta x_n, x_p, \theta p, \Delta x_{n-1}, \ldots\ldots\ldots\ldots, \Delta x_{i+1}, \Delta x_i, \theta_i, \Delta x_{i-1}, \ldots\ldots, \Delta x_2, \Delta x_Q, \theta_Q\}^T$$

$[K]$; complex stiffness matrix defined with the displacements from Axis-III
The accelerations and displacements of i-th mass point are expressed by eg. (4)

$$\left.\begin{aligned} \ddot{x}_i + \ddot{y}_o &= \Delta\ddot{\bar{x}}_i + \frac{Hi-Hp}{H_Q-Hp}(\ddot{x}_Q + \ddot{y}_o) + \frac{Hi-HQ}{Hp-HQ}(\ddot{x}_p + \ddot{y}_o) \\ \Delta x_i &= \Delta\bar{x}_i + \frac{Hi-Hp}{H_Q-Hp}\Delta x_Q + \frac{Hi-HQ}{Hp-H_Q}\Delta x_p \end{aligned}\right\} \quad (4)$$

Substituting eg. (4) and $\Delta\bar{x}_p = 0$, $\Delta\bar{x}_Q = 0$ into eq. (3), we obtain the following equations for the structure supported at the points "P" and "Q".

$$\{\bar{F}\} = [\bar{K}]\left\{\begin{matrix}\overline{\Delta x} \\ \theta\end{matrix}\right\} + [L_3, L_7]\left\{\begin{matrix}\Delta x_p \\ \Delta x_Q\end{matrix}\right\} + [L_4, L_8]\left\{\begin{matrix}\theta p \\ \theta_Q\end{matrix}\right\} \quad (5)$$

where $[\bar{K}]$ denotes the stiffness matrix defined with the displacements from Axis-IV, and

$$[L_3, L_7, L_4, L_8] = \left[\begin{matrix} \sum\limits_{j=2}^{n} k_{nj}\dfrac{Hj-Hp}{HQ-Hp}, & \sum\limits_{j=2}^{n} k_{nj}\dfrac{Hj-HQ}{Hp-HQ}, & k_{np}, & k_{nQ} \\ \cdots\cdots\cdots & & \cdots\cdots\cdots & \\ \sum\limits_{j=2}^{n} k_{ij}\dfrac{Hj-Hp}{HQ-Hp}, & \sum\limits_{j=2}^{n} k_{ij}\dfrac{Hj-HQ}{Hp-HQ}, & k_{ip}, & k_{iQ} \\ \cdots\cdots\cdots & & \cdots\cdots\cdots & \\ \sum\limits_{j=2}^{n} k_{2j}\dfrac{Hj-Hp}{HQ-Hp}, & \sum\limits_{j=2}^{n} k_{2j}\dfrac{Hj-HQ}{Hp-HQ}, & k_{2p}, & k_{2Q} \end{matrix}\right]$$

$$\left\{\begin{matrix}\overline{\Delta x} \\ \theta\end{matrix}\right\} = \{\Delta\bar{x}_n, \Delta\bar{x}_{n-1}, \ldots\ldots, \Delta\bar{x}_{i+1}, \Delta\bar{x}_i, \theta i, \Delta\bar{x}_{i-1}, \ldots\ldots, \Delta\bar{x}_2\}^T$$

Whereas, inertia forces can be expressed by the following equation.

$$\{\bar{F}\} = -[\bar{M}]\left\{\begin{matrix}\ddot{\overline{\Delta x}} \\ \ddot{\theta}\end{matrix}\right\} - [L1, L5]\left\{\begin{matrix}\ddot{x}_p + \ddot{y}o \\ \ddot{x}_Q + \ddot{y}o\end{matrix}\right\} - [L2, L6]\left\{\begin{matrix}\ddot{\theta}p \\ \ddot{\theta}_Q\end{matrix}\right\} \quad (6)$$

where $[\bar{M}]$ denotes the mass matrix related to $[\bar{K}]$, and

$$[L_1, L_5, L_2, L_8] = \left[\begin{matrix} \sum\limits_{j=2}^{n} m_{nj}\dfrac{Hj-Hp}{HQ-Hp}, & \sum\limits_{j=2}^{n} m_{nj}\dfrac{Hj-HQ}{Hp-HQ}, & m_{np}, & m_{nQ} \\ \cdots\cdots\cdots & & \cdots\cdots\cdots & \\ \sum\limits_{j=2}^{n} m_{ij}\dfrac{Hj-Hp}{HQ-Hp}, & \sum\limits_{j=2}^{n} m_{ij}\dfrac{Hj-HQ}{Hp-HQ}, & m_{ip}, & m_{iQ} \\ \cdots\cdots\cdots & & \cdots\cdots\cdots & \\ \sum\limits_{j=2}^{n} m_{2j}\dfrac{Hj-Hp}{H_Q-Hp}, & \sum\limits_{j=2}^{n} m_{2j}\dfrac{Hj-HQ}{Hp-H_Q}, & m_{2p}, & m_{2Q} \end{matrix}\right]$$

$$\left\{\begin{matrix}\ddot{\overline{\Delta x}} \\ \ddot{\theta}\end{matrix}\right\} = \{\Delta\ddot{\bar{x}}_n, \Delta\ddot{\bar{x}}_{n-1}, \ldots\ldots, \Delta\ddot{\bar{x}}_{i+1}, \Delta\ddot{\bar{x}}_i, \ddot{\theta}_i, \Delta\ddot{\bar{x}}_{i-1}, \ldots\ldots, \Delta\ddot{\bar{x}}_2\}^T$$

Therefore, by combination of eq. (5) and eq. (6), the final equations of motion can be obtained as follows:

$$[\bar{M}]\left\{\begin{matrix}\ddot{\overline{\Delta x}}\\ \ddot{\theta}\end{matrix}\right\} + [\bar{K}]\left\{\begin{matrix}\overline{\Delta x}\\ \theta\end{matrix}\right\} = -\{F_2\} \qquad (7)$$

where

$$\{F_2\} = [L_1, L_2, L_3, L_4, L_5, L_6, L_7, L_8]\left\{\begin{matrix}\alpha_p\\ \alpha_Q\end{matrix}\right\}$$

$$\left\{\begin{matrix}\alpha_p\\ \alpha_Q\end{matrix}\right\} = \{\ddot{x}_p + \ddot{y}_o, \ddot{\theta}_p, \Delta x_p, \theta_p, \ddot{x}_Q + \ddot{y}_o, \ddot{\theta}_Q, \Delta x_Q, \theta_Q\}^T \qquad (8)$$

The known values in the time domain, such as α_p, α_Q in eq. (8) are those which were already computed in PHASE-1 analysis.

However, if the boundary conditions of points "P" and "Q" are others, the force vectors $\{F_2\}$ on the right hand of eq. (7) can be formed as follows:

Pin supported at "P" and fixed at "Q":

$$\{F_2\} = [L_1, L_5, L_7]\left\{\begin{matrix}\ddot{x}_p + \ddot{y}_o\\ \ddot{x}_Q + \ddot{y}_o\\ \Delta x_p\end{matrix}\right\}$$

Pin supported at "P" and "Q":

$$\{F_2\} = [L_1, L_5]\left\{\begin{matrix}\ddot{x}_p + \ddot{y}_o\\ \ddot{x}_Q + \ddot{y}_o\end{matrix}\right\}$$

4. CALCULATION OF EIGENVALUES AND RESPONSE VALUES

4.1 Eigenvalue Analysis

In order to obtain circular frequencies, corresponding damping ratios and eigenvalues for response analysis, we get the following eigenvalues as solution of the characteristic equation. The characteristic equation eq. (10) is formed by substituting $\{x\} = \{X\}e^{\lambda t}$ into the free vibration equation eq. (9), and frequency equation can be expressed by eq. (11).

$$[M]\{\ddot{x}\} + [K]\{x\} = \{0\} \qquad (9)$$

$$[\lambda^2[M] + [K]]\{X\} = \{0\} \qquad (10)$$

$$det.|\lambda^2[M] + [K]| = \{0\} \qquad (11)$$

where λ, $\{X\}$ are eigenvalues and eigenvectors, respectively.

Assuming the j-th eigenvalue as λ_j, it is expressed in the complex form, as follows:

$$\lambda_j = -\lambda_{Rj} + i\,\lambda_{Ij} \quad (\text{where } \lambda_{Rj} > 0,\ \lambda_{Ij} > 0)$$

and the corresponding eigenvectors can be obtained by solving the simultaneous equations, which are formed by substituting λ_j into eq.(10), and the j-th eigenvectors are expressed in the complex form which is the same as λ_j, as follows: $\{X_j\} = \{X_{Rj}\} + i\,\{X_{Ij}\}$

Finally, we can define the j-th circular frequency ω_j, and the corresponding damping ratio h_j, respectively, by using the following equations.

$$\omega_j = \sqrt{\lambda_{Rj}^2 + \lambda_{Ij}^2}\ ,\quad h_j = \lambda_{Rj}/\omega_j$$

4.2 Response Analysis

Assuming the eigenvectors to be linearly independent, accelerations, velocities, displacements and constant vectors are expressed by eq.(12) ∿ eq.(14), respectively [2].

$$\left\{\begin{matrix}\ddot{x}\\ \dot{x}\end{matrix}\right\} = \sum_{j=1}^{n} \{\dot{q}_j\, Z_j + \dot{\overline{q}}_j\, \overline{Z}_j\} \tag{12}$$

$$\left\{\begin{matrix}\dot{x}\\ x\end{matrix}\right\} = \sum_{j=1}^{n} \{q_j\, Z_j + \overline{q}_j\, \overline{Z}_j\} \tag{13}$$

$$\{f\} = \sum_{j=1}^{n} \{\beta_j\, Z_j + \overline{\beta}_j\, \overline{Z}_j\} \tag{14}$$

$$\{Zj\} = \{\lambda_j\, \chi_j,\ \chi_j\}^T, \quad \{\overline{Zj}\} = \{\overline{\lambda}_j\overline{\chi}_j, \overline{\chi}_j\}^T$$

where $\dot{q}_j$, q_j are the j-th generalized coordinates and β_j is the j-th modal participation factors, and $\dot{\overline{q}}_j$, $\overline{q}_j$ and $\overline{\beta}_j$ are the conjugate values of $\dot{q}_j$, q_j and β_j, respectively.

According to the orthogonality properties of eigenvectors in the regions of $\omega > 0$ and $\omega < 0$, we obtain the following equations:

$$\{\dot{q}\} - [\ \lambda\ ]\ \{q\} = -\ddot{y}o\ \{\beta\}\ ,\ \{\dot{\overline{q}}\} - [\overline{\lambda}]\{\overline{q}\} = -\ddot{y}o\ \{\overline{\beta}\} \tag{15}$$

where

$$\{\beta\} = [\overline{Y}^T\ D\ Z]^{-1}[\overline{Y}^T\ D\ f]\ ,\ \{\overline{\beta}\} = [Y^T\ D\ \overline{Z}]^{-1}[Y^T\ D\ f] \tag{16}$$

and

$$[Y] = [-\lambda X,\ X]^T \quad , \quad [\overline{Y}] = [-\overline{\lambda}\overline{X},\ \overline{X}]^T$$

In this method of equations of motion with complex stiffness, it is characteristic to obtain modal participation factors by computing the simultaneous equations of eq.(16).

Constant Vector $\{f\}$:

The constant vector $\{f\}$ of the response calculation of PHASE-1 is formed by a vector $\{Co\}$. On the other hand, the constant vector of PHASE-2 is formed by vector $\{L_1, L_2, L_3, L_4, L_5, L_6, L_7, L_8\}$. Consequently, the modal participation factor $\{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7, \beta_8\}$ should be calculated against disturbances, which are constituted by response values, α_p and α_Q in eq.(8), in the time domain of PHASE-1.

Generalized Coordinates $\{q_j\}$:

The j-th generalized coordinates can be expressed by the following equations using impulse response and the convolution theory.

For PHASE-1 $\quad q_j\ (t) = q_j(o)\ e^{\lambda_j t} - \beta_j \int_o^t \ddot{y}o\ (\tau) e^{\lambda_j(t-\tau)} d\tau$

For PHASE-2 $\quad g_j\ (t) = q_j(o)\ e^{\lambda_j t} - \int_o^t \{\beta p,\ \beta_Q\} \left\{\begin{matrix}\alpha_p\ (\tau)\\ \alpha_Q\ (\tau)\end{matrix}\right\} e^{\lambda_j(t-\tau)}\ d\tau$

Response Values:

From the results of the conjugate generalized coordinates, we calculate response values in the time domain by using the following equations:

For PHASE-1

$$\{\ddot{x}\} = \sum_{j=1}^{n} \dot{q}_j \lambda_j \{X_j\} + \sum_{j=1}^{n} \dot{\bar{q}}_j \bar{\lambda}_j \{\bar{X}_j\}$$

$$\{\dot{x}\} = \sum_{j=1}^{n} \dot{q}_j \{X_j\} + \sum_{j=1}^{n} \dot{\bar{q}}_j \{\bar{X}_j\}$$

$$\{x\} = \sum_{j=1}^{n} q_j \{X_j\} + \sum_{j=1}^{n} \bar{q}_j \{\bar{X}_j\}$$

For PHASE-2 (from the Axis-IV)

$$\{\Delta\ddot{\bar{x}}\} = \sum_{j=1}^{n} \dot{q}_j \lambda_j \{\Delta X_j\} + \sum_{j=1}^{n} \dot{\bar{q}}_j \bar{\lambda}_j \{\bar{X}_j\}$$

$$\{\Delta\dot{\bar{x}}\} = \sum_{j=1}^{n} q_j \{\Delta X_j\} + \sum_{j=1}^{n} \dot{\bar{q}}_j \{\Delta\bar{X}_j\}$$

$$\{\Delta\bar{x}\} = \sum_{j=1}^{n} q_j \{\Delta X_j\} + \sum_{j=1}^{n} \bar{q}_j \{\Delta\bar{X}_j\}$$

Consequently, the response values of PHASE-2 from the Axis-II are as follows:

$$\{\ddot{x} + \ddot{y}o\} = \{\Delta\ddot{\bar{x}}\} + \{\frac{Hi-Hp}{HQ-Hp}\} (\ddot{x}_Q + \ddot{y}o) + \{\frac{Hi-HQ}{Hp-HQ}\} (\ddot{x}_p + \ddot{y}o)$$

$$\{\dot{x}\} = \{\Delta\dot{\bar{x}}\} + \{\frac{Hi-Hp}{H_Q-Hp}\} \dot{x}_Q + \{\frac{Hi-HQ}{Hp-H_Q}\} \dot{x}_p$$

$$\{x\} = \{\Delta\bar{x}\} + \{\frac{Hi-Hp}{H_Q-Hp}\} x_Q + \{\frac{Hi-HQ}{Hp-H_Q}\} x_p$$

5. EXAMINATION OF THEORETICAL VALIDITY

In order to examine the theoretical validity of this method as described above we made a case study of a certain mathematical model, which is shown in Fig.-5. In this figure Model-C-30 is a PHASE-1 model of the entire assemblages of structures consisting of four structural components. While Model-C-15 is a PHASE-2 model of R.P.V. and fuel assemblages which can be assumed to be the light weight structures among the four components. The mass points 1, (2) and 6, (7) of Model-C-15 are the input points of PHASE-2. Model-C-30 is a twenty degrees of freedom system including rocking and swaying, and Model-C-15 is an eight degrees of freedom system because the rotational angle of any mass point is assumed to be free from inertia forces.

Table-I shows the weight of each mass point in the mathematical model of PHASE-1 and PHASE-2. Making analysis of eigenvalues for two models, we obtained natural periods, damping ratio and participation functions, which are shown in Table-II and Fig.-6, respectively.

Next, we calculated response values to EL CENTRO 1940 NS during 3.0 seconds using our proposed method and attempted to compare the computed values of mass points 6 to 23 in PHASE-

1 with these of mass points 1 to 15 in PHASE-2. In the first step, we calculated the time hysteresis response values of Model-C-30 considering rocking and swaying of the foundation. At the second step, taking the computed results of mass points 16, 17, 21 and 22 in PHASE-1 for the already known response values of mass points 1, 2, 6 and 7 in PHASE-2, we made the response calculations for Model-C-15.

Table-III shows the maximum response values of displacements, velocities and absolute accelerations. In this table, column-A shows the values of PHASE-1 and column-B shows those of PHASE-2. In comparison with A and B, it can be judged that the computed results show good agreement between A and B, there being the reason for the differences between them being less than 1% and assumed as simply computational errors.

Judging from the above-mentioned examinations, this method can be considered to have theoretical validity. However, in case of applying this method to any response analysis of actual structures, we have to set up two mathematical models under sufficient considerations because it is very important that there are little changes of response values in PHASE-1 analysis for any mathematical model, for example, structures like G-3 or 4 in Model-C-30 would be idealized with either degrees of freedom less than those in this case or an extremely springless model.

6. NUMERICAL EXAMPLE OF ACTUAL NUCLEAR POWER PLANT

As we obtained theoretical validity of this method, we applied this method to the response analysis of fuel assemblages of the actual BWR type nuclear power plant. Fig.-7 shows R.P.V. and various kinds of inner components of fuel assemblages. The mathematical model for these assemblages is already shown in Fig.-3.

PHASE-1 of Dynamic Response Analysis:

Initially, we evaluated the dynamical properties such as mass, stiffness and damping with respect to both aboveground structures and soil-structure interaction according to the proposed evaluation techniques, and then calculated natural periods, damping ratios and the corresponding eigenvectors.[1] Next, putting TAFT 1952 EW (maximum acceleration is 300 GAL) into the base of foundation in the PHASE-1 model shown in Fig.-1, we calculated the time hysteresis response values for the inputs to the PHASE-2 model.

PHASE-2 of Dynamic Response Analysis:

The weight of each mass point is shown in Table-IV. We calculated the stiffness submatrix of individual structural component of the PHASE-2 model by means of the Bending Shear Deflection Theory and the corresponding complex stiffness sub-matrix using the damping ratio of individual groups shown in Table-V. Table-VI shows the natural periods and corresponding damping ratios of this PHASE-2 model. Some predominant modal participation functions among computed results are illustrated in Fig.-8. Next, we took the computed results of PHASE-1 as input motions of PHASE-2 and calculated only the response values of the fuel assemblages. The maximum response values of displacements, absolute accelerations, bending moments and shear forces are shown in Fig.-9.

7. CONCLUSIONS

The authors proposed new method of the dynamic response analysis which is considered best with respect to light-weight structures such as fuel assemblages and shell structures including, for example, drywells, which have dynamical membranous properties, and must be computed with multidegrees of freedom.

On the basis of this studies some conclusions may be reached as follows: By our proposed method, we can make analyses of the response of composite structures such as nuclear power plants to earthquake motions, and we can also determine the numerical accuracy of the computed results. It is clear that this method has sufficient applicability to other response analyses with multi-input motions. However, as stated previously, it would be necessary to study the establishment of a mathematical model for PHASE-1 with the lowest numerical errors.

Acknowledgement

The authors wish to thank the engineers of the Atomic Power Division of Tokyo Shibaura Electric Co., Ltd. for their helpful engineering comments concerning reactor internals, and to acknowledge the cooperation of structural engineers in the Nuclear Engineering Project Office of Takenaka Komuten Co., Ltd., engaged in the design of the nuclear reactor buildings mentioned in this paper.

References

[1] Y. Tsushima, J. Jido, "Aseismic Design of Structures with Nuclear Reactors Method of Earthquake Response Analysis for Composite Structures Evaluated for Damping Efficiencies by Material and Structure Type", Proc. the 1st Intl. Conf. on Structural Mechanics in Reactor Technology, Berlin, Germany, September 20 - 24, 1971.

[2] Y. Tsushima, J. Jido, "Analysis of Multi-Degree of Freedom Systems with Complex Stiffness and It's Applications (Part 1 Modes Super-position Method Applied to Multi-Degree of Freedom Systems)", Transactions of the Architectural Institute of Japan, No.220, June 1974. (Japanese)

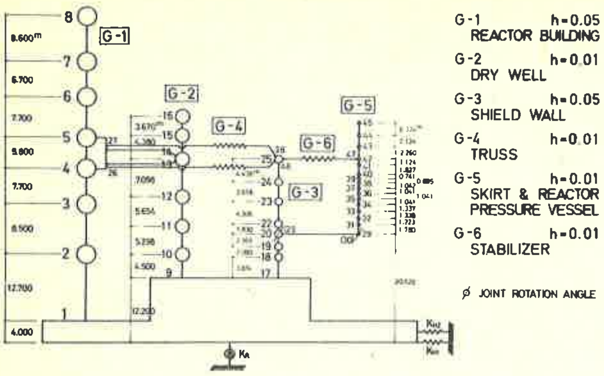


Fig.-1 PHASE-1 Model for Actual Nuclear Power Plant

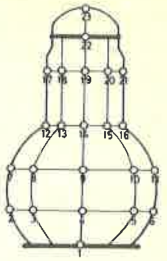


Fig.-2

PHASE-2 Model for Drywell

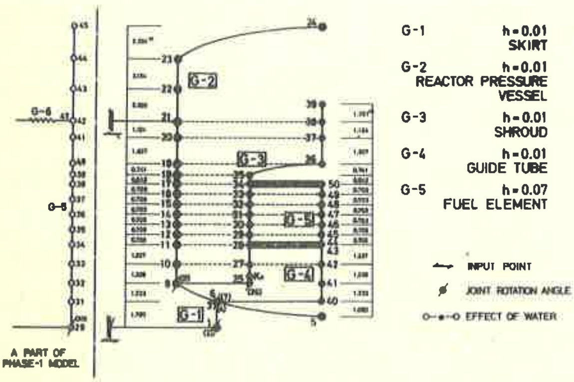


Fig.-3 PHASE-2 Model for R.P.V. and Fuel Assemblages

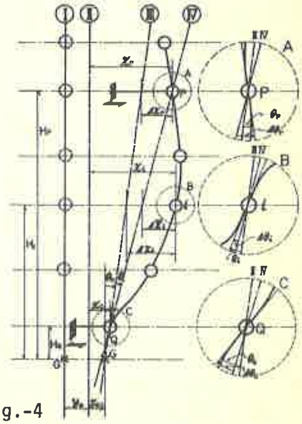


Fig.-4

Coordinates and Parameters of Mathematical Model

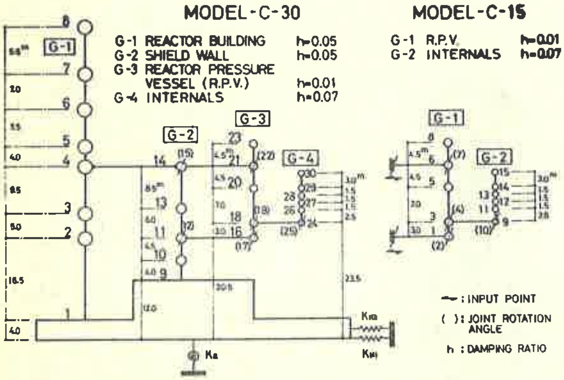


Fig.-5 Mathematical Model for Examining Theoretical Validity

Table-I

Weights of Example Model (ton)

MASS №	MODEL C - 30	MODEL C - 15
1		100.0
2	35000.0	
3	21550.0	200.0
4	14500.0	
5	10500.0	200.0
6	7000.0	200.0
7	1500.0	
8	1000.0	100.0
9		100.0
10	500.0	
11	300.0	100.0
12		100.0
13	500.0	100.0
14	100.0	100.0
15		60.0
16	100.0	
17		
18	250.0	
19		
20	300.0	
21	280.0	
22		
23	100.0	
24	100.0	
25		
26	100.0	
27	100.0	
28	100.0	
29	100.0	
30	50.0	

Table-II
Natural Periods and Damping Ratios of Example Model

MODE	MODEL-C-30		MODEL-C-15	
	T	h	T	h
1	0.2457	0.0797	0.1542	0.0632
2	0.1434	0.0579	0.0404	0.0589
3	0.1185	0.0907	0.0241	0.0258
4	0.0816	0.0464	0.0214	0.0100
5	0.0811	0.1295	0.0195	0.0524
6	0.0754	0.0529	0.0178	0.0255
7	0.0595	0.0715	0.0152	0.0406
8	0.0480	0.0487	0.0184	0.0496
9	0.0469	0.0570		
10	0.0360	0.0625		
11	0.0347	0.0504		
12	0.0312	0.0498		
13	0.0292	0.0462		
14	0.0280	0.0287		
15	0.0239	0.0307		
16	0.0194	0.0317		
17	0.0192	0.0564		
18	0.0147	0.0276		
19	0.0132	0.0688		
20	0.0104	0.0696		

T: PERIOD (sec)
h: DAMPING RATIO

Table-III
Maximum Response Values of Example Model
(for EL CENTRO 1940 NS)

	MASS №		DISPLACEMENT (cm)			VELOCITY (cm/sec)			ACCELERATION (GAL)		
	A	B	A	B	B/A	A	B	B/A	A	B	B/A
O	16	1	0.557	0.557	1.0	15.77	15.77	1.0	591.0	591.0	1.0
	18	3	0.665	0.665	1.000	14.82	14.58	0.983	431.4	431.0	0.999
	20	5	0.921	0.921	1.000	20.11	20.08	0.998	618.0	617.3	0.997
O	21	6	1.065	1.065	1.0	23.08	23.08	1.0	745.6	745.6	1.0
	23	8	1.228	1.228	1.000	24.43	24.45	1.002	880.0	874.4	0.987
	24	9	0.665	0.665	1.000	14.82	14.38	0.983	431.4	431.0	0.999
	26	11	0.843	0.844	1.001	18.70	18.65	0.998	527.0	527.7	0.999
	27	12	0.967	0.970	1.003	22.22	22.21	0.999	588.0	587.4	0.999
	28	13	1.098	1.103	1.004	25.69	25.80	1.004	706.6	707.5	1.001
	29	14	1.224	1.229	1.004	25.59	25.74	1.005	854.7	856.4	1.003
	33	15	1.472	1.480	1.005	35.17	35.37	1.005	1146.5	1147.3	1.000

A: MODEL-C-30 B: MODEL-C-15 O: INPUT POINT

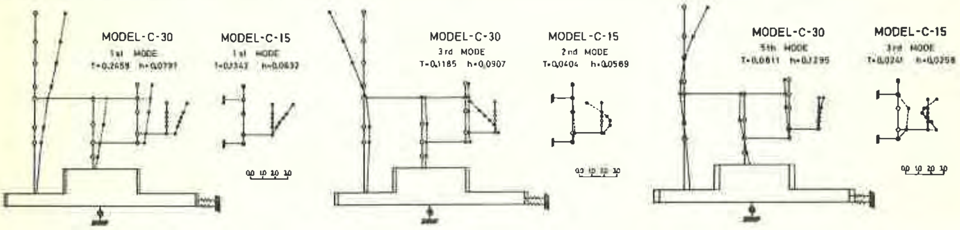


Fig.-6 Participation Functions of Example Model

Table-IV
Weights of PHASE-2 Model (R.P.V. and Fuel Assemblages)

Mass point	Weight (ton)	Mass point	Weight (ton)
3	37.4	28	50.7
5	19.9	29	38.1
8	56.9	〜	〃
10	79.6	34	38.1
11	72.9	35	28.1
12	41.0	36	16.6
〜	〃	37	14.7
16	41.0	38	14.5
17	42.2	39	4.3
18	40.7	41	18.1
19	50.3	42	18.7
20	53.7	45	32.2
22	83.4	〜	〃
23	72.7	48	32.2
24	9.7	49	30.4
27	55.9		

Table-V Damping Ratio of Each Group of PHASE-2 Model

Group (G)	Damping ratio (GH)	Group (G)	Damping ratio (GH)
1	0.01	4	0.01
2	0.01	5	0.07
3	0.01		

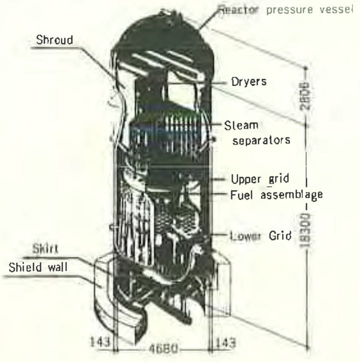
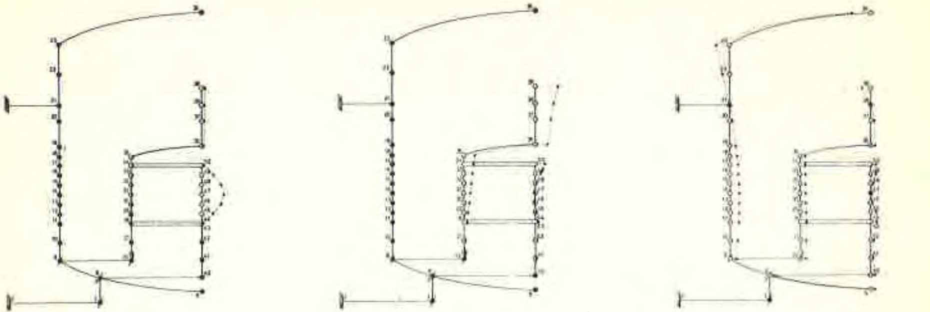


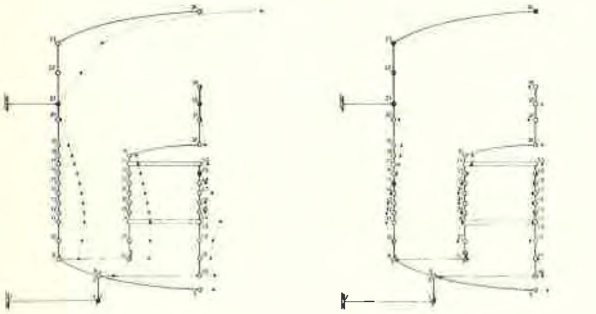
Fig.-7 Figures of Actual R.P.V. and Fuel Assemblages

Table-VI
Natural Periods and Damping Ratios of PHASE-2 Model

Mode	Periods (sec.)	Damping ratios
1	0.2600	0.0660
2	0.1498	0.0135
3	0.0645	0.0690
4	0.0543	0.0101
5	0.0516	0.0101
6	0.0368	0.0106
7	0.0295	0.0651
8	0.0284	0.0130
9	0.0245	0.0106
10	0.0180	0.0634
11	0.0176	0.0150
12	0.0150	0.0100
13	0.0133	0.0678
14	0.0131	0.0102
15	0.0124	0.0106
16	0.0095	0.0100
17	0.0084	0.0100
18	0.0082	0.0100
19	0.0074	0.0100
20	0.0739	0.0101



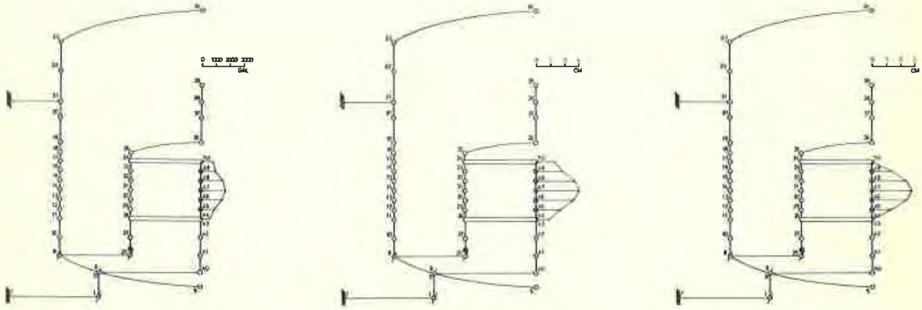
1st Mode 2nd Mode 6th Mode



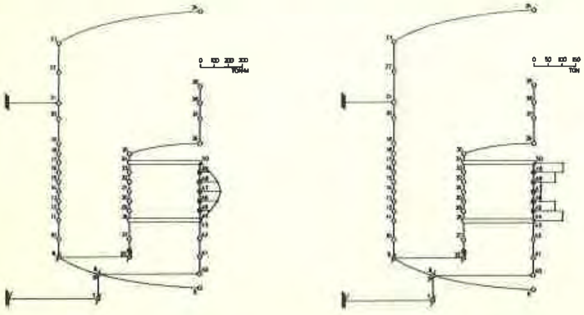
9th Mode 18th Mode

Fig.-8

Participation Functions of PHASE-2 Model



Absolute Acceleration Displacement from AXIS-II Displacement from AXIS-IV



Bending Moment Shear Force

Fig.-9

Maximum Response Values of PHASE-2 Model

(for TAFT 1952 EW 300 GAL)