Maternal Age At Last Birth And Reproductive Span: An Analysis of Egyptian Fertility

by

Amelia Dale Horne

Department of Biostatistics
University of North Carolina at Chapel Hill

Institute of Statistics Mimeo Series No. 1495T

December 1985

MATERNAL AGE AT LAST BIRTH AND REPRODUCTIVE SPAN: AN ANALYSIS OF EGYPTIAN FERTILITY

by

Amelia Dale Horne

A Dissertation submitted to the faculty of the University of North Carolina at Chapel Hill in partial fulfillment of the requirements for the degree of Doctor of Public Health in the Department of Biostatistics.

Chapel Hill

1985

Approved by:

Adviser

Reader

Reader

ABSTRACT

AMELIA DALE HORNE. Maternal Age at Last Birth and Reproductive Span: An Analysis of Egyptian Fertility. (Under the direction of C. M. SUCHINDRAN.)

Studies of events of the life cycle relating to human reproduction have concentrated on the beginning of the childbearing period, i.e, age at marriage and age at first birth. The present research initiates analysis of the end of the reproductive period. The variables age at last birth and reproductive span are examined for Egypt with the 1980 Egyptian Fertility Survey data.

The hazard of the last birth occurring by age x is modeled via proportional hazards regression. While age at marriage and parity are controlled, the effects of women's education, rural or urban residence, and marital dissolution are assessed. The effects of these independent variables on the length of the reproductive span are estimated by multiple regression.

Both of the above methods of modeling require that detailed micro-level data be available. Theoretical formulations are derived for expressing the summary measures mean maternal age at last birth and mean reproductive span, and their variances, when the only data available are a set of age-specific fertility rates and perhaps a life table for women.

Projections of mean maternal age at last birth are done by two methods. The first method is the construction of a parity-specific fertility table, and the second involves the fitting of a Poisson

covariate model to the EFS data.

Finally, policy implications for Egypt of the results of this study are considered.

ACKNOWLEDGMENTS

I am greatly indebted to my adviser, Dr. C. M. Suchindran, for stimulating my initial interest in this topic and for encouraging me throughout the various stages of research.

I also wish to express my sincere gratitude to my doctoral committee, Dr. James R. Abernathy, Dr. Moye Freymann, Dr. Michael Symons, Dr. Amy Tsui, and Dr. Peter Uhlenberg, for their helpful comments and suggestions.

The financial aid given to me by the National Institute of Child Health and Human Development, Grant Number 5-T32-HD07237, made my doctoral study possible. This support is very much appreciated.

I also owe thanks to the Central Agency for Population Mobilization and Statistics, Cairo, Egypt, which granted me permission to use its 1980 Egyptian Fertility Survey data.

Thankful appreciation is also due to Doug Vass for his skillful typing of the dissertation manuscript.

Finally, I owe a very special debt of gratitude to my husband, Dr. Nabil El-Khorazaty, who introduced me to the subject of biostatistics and encouraged me in every way to reach this goal.

TABLE OF CONTENTS

LIST OF TABLES....................vii

LIST OF FIGURES....................ix

CHAPTER

I. INTRODUCTION....................1

1.0 The Problem and Its Interest....................1
1.1 Review of the Literature....................4
1.2 Purpose of the Present Study....................5
1.3 The Data....................7

II. A DESCRIPTIVE PREVIEW....................9

2.0 Introduction....................9
2.1 Age at First Birth....................9
2.2 Age at Last Birth....................12
2.3 Reproductive Span....................23
2.4 Conclusion....................39

III. MATERNAL AGE AT LAST BIRTH....................44

3.0 Introduction....................44
3.1 Direct Modeling....................45
3.2 Indirect Modeling....................61
3.3 Conclusion....................73

IV. REPRODUCTIVE SPAN....................75

4.0 Introduction....................75
4.1 Direct Modeling....................75
4.2 Indirect Modeling....................80
4.3 Conclusion....................86

V. PROJECTIONS OF MATERNAL AGE AT LAST BIRTH........................88

5.0 Introduction........................88
5.1 Projection Based on Parity........................88

5.1.1 Nour's Model........................89
5.1.2 Application of Nour's Model to the EFS Data........................93

5.2 Projection Based on Age........................100

5.2.1 The Model........................100
5.2.2 Application of the Model to the EFS Data........................101

5.3 Conclusion........................107

VI. SUMMARY AND POLICY IMPLICATIONS........................109

6.0 Introduction........................109
6.1 Summary........................109

6.1.1 Maternal Age at Last Birth........................109
6.1.2 Reproductive Span........................112
6.1.3 Projections of Maternal Age at Last Birth........................114

6.2 Policy Implications........................117

VII. SUGGESTIONS FOR FURTHER RESEARCH........................123

7.1 Theory........................123
7.2 Applications........................123

7.2.1 Data Collection........................123
7.2.2 Replication........................124
7.2.3 Analyses by Different Geographic Regions........................124
7.2.4 Use of Indirect Modeling for Comparative Purposes........................125
7.2.5 Projection of Age at First Birth........................125
7.2.6 Use of Additional Time-Dependent Covariates in Hazard Modeling........................126

REFERENCES........................127

LIST OF TABLES

TABLE

2.1 Percentage Distribution of Age at First Birth by Current Age, Egypt, 1980 10

2.2 Percentage Distribution of Age at Last Birth by Current Age for Women Aged 35 and Over, Egypt, 1980 13

2.3 Mean Age at Last Birth by Current and Childhood Residence and Final Parity for Women Aged 45 to 49, Egypt, 1980 15

2.4 Mean Age at Last Birth by Number of Times Married and Status of First Marriage for Women Aged 45 to 49, Egypt, 1980 21

2.5 Mean Age at Last Birth by Current and Childhood Residence and Women's Education for Women Aged 45 to 49, Egypt, 1980 22

2.6 Percentage Distribution of Reproductive Span by Current Age for Women Aged 35 and Over, Egypt, 1980 27

2.7 Mean Reproductive Span by Current Age and Parity for Women Aged 35 and Over, Egypt, 1980 28

2.8 Mean Reproductive Span by Current Age and Age at Marriage for Women Aged 35 and Over, Egypt, 1980 31

2.9 Mean Reproductive Span in Years by Current Age and Current Marital Status for Women Aged 35 and Over, Egypt, 1980 35

2.10 Mean Reproductive Span in Years by Current Age and Years Spent in the Married State for Women Aged 35 and Over, Egypt, 1980 36

2.11 Mean Reproductive Span by Current Age and Status of First Marriage for Women Aged 35 and Over, Egypt, 1980 38

2.12 Mean Reproductive Span in Years by Current and Childhood Residence, for Women Aged 35 and Over, Egypt, 1980 40

3.1 Spearman Correlation Coefficients for independent Variables for Women Aged 45 to 49, Egypt, 1980 48

3.2 Results From Proportional Hazards Regression Model 3.1 For Women Aged 45 to 49, Egypt, 1980 53

3.3 Results from Hazard Regression Model 3.7 with Time-dependent Covariate for Women Aged 45 to 49, Egypt, 1980 57

3.4 Cumulative Life Table Probabilities of Last Birth Occurring by Beginning of Age Interval for Women Aged 45 to 49, Egypt, 1980 59

3.5 Illustrative Calculation of Mean Age at Last Birth, Egypt, 1980 72

4.1 Results of Regression Analysis for Women Aged 45 to 49, Egypt, 1980 79

4.2 Illustrative Calculation of Mean Age at First Birth, Egypt, 1980 85

5.1 Illustrative Calculation of Parity Progression Ratio for Parity 0, Egypt, 1980 94

5.2 Percentage Distribution of Women by Current Age and Parity, Egypt, 1980 96

5.3 Illustrative Calculation of C_{ij} for Parity 0, Egypt, 1980 97

5.4 Projections of Mean Age at Last Birth by Current Parity, Egypt, 1980 98

5.5 Maximum Likelihood Parameter Estimates and Standard Deviations for Ever-married Women Aged 25 and Over, Egypt, 1980 103

5.6 Calculation of Projected Age-specific Fertility Rates, 25-29 Cohort, Egypt, 1980 104

5.7 Projected Age at Last Birth for Women Aged 25 to 29, Egypt, 1980 106

LIST OF FIGURES

FIGURE

2.1 Mean Age at Last Birth by Parity, for Women 45-49 Years of Age, Egypt, 1980.............................16

2.2 Percentage of Women 45-49 Having Last Birth During Age Interval X, by Parity, Egypt, 1980...............17

2.3 Mean Age at Last Birth by Age at Marriage, for Women Aged 45-49, Egypt, 1980...............................19

2.4 Mean Age at Last Birth by Years Spent in Married State, for Women 45-49, Egypt, 1980....................20

2.5 Mean Age at Last Birth by Woman's Education, for Women 45-49, Egypt, 1980................................24

2.6 Mean Age at Last Birth by Women's Years of Schooling, for Women 45-49, Egypt, 1980.....................25

2.7 Mean Reproductive Span by Parity, for Women 45-49, Egypt, 1980..30

2.8 Mean Reproductive Span by Age at Marriage for Women 45-49, Egypt, 1980.................................32

2.9 Mean Reproductive Span by Maternal Age at First Birth, for Women 45-49, Egypt, 1980...................33

2.10 Mean Reproductive Span by Maternal Age at Last Birth for Women 45-49, Egypt, 1980.....................34

2.11 Mean Reproductive Span by Years Spent in Married State, for women 45-49, Egypt, 1980.................37

2.12 Mean Reproductive Span by Education, for Women, 45-49, Egypt, 1980.....................................41

2.13 Mean Reproductive Span by Wife's Years of Education, for Women 45-49, Egypt, 1980.....................42

3.1 Plot of Long Minus Log Survival Function...................51

3.2 Plot of Cumulative Hazard Function of Residuals............52

4.1 Graphical Diagnostics of Regression Model 4.1 for Women Aged 45 to 49, Egypt, 1980.......................78

CHAPTER I

INTRODUCTION

1.0 The Problem and Its Interest

As persons progress through their life course, they are expected to move through various stages or roles. The usual progression is through periods of early childhood, years devoted to education, entrance into the labor force, marriage and reproduction, and retirement. These stages are age related and prescribed by social norms. Entering a period of the life cycle early or late, as deemed by prevailing norms, may be considered socially ill timed. (See Neugarten et al., 1965; Elder, 1975; Neugarten and Hagestad, 1976; and Featherman, 1982).

Important among the events of the life cycle are the beginning and ending of childbearing. While age at marriage and the onset of childbearing have been the subjects of numerous studies in Egypt and elsewhere (e.g., El-Guindy (1971), Kafafi (1983), Kiernan and Diamond (1983), Trussell and Bloom (1983)), the completion of the reproductive period, and consequently the length of the reproductive span, have received little attention. For Egypt, the geographic focus of this study, there has been no research on the end of the childbearing period.

Yet there are important reasons why this topic should be studied in order to gain some understanding of the differences in completion of childbearing among various population subgroups. First, the completion of reproduction is of significance to the majority of women who are mothers. For them, the exit from this stage of the life cycle brings them close to the end of intensive child care and related matters, and allows them to devote more of their time to other pursuits. Having the last child relatively late in life may limit the roles and activities a woman can explore upon the completion of childbearing.

Secondly, significant changes in the timing of completing childbearing and in the length of the reproductive span may have important consequences regarding peer and family relationships. A shift to older (or to younger) ages at last birth may result in a larger number of women/couples having little in common with their peers who are in another stage of the life cycle. Older ages at the birth of the last child also may affect parent-child relationships, particularly that involving the last child, since older parents tend to be more mature and financially secure than younger parents. On the other hand, older, less energetic parents may find it difficult to keep pace with the youngest, active child, making it necessary to relegate a portion of the parental duties to an older sibling. If older age at last birth is coupled with high parity, the youngest child, and perhaps all the children, may find it difficult to compete for their parents' attention. If old age at last childbirth also means a long reproductive span, a consequence would be considerable differences in age among siblings, thus affecting sibling relationships. Ending the childbearing period late in life can also affect the husband-wife relationship by giving them less time together free of the responsibilities of childbearing before the period of retirement and old age.

Moreover, a change to older ages at last birth will mean that more offspring are still relatively young during their parents' old age and thus less able to provide financial and other support for them. The older parents will be less likely to live long enough to see all their grandchildren born, or to see them grow to young adulthood. The quality of the relationships between grandparents and grandchildren could thus be affected.

Thirdly, there are reasons to expect that age at last birth has and will continue to undergo changes in Egypt because of recent demographic and social trends. For example, age at marriage is expected to continue to rise, as well as education and increased enrollment of women in secondary schools and in universities (El-Guindy, 1971; Khalifa, 1974; Ibrahim, 1981). Both these factors tend to delay age at first birth and to relate to age at last birth. In addition, the divorce rate in Egypt is relatively high (Hanna, 1971; Khalifa, 1974), and most divorced women remarry (Hanna, 1971). If women tend to produce offspring from their latest marriage, marital disruption that leads to remarriage is expected to increase age at conclusion of childbearing.

Fourthly, as Rindfuss and Bumpass (1976) and Fallo-Mitchell and Ruff (1982) pointed out, there are probably social norms regarding the age for ending childbearing. Additionally, there appear to be evolving health norms regarding the maternal age for having the last birth. Nortman (1974) summarized the increased risks to maternal and fetal/perinatal health of bearing children at older ages. There is evidence that in Egypt maternal mortality increases sharply at maternal ages over 35 (Younis, N. et al., 1979). Fortney et al. (1982) found that in Egypt, where infant mortality is relatively high, babies born to women over age 35 had significantly lower survival rates than did infants born to younger mothers. Omran and Johnston (1984) have further documented in African countries the deleterious effects to the health of the entire family of childbearing at maternal ages less than 20 and over 35. Moreover, a conference organized in 1978 by Al Azhar University in Cairo urged that because of high perinatal and infant mortality rates in the Middle East, women should avoid childbirths at high risk age groups (People, 1979). The analysis of variations in age at last birth and in reproductive span among different sociodemographic subgroups of women hopefully will contribute to our understanding of the norms regarding these events of the life cycle.

Finally, the ages at which women stop having children and the length of the reproductive span can be important factors affecting population growth. High growth rates and overpopulation have long been of concern to planners and policymakers in Egypt. Since birth control in Egypt is used mostly to limit rather than to space births (CAPMAS, 1983 c; Khalifa et al., 1982), old age at last birth and long reproductive spans are expected to accompany high parity, and thereby contribute to rapid population growth.

All the above reasons point to the need to fill the current void of research on the end of the childbearing period. Analysis of the Egyptian Fertility Survey data can provide insights for the case of Egypt.

1.1 Review of the Literature

There is no previous or ongoing research on the change, variation in, or determinants of age at last birth or of the reproductive span. This area of inquiry appears to be wide open for investigation.

Krishnamoorthy (1979) developed mathematical expressions for various life cycle descriptive measures, including calculations of mean age at first and last birth, from which the span of childbearing can be deduced. He pointed out that his work is "only an attempt to stimulate interest in further work in this potential area incorporating other variables, such as marital status, economic status, etc.," and that detailed models need to be developed to explain the relevance of other variables and for testing hypotheses.

Nour (1984) demonstrated the construction of fertility tables using essentially life table type techniques. Using a woman's parity rather than her age as the criterion, he showed how to summarize the reproductive experience of a population based, on the one hand, on observed sets of current parity-specific rates of reproduction, and, on the other hand, on current parity distributions. Among the statistics computed were the distribution of the number of children remaining to be born to women of different parities, the corresponding distribution of completed family size, and the expected length of waiting time until completion of childbearing.

Both these authors contributed methods for obtaining useful and informative descriptive measures. But, as Krishnamoorthy noted, descriptive statistics do not allow for analysis incorporating the effects of other variables simultaneously. The present analytic as well as descriptive approach to the study of the end of childbearing examines the effects of sociodemographic variables and tests hypotheses about them.

1.2 Purpose of the Present Study

The present research attempts to determine variation in completion of childbearing and in length of the reproductive span among Egyptian women, by examining the effects of several social and demographic variables. Variables whose effects are modeled are marital experience, women's education, and rural/urban residence, while the effects of age at marriage and parity are controlled. Little is currently known about the relationships of these variables to age at last birth and consequently to the span of childbearing.

For the sake of simplicity, the study of the effect of marital experience concentrated on the status of the first marriage. Several measures of marital experience are available in the dataset used, e.g., number of times married, current marital status, number of years spent in marital union, and these are examined descriptively in chapter II. As mentioned earlier, Egypt has a relatively high crude divorce rate, about 2.0 per 1000 population for 1975-80 (CAPMAS, 1983 d), the effects of which are of particular social and demographic interest. It is expected that marital disruption that leads to remarriage will tend to increase age at last birth and the length of the reproductive span. On the other hand, marital disruption without remarriage is expected to have the reverse effect.

Education of women, especially at higher levels, has been found to be an important determinant of fertility behavior in numerous parts of the world, and Egypt is no exception (El-Sayeh, 1971). Especially since women's education tends to be positively related to age at marriage in Egypt (Kafafi, 1983), it is conceivable that postponement of marriage leads to delayed reproduction and age at last birth. On the other hand, the more highly educated the woman, the more likely she is to be aware of the social and health disadvantages of bearing children at old maternal ages. Also, it is anticipated that the more highly educated the woman, the higher her expectation of standard of living and the smaller her completed family size are likely to be. Thus, a negative relationship between education of women and age at last birth, as well as length of reproductive span, is expected.

The difference between residence in urban and in rural areas also has been found to manifest itself in fertility behavior. In Egypt, the difference between life in urban and in rural areas represents a "cultural gulf" (Loza, 1982). In rural areas, children have greater utility, and the prevalence of contraceptive use is lower than in urban areas (El-Deeb and Casterline, 1983). Also, of those currently aged 45 to 49, rural women desire an average of 0.9 additional child compared to 0.1 in urban regions (CAPMAS, 1983 c, p. 86). For these reasons it is anticipated that age at last birth tends to be older and reproductive span longer in rural than in urban areas.

The present study consists of three phases of work. First, a descriptive exploration was undertaken to gain some preliminary insight into the data, mainly concerning univariate relationships. This phase of research, presented in chapter II, is accompanied by graphical illustrations to provide visual detection of relationships. The second stage of the study, presented in chapters III and IV, is the development and use of models applied to data on individuals to investigate the effects of the independent variables mentioned before, on age at last birth and reproductive span. The third stage of work is the projection of expected future ages at completion of childbearing, based first on current parity and then on current age. These projections are given in chapter V. Methodologies of this study and results are summarized, and policy implications considered, in chapter VI. Chapter VII gives suggestions for future research.

1.3 The Data

The data analyzed here are from the 1980 Egyptian Fertility Survey (EFS) executed by the Egyptian Central Agency for Public Mobilization and Statistics (CAPMAS). The survey was a par of the World Fertility Survey and was done in collaboration with the World Bank. (CAPMAS, 1983 a.)

The selected sample represents a national survey, excluding only the Sinai population, nomads, and non-Egyptian nationals. The sample of 10,000 households selected from 200 primary sampling units (towns and villages) was self-weighted.

Within urban and rural regions, a second stage of cluster sampling was conducted, in which ultimate area units (UAU's) were selected. In the last stage of the sampling process, systematic samples of dwellings from each UAU were chosen. All ever-married women under age 50 in all households in selected dwellings were to be interviewed. The final sample contains pregnancy, birth, and marital histories, and data on infant/child mortality, contraceptive practice, and socioeconomic characteristics of 8,788 women. (CAPMAS, 1983 b.)

For the present study, women who had experienced no childbirths by the time of the interview were excluded from analysis. Thus, the study sample size was reduced to 7,817 women, 3,343 from urban areas and 4,474 from rural regions.

In any data analysis, some consideration must be given to the quality of the data. In particular, since this research deals with maternal age at last birth and, in order to compute length of the reproductive span, age at first birth, the accuracy of the reporting of births and dates of births is of interest. Any survey dataset of the magnitude of the EFS, undertaken in a society that is not highly literate cannot be expected to be totally free from error. However, evaluation of the quality of the EFS fertility data determined "that omission of births was negligible and that misplacement of dates of birth was of a modest magnitude" (CAPMAS, 1983 c, p. 31).

CHAPTER II

A DESCRIPTIVE PREVIEW

2.0 Introduction

An examination of descriptive statistics from the Egyptian Fertility Survey data provides preliminary insights for this study. In particular, it displays simple, visual relationships between variables in a mostly univariate manner. Analyses of the effects of several variables simultaneously are presented in chapters III and IV.

Descriptive results on age at first birth, age at last birth, and reproductive span are given below. Women with no births at the time of the survey were excluded from computations, as these women had no observed ages at birth.

2.1 Age at First Birth

The percentage distribution of age at first birth by current age and residence is given in table 2.1. Since only women who had had at least one child were included for analysis, younger cohorts of women necessarily had their first births at young ages. No information, therefore, is available about women in the younger cohorts who had not had their first child by the time of the interview. Thus, data for the oldest cohorts provide more complete information about the distribution of age at first birth.

For the total group of women, of current age 45 to 49, the largest percentage (about 53%) experienced their first births some time between

TABLE 2.1

PERCENTAGE DISTRIBUTION OF AGE AT FIRST BIRTH BY CURRENT AGE

EGYPT, 1980

Current Age	Age at First Birth								
	<15	15-19	20-24	25-29	30-34	35-39	40-44	45+	N
				A. Urban					
15-19	5.7	94.3	-	-	-	-	-	-	106
20-24	3.0	57.5	39.4	-	-	-	-	-	464
25-29	3.8	37.6	44.3	14.3	-	-	-	-	718
30-34	4.2	45.1	30.3	17.7	2.8	-	-	-	650
35-39	5.8	46.1	32.8	11.6	3.1	0.5	-	-	551
40-44	8.1	49.8	28.2	10.5	3.3	0.2	-	-	458
45-49	8.8	51.8	26.0	8.8	2.8	1.5	0.3	0.0	396
Total	5.3	48.4	32.6	10.9	1.8	0.3	0.03	0.0	3343
				B. Rural					
15-19	13.9	86.1	-	-	-	-	-	-	216
20-24	6.4	69.9	23.7	-	-	-	-	-	845
25-29	8.2	52.5	34.2	5.2	-	-	-	-	833
30-34	7.9	56.1	27.0	7.6	1.4	-	-	-	811
35-39	9.0	57.8	25.0	6.3	1.5	0.4	-	-	735
40-44	8.8	52.1	29.1	6.6	2.5	0.9	0.0	-	560
45-49	7.2	53.4	28.5	7.8	1.5	0.8	0.6	0.2	474
Total	8.2	59.0	26.5	5.0	1.0	0.3	0.1	0.02	4474
				C. Total					
15-19	11.2	88.8	-	-	-	-	-	-	322
20-24	5.2	65.5	29.3	-	-	-	-	-	1309
25-29	6.1	45.6	38.9	9.4	-	-	-	-	1551
30-34	6.2	51.2	28.5	12.1	2.0	-	-	-	1461
35-39	7.6	52.8	28.4	8.6	2.2	0.5	-	-	1286
40-44	8.4	51.1	28.7	8.3	2.8	0.6	0.0	-	1018
45-49	7.9	52.6	27.4	8.3	2.1	1.1	0.5	0.1	870
Total	6.9	54.4	29.1	7.5	1.3	0.3	0.05	0.01	7817

Source: 1980 Egyptian Fertility Survey, Standard recode, version 3.

15 and 19 years of age. The next most popular age-at-first-birth interval was 20 to 24 years of age, with percentages experiencing their first births dropping sharply thereafter and then declining steadily. Similar trends hold for younger cohorts as well, and for urban and rural women.

Differences between urban and rural women are small for the older cohorts, but tend to diverge among the younger groups of women. For example, among those currently aged 45 to 49, 52% of the urban and 53% of the rural had their first child between 15 and 19 years of age. For those currently aged 25 to 29, however, the respective percentages were 38 and 53. This divergence in proportions having first births at an early age suggests that conditions in the urban areas in recent years have tended to delay the onset of childbearing. Could this trend be due to higher education of women in urban areas, their changing roles in the cities, or the recent urban housing shortage?

Under the assumption that all of the 35-39, 40-44, and 45-49 cohorts had had their first births, a test for differences among the distributions of these cohorts for the total group of women can be made. With the use of frequencies, and with the last three categories combined, a chi-square with ten degrees of freedom of 13.59 is obtained, indicating no significant cohort differences in age at first birth for these three oldest groups of women, at the .05 or even .10 level of significance.

A test for urban/rural differences in the oldest cohort also can be done. The chi-square with five degrees of freedom is 3.38, indicating no significant residential difference among women aged 45 to 49. Comparing urban and rural women in the 25-29 age group, however, yields a chi-square with three degrees of freedom of 75.49, indicating a significant difference in the distribution of age at first birth by residence for this younger cohort of women.

2.2 Age at Last Birth

Data on age at last birth for women in the younger cohorts may be considered incomplete, since most of these women had not had their actual last births by the time of the interview. For this reason data for the younger groups of women are omitted in the following tables.

Table 2.2 shows the percentage distribution of age at last birth by current age and current residence for women aged 35 and over. It may be assumed that almost all the women in the age group 45 to 49 had finished their childbearing, and thus had relatively complete information. A majority of these women experienced their last births between ages 30 and 39. Another 2.6% completed childbearing between ages 45 and 49. (It will be recalled that women aged 50 and older were not interviewed in the survey, so no information is available about births occurring beyond age 49.)

Under the assumption that women aged 40 and older had completed their childbearing, a test for differences between the last two cohorts for the total group of women can be done. With the first two and the last two categories combined, a chi-square with five degrees of freedom (based on frequencies) of 40.12 is obtained, indicating a significant difference in the distribution of age at last birth between the two age groups. This result, if the above assumption is true, could indicate an actual cohort difference, or it could mean that the assumption is false--that women in the 40-44 age group had not completed their childbearing.

Urban and rural figures for the oldest cohort show that urban women tended to have their last children earlier than rural women: about

TABLE 2.2

PERCENTAGE DISTRIBUTION OF AGE AT LAST BIRTH BY CURRENT AGE

FOR WOMEN AGED 35 AND OVER

EGYPT, 1980

Current Age	Age at Last Birth								
	<15	15-19	20-24	25-29	30-34	35-39	40-44	45-49	N
				A. Urban					
35-39	0.2	0.9	8.5	19.8	45.2	25.4	-	-	551
40-44	0.2	1.7	5.7	21.6	34.1	29.9	6.8	-	458
45-49	0.3	1.0	3.8	16.2	33.6	28.5	14.6	2.0	396
Total	0.2	1.2	6.3	19.4	38.3	27.7	6.3	0.6	1405
				B. Rural					
35-39	0.0	1.0	5.0	16.6	48.4	29.0	-	-	735
40-44	0.2	1.1	4.1	11.3	26.1	42.9	14.5	-	560
45-49	0.0	1.9	3.4	9.5	24.9	35.2	21.9	3.2	474
Total	0.1	1.3	4.3	13.0	35.0	35.1	10.5	0.9	1769
				C. Total					
35-39	0.1	0.9	6.5	18.0	47.0	27.4	-	-	1286
40-44	0.2	1.4	4.8	15.9	29.7	37.0	11.0	-	1018
45-49	0.1	1.5	3.6	12.5	28.9	32.2	18.6	2.6	870
Total	0.1	1.3	5.2	15.8	36.5	31.8	8.6	0.8	3174

Source: 1980 Egyptian Fertility Survey.

55% of the urban women had their last child by age 34, compared to 40% of the rural women. Again, with frequencies for the first two and the last two categories combined, a test for residential differences in the oldest cohort can be done. The chi-square with five degrees of freedom is 24.23, indicating a significant difference between the distributions for urban and rural women aged 45 to 49.

Table 2.3 shows mean age at last birth by current and childhood residence and final parity for women aged 45 to 49. For all women and for both urban and both rural subgroups, mean age at last birth is positively related to final parity. Figure 2.1 displays this relationship graphically for the total group of women in this oldest cohort. This result indicates that women in Egypt who end childbearing at older ages tend to have large numbers of children.

For women with one or two children, the difference between urban and rural childhood residence appears to be negligible. However, women with current urban residence ended childbearing an average of about two years later than current rural women (age 29.58 vs. 27.66). This result raises the question as to whether current rural women in this low parity group tended to end childbearing because of some type of marital dissolution (divorce, separation, or widowhood). In all other groups, rural women had their last children later than urban women.

In figure 2.2 the percentage of women aged 45 to 49 having their last births during various intervals is shown, with parity level controlled. The distributions for women with 1-2 children and with 3-5 children are similar in location, except that the former is flatter and more spread out than the latter. The distribution for women with 6 or more children is shifted to the right of those with fewer children, indicating that women who end reproduction later tend to have more children.

TABLE 2.3

MEAN AGE AT LAST BIRTH BY CURRENT AND CHILDHOOD RESIDENCE AND FINAL PARITY FOR WOMEN AGED 45 TO 49

EGYPT, 1980

Residence	Final Parity		
	1-2	3-5	6+
Current			
Urban	29.58 (7.961)[a] N=31	31.78 (5.299) N=91	35.62 (4.737) N=274
Rural	27.66 (7.906) N=44	32.86 (5.689) N=92	37.55 (4.382) N=338
Childhood			
Urban	28.26 (7.773) N=45	31.34 (5.057) N=92	35.73 (4.615) N=262
Rural	28.74 (8.293) N=30	33.31 (5.793) N=91	37.40 (4.536) N=350
Total	28.45 (7.933) N=75	32.32 (5.510) N=183	36.69 (4.641) N=612

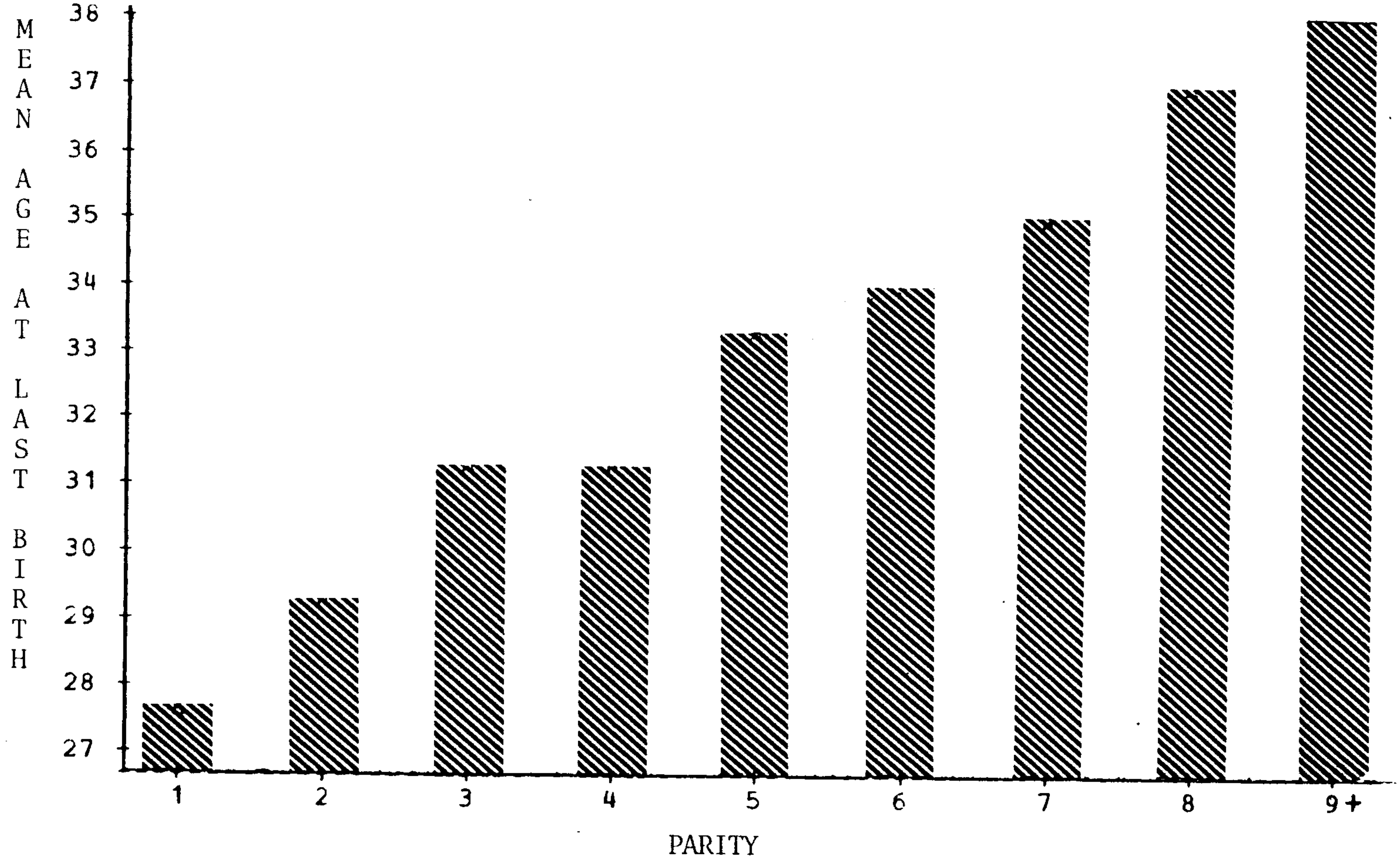
[a]Figures in parentheses are standard deviations.

Source: 1980 Egyptian Fertility Survey.

FIGURE 2.1

MEAN AGE AT LAST BIRTH BY PARITY, FOR WOMEN 45-49 YEARS OF AGE

EGYPT, 1980

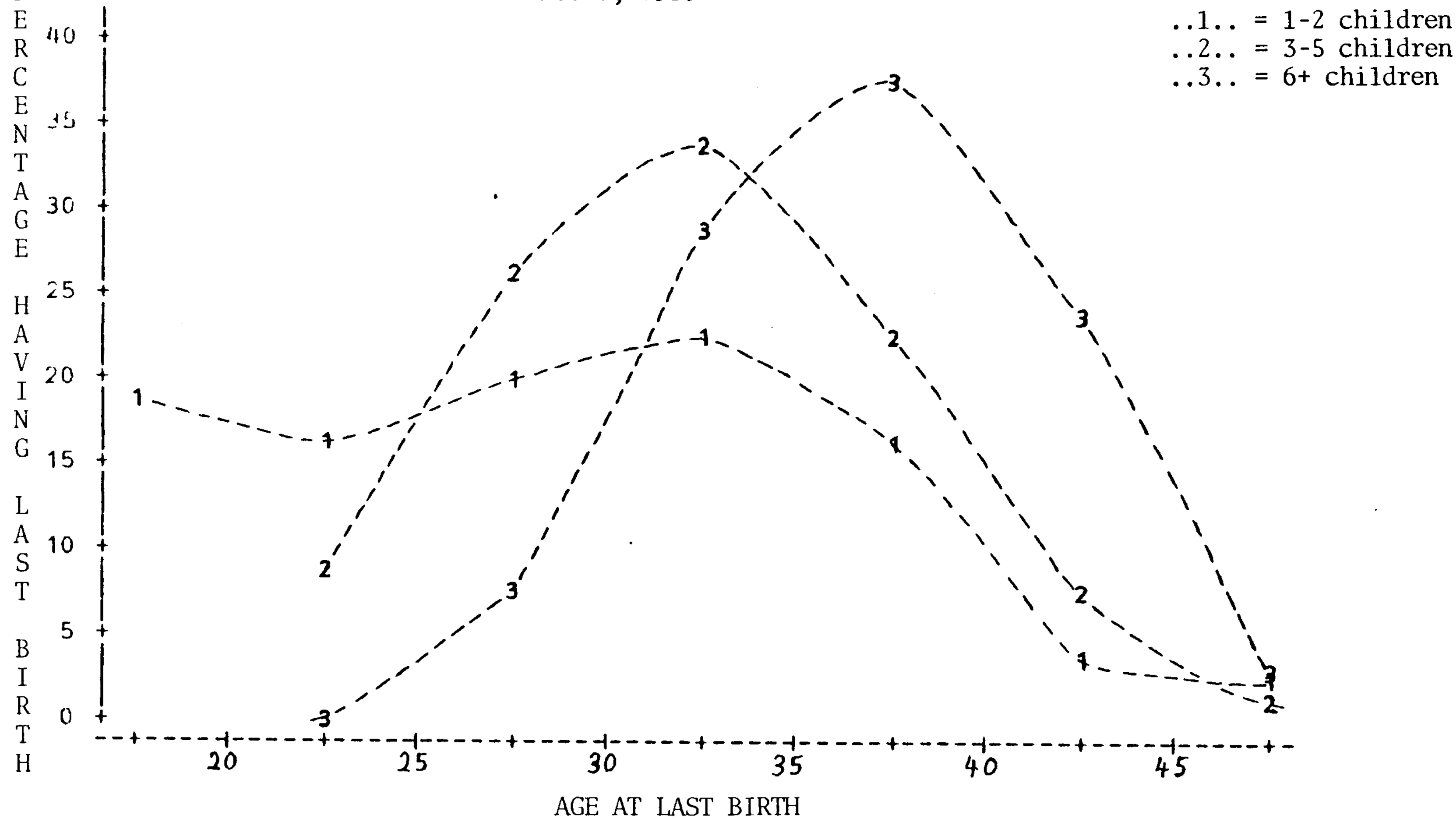


Source: 1980 Egyptian Fertility Survey.

FIGURE 2.2

PERCENTAGE OF WOMEN 45-49 HAVING LAST BIRTH DURING AGE INTERVAL X, BY PARITY

EGYPT, 1980



Source: 1980 Eyptian Fertility Survey.

Figure 2.3 shows mean age at last birth by age at first marriage, for women aged 45 to 49. For women married at ages less than 25 years there was little variation in mean age at last birth, the mean being around 35 years of age. Women who were married at later ages, however, ended childbearing, on average, at increasing older ages.

Figure 2.4 shows mean age at last birth for women currently aged 45 to 49, by number of years spent in the married state, without regard to the number of times married. There appear to be two patterns, one for women who spent less than 15 years in marital union and another for those who were in union for more than 15 years. The former group had their last births, on average, around age 31, while those in the latter group tended to have last births between ages 33 and 35.

In table 2.4 mean age at last birth by number of times married and status of the first marriage is given for women in the oldest cohort. For these women the average age at the end of childbearing increased with number of unions. This finding suggests that the women tended to produce children by their new husbands.

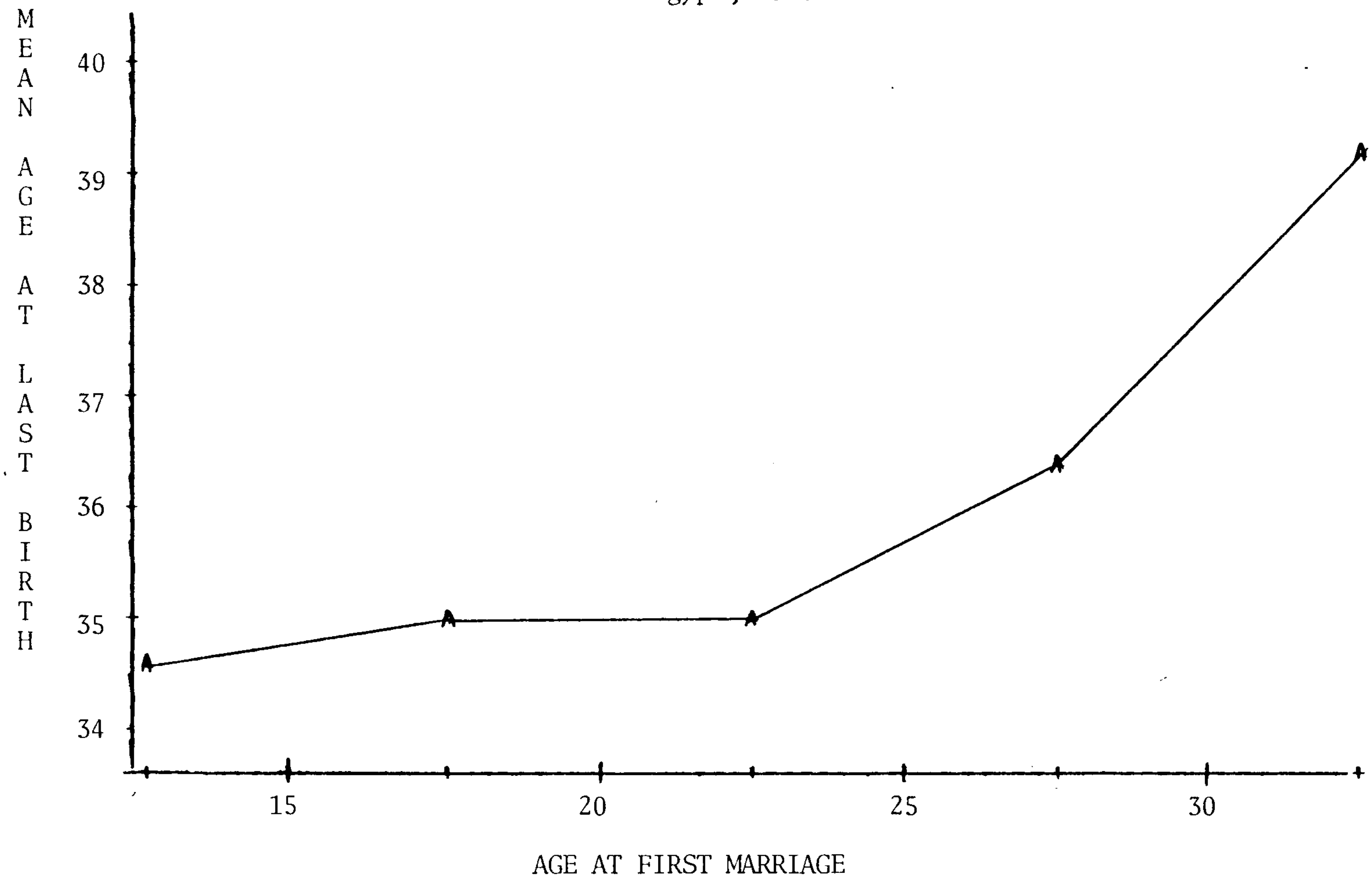
There appears to be little difference in age at last birth between women with intact first marriages and those whose dissolved first unions were followed by remarriage. This apparent disparity with the above result is because women who were married once include women with intact first unions and also those with dissolved first marriages who had not remarried. Women with dissolved first marriages who had not remarried ended childbearing, on average, earlier than the other women.

Table 2.5 gives mean age at last birth by current and childhood residence and education, for women aged 45 to 49. For the total group of women, mean age at the end of childbearing is inversely related to

FIGURE 2.3

MEAN AGE AT LAST BIRTH BY AGE AT MARRIAGE, FOR WOMEN AGED 45-49

Egypt, 1980

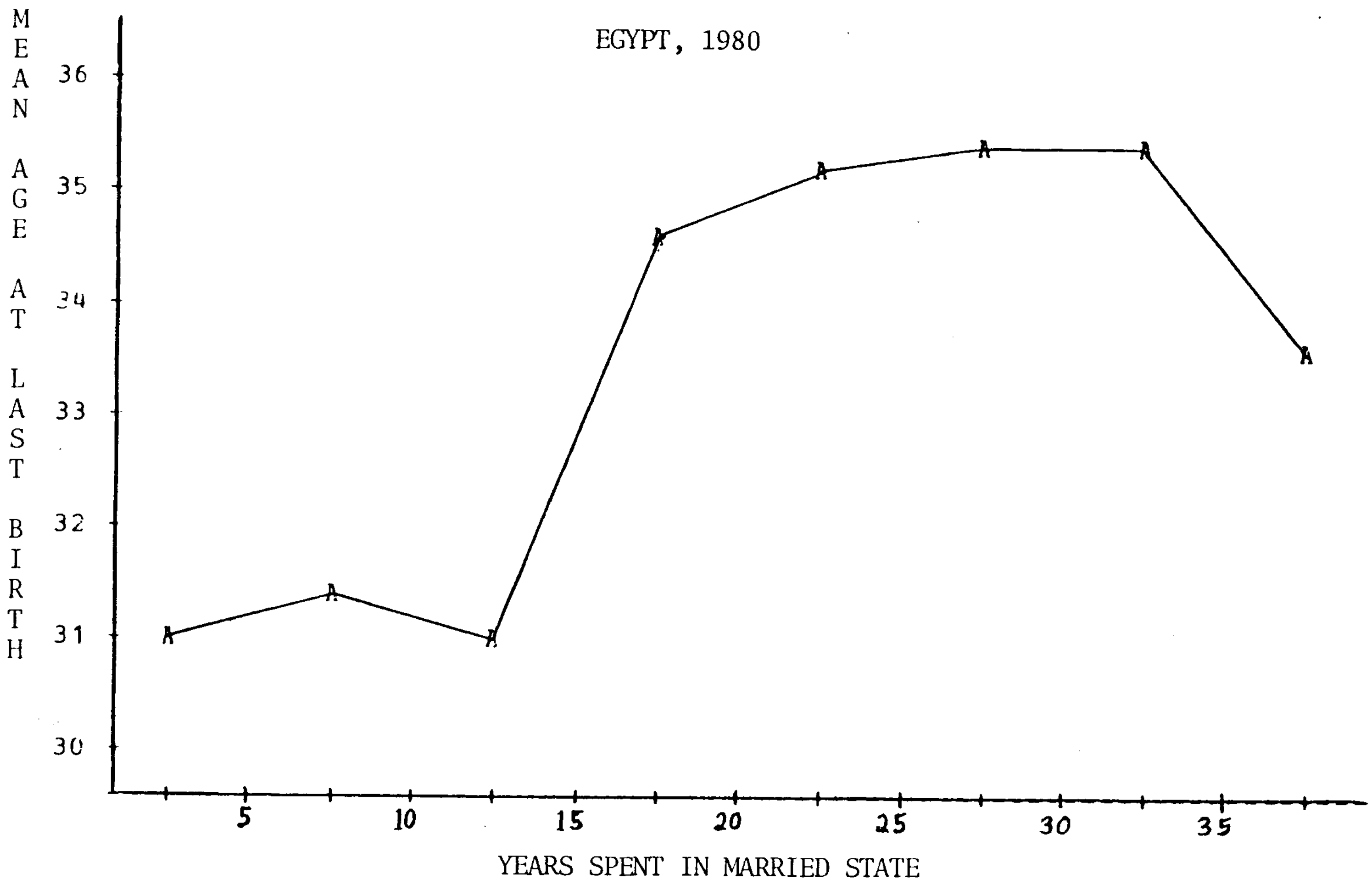


Source: 1980 Egyptian Fertility Survey.

FIGURE 2.4

MEAN AGE AT LAST BIRTH BY YEARS SPENT IN MARRIED STATE FOR WOMEN 45-49

EGYPT, 1980



Source: 1980 Egyptian Fertility Survey.

TABLE 2.4

MEAN AGE AT LAST BIRTH BY NUMBER OF TIMES

MARRIED AND STATUS OF FIRST MARRIAGE

FOR WOMEN AGED 45 TO 49

EGYPT, 1980

Characteristics	N	Mean Age at Last Birth	STD. DEV.
Number Times Married			
1	741	34.98	5.791
2	122	35.41	6.149
3-4	7	37.82	4.859
Status of First Marriage			
Intact	596	35.77	5.564
Dissolved & Remarried	129	35.54	6.095
Dissolved, No Remarriage	145	31.71	5.577

Source: 1980 Egyptian Fertility Survey.

TABLE 2.5

MEAN AGE AT LAST BIRTH BY CURRENT AND CHILDHOOD RESIDENCE AND WOMEN'S EDUCATION FOR WOMEN AGED 45 TO 49

EGYPT, 1980

Residence	Education Level Illiterate	Literate	Primary	Secondary +
Current				
Urban	35.26 (5.602)[a] N=251	33.44 (4.906) N=82	31.58 (5.087) N=28	31.23 (5.361) N=35
Rural	35.87 (5.968) N=445	33.78 (5.058) N=22	32.49 (7.485) N=7	-
Childhood				
Urban	34.84 (5.956) N=255	33.07 (4.883) N=81	30.89 (5.221) N=29	31.10 (5.389) N=34
Rural	36.12 (5.729) N=441	35.05 (4.817) N=23	35.97 (5.436) N=6	35.50 (-) N=1
Total	35.65 (5.842) N=696	33.51 (4.916) N=104	31.76 (5.530) N=35	31.23 (5.361) N=35

[a]Figures in parentheses are standard deviations.

Source: 1980 Egyptian Fertility Survey.

level of education, although the difference between women with primary and those with secondary or higher education appears to be small. (See also figure 2.5.) This trend is visible for women currently living in both urban and rural areas. Within the individual categories of education, however, differences between current urban and current rural residence are negligible.

When the data are examined by childhood place of residence, the inverse relationship between age at last birth and education for urban women still exists, except that women with primary and those with secondary or higher education show little difference. Education tended to make the least difference in age at last birth for women with childhood residence in rural areas. When education is controlled, women with rural childhood residence tended to end childbearing later than those with urban residence, and the differences are greater than the current urban/rural figures. Obviously, there was some migration.

These findings suggest that while educational level can account for current residential differences in age at last birth, it does not account for all the childhood residential differences. One might then speculate from these data about whether attitudes and values acquired during childhood tend to override the effect of education.

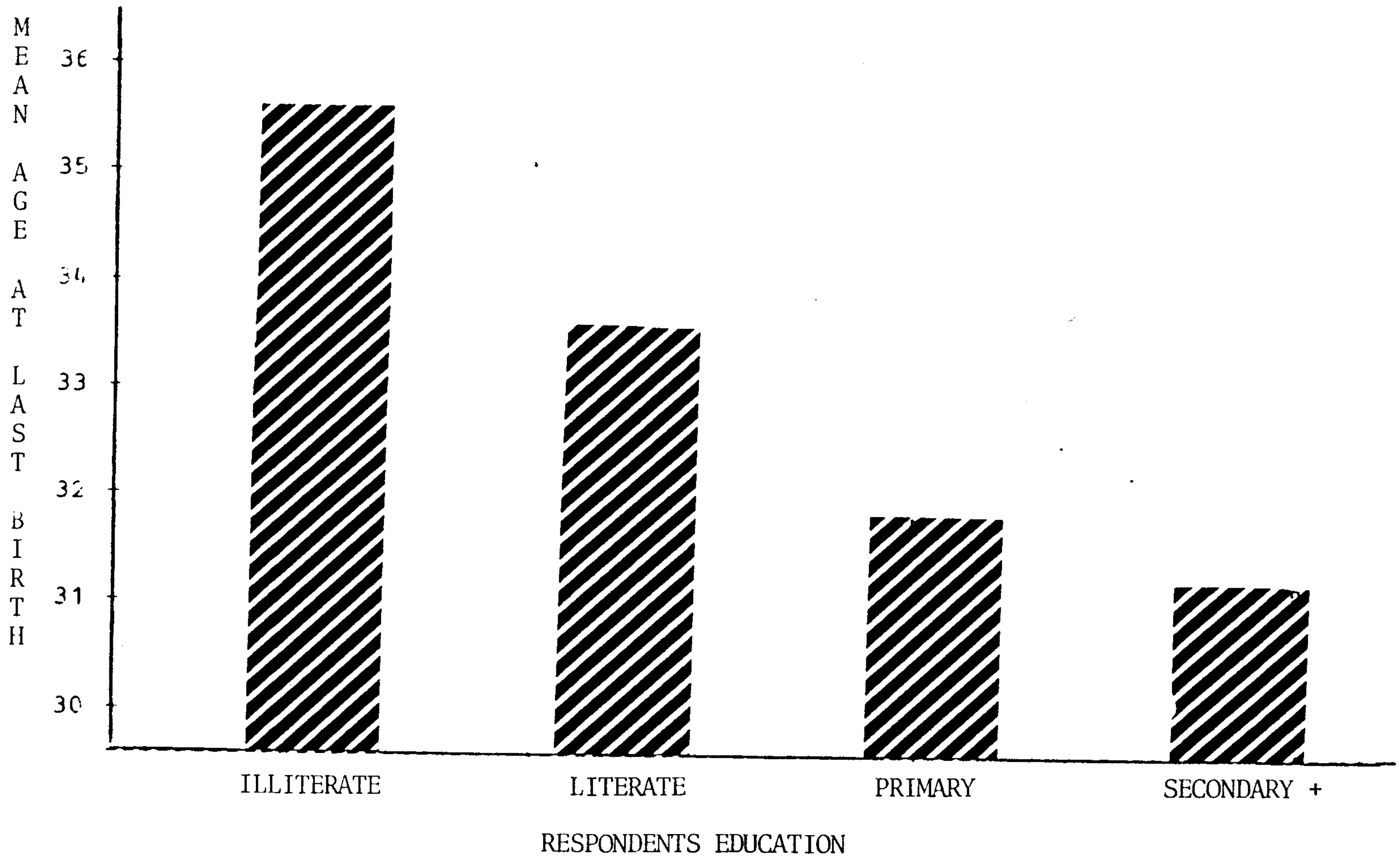
Figure 2.6 shows mean age at last birth by women's years of formal schooling. The picture displays a general tendency for the mean age at the end of childbearing to decline with an increase in the number of years of education.

2.3 Reproductive Span

Reproductive span, computed in years, was obtained by subtracting age at first birth from age at last birth. Thus, women who had had less

FIGURE 2.5

MEAN AGE AT LAST BIRTH BY WOMAN'S EDUCATION, FOR WOMEN 45-49, EGYPT, 1980

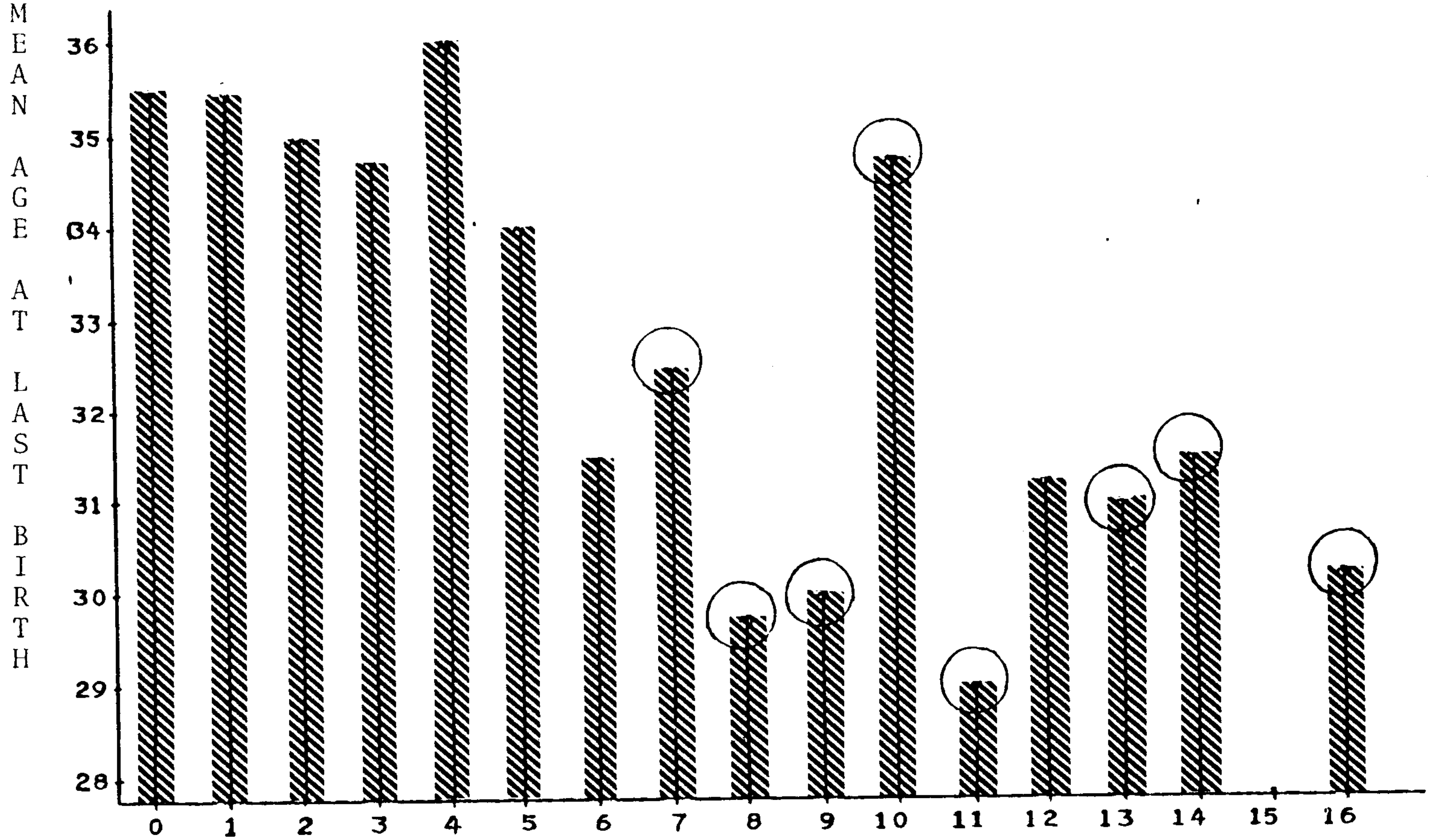


Source: 1980 Egyptian Fertility Survey.

FIGURE 2.6

MEAN AGE AT LAST BIRTH BY WOMEN'S YEARS OF SCHOOLING, FOR WOMEN 45-49

EGYPT, 1980



Note: Circles represent frequencies less than 10.

Source: 1980 Egyptian Fertility Survey.

than two children at the time of the survey were excluded from analysis. Also excluded were women who had had only two children which were twin births.

Table 2.6 gives the percentage distribution of reproductive span by current age and residence for women aged 35 and over. The mode of the distribution for the oldest age group among the total group of women was 15 to 19 years. The mode for urban women occurred at reproductive spans between 10 and 14 years in length, with spans of 15 to 19 years occurring second most often. The same was true for women of current age 40 to 44. For rural women, the mode occurred at spans of from 15 to 19 years, for both women 45 to 49 and 40 to 44 years of age.

As before, a test for differences in distribution between the last two cohorts of the total group can be done. With frequencies for the last two categories combined, the chi-square with five degrees of freedom is 35.96, indicating a significant difference between the two cohorts. Again, this result can indicate either a true cohort difference, if the younger cohort had completed childbearing, or it may indicate that women in the 40 to 44 age group had not yet had all their children.

A comparison between the urban and rural distributions for the oldest cohort also can be made. With frequencies combined as before, a chi-square with five degrees of freedom equal to 18.68 is obtained, indicating significant differences between the distributions of reproductive span by current residence. This result and the above urban/rural comparison connote that rural women tended to experience longer spans of childbearing than did women in urban areas.

In table 2.7 mean reproductive span by current age and parity is given for women aged 35 and over. For the three age groups, the mean

TABLE 2.6

PERCENT DISTRIBUTION OF REPRODUCTIVE SPAN BY CURRENT AGE

FOR WOMEN AGED 35 AND OVER

EGYPT, 1980

Current Age	Reproductive Span							
	<5	5-9	10-14	15-19	20-24	25-29	30+	N
				A. Urban				
35-39	15.3	22.0	30.6	27.5	4.2	0.4	-	523
40-44	8.6	20.9	31.1	26.1	11.9	1.1	0.2	444
45-49	5.5	15.8	28.2	26.1	20.6	3.7	0.0	379
Total	10.3	19.9	30.1	26.6	11.4	1.6	0.1	1346
				B. Rural				
35-39	6.0	14.5	34.6	38.5	6.5	0.0	-	712
40-44	4.9	13.0	24.3	32.8	23.5	1.5	0.0	548
45-49	4.0	9.8	22.0	32.4	25.1	6.2	0.7	451
Total	5.1	12.8	28.0	35.1	16.8	2.1	0.2	1711
				C. Total				
35-39	10.0	17.7	32.9	33.8	5.5	0.2	-	1235
40-44	6.6	16.5	27.3	29.8	18.3	1.3	0.1	992
45-49	4.7	12.5	24.8	29.5	23.0	5.1	0.4	830
Total	7.4	15.9	28.9	31.4	14.4	1.9	0.2	3057

Source: 1980 Egyptian Fertility Survey.

TABLE 2.7

MEAN REPRODUCTIVE SPAN BY CURRENT AGE AND PARITY FOR

WOMEN AGED 35 AND OVER

EGYPT, 1980

Current Age	Parity (No. Children)		
	2	3-5	6+
35-39	3.598	9.600	15.588
	(3.077)[a]	(4.087)	(3.342)
	N=79	N=410	N=746
40-44	3.646	10.012	17.028
	(2.705)	(4.214)	(4.320)
	N=52	N=260	N=680
45-49	6.929	10.418	18.091
	(5.989)	(4.791)	(4.800)
	N=35	N=183	N=612

[a]Figures in parentheses are standard deviations.

Source: 1980 Egyptian Fertility Survey.

span of childbearing showed a positive relationship with parity. This result is as expected, and is shown graphically in figure 2.7 for women 45 to 49 years of age.

Table 2.8 gives mean reproductive span by current age and age at marriage for women aged 35 and over. The average span shows an inverse relationship with age at marriage for the three cohorts of women. This relationship is portrayed graphically for women in the oldest cohort in figure 2.8. In figure 2.9 mean reproductive span is plotted by age at first birth for women 45 to 49, indicating also a negative relationship. Since length of reproductive span is positively related to parity, these results corroborate the contention that early age at marriage tends to produce large family sizes.

Figure 2.10 shows mean reproductive span by age at last birth for women 45 to 49 years of age. A positive relationship emerges.

Table 2.9 gives the same data by current marital status for women aged 35 and over. Women currently married at the time of the interview experienced longer reproductive spans, on average, than did those in the other status groups. Those widowed showed the next longest spans, while those who were divorced tended to have the shortest reproductive spans.

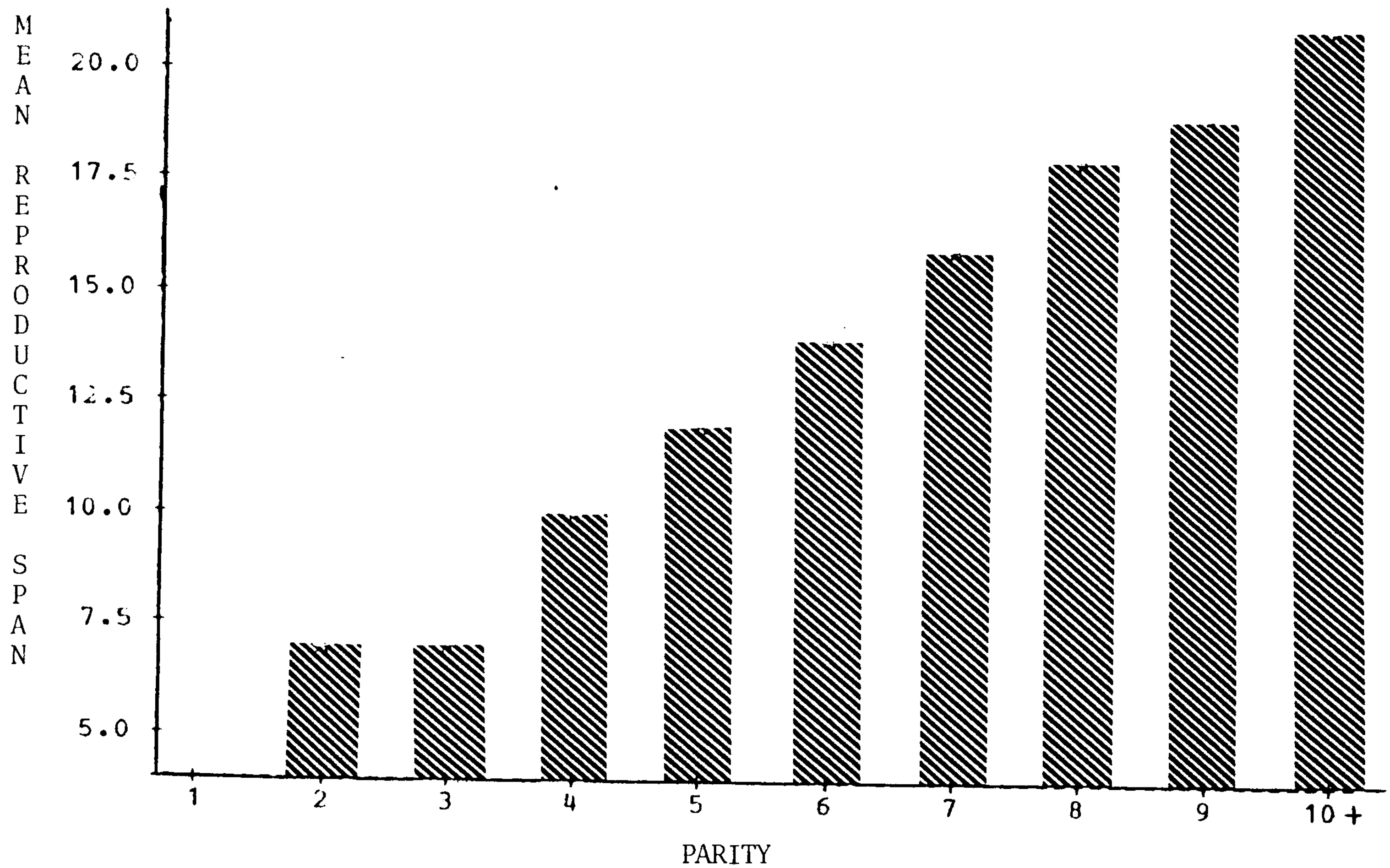
Another measure of marital experience is presented in table 2.10, which gives mean reproductive span by current age and years spent in the married state for women aged 35 and over. The mean span tended to increase with the number of years spent in marital union. This trend, not unexpected, is portrayed graphically in figure 2.11 for women in the oldest cohort.

Table 2.11 contains data on mean span of childbearing by the status

FIGURE 2.7

MEAN REPRODUCTIVE SPAN BY PARITY, FOR WOMEN 45-49

EGYPT, 1980



Source: 1980 Egyptian Fertility Survey.

TABLE 2.8

MEAN REPRODUCTIVE SPAN BY CURRENT AGE AND AGE AT MARRIAGE

FOR WOMEN AGED 35 AND OVER

EGYPT, 1980

Current Age	Age at Marriage				
	<15	15-19	20-24	25-29	30+
35-39	15.526	13.059	9.459	5.708	2.861
	(4.743)[a]	(4.605)	(3.930)	(2.696)	(2.211)
	N=348	N=634	N=197	N=50	N=6
40-44	17.264	14.624	12.023	7.011	3.871
	(5.365)	(5.374)	(4.728)	(3.446)	(2.732)
	N=271	N=517	N=147	N=46	N=11
45-49	18.541	16.160	11.996	8.552	6.555
	(5.495)	(5.566)	(5.489)	(4.232)	(3.957)
	N=257	N=414	N=123	N=27	N=9

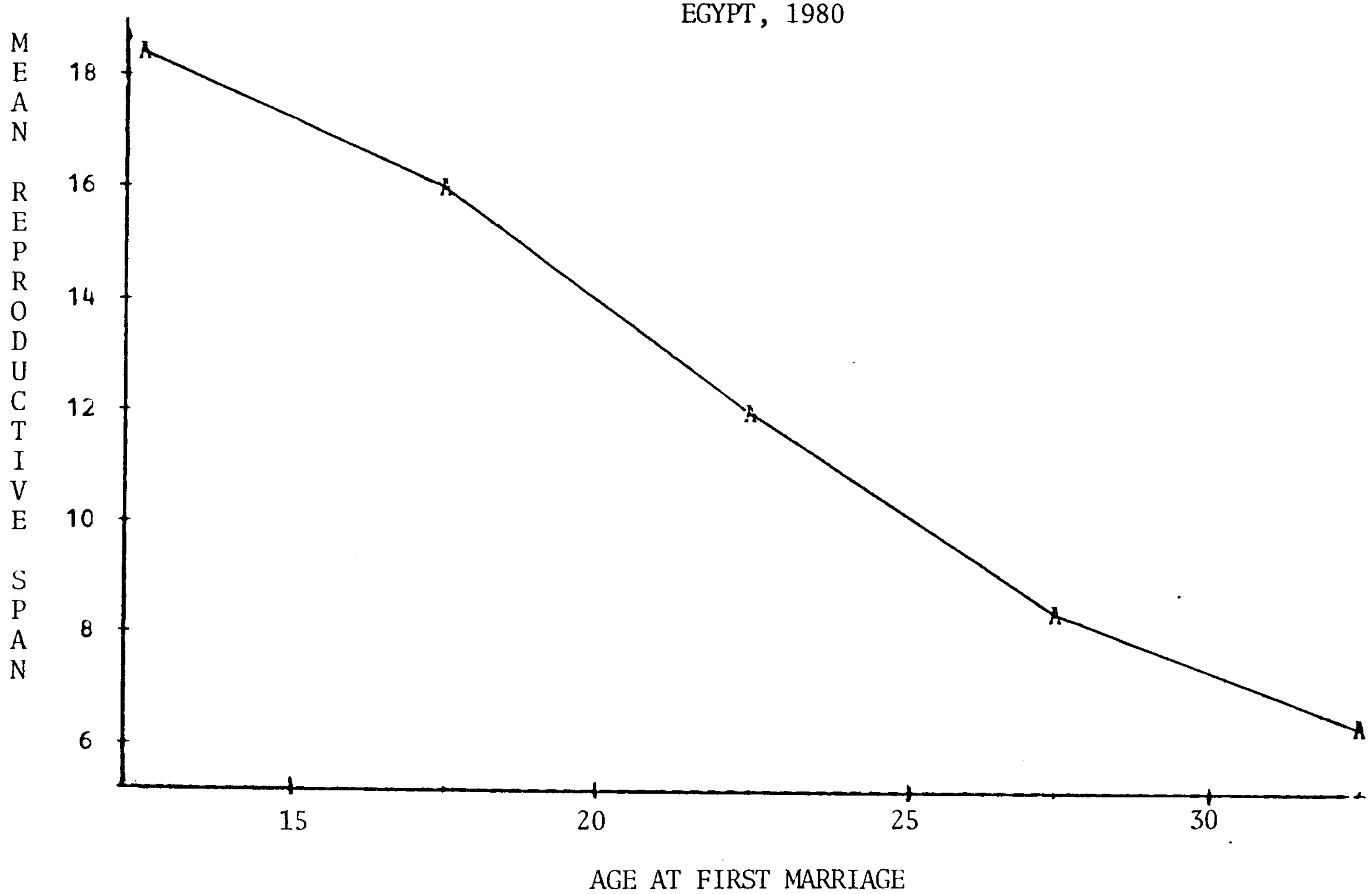
[a]Figures in parentheses are standard deviations.

Source: 1980 Egyptian Fertility Survey.

FIGURE 2.8

MEAN REPRODUCTIVE SPAN BY AGE AT MARRIAGE FOR WOMEN 45-49

EGYPT, 1980

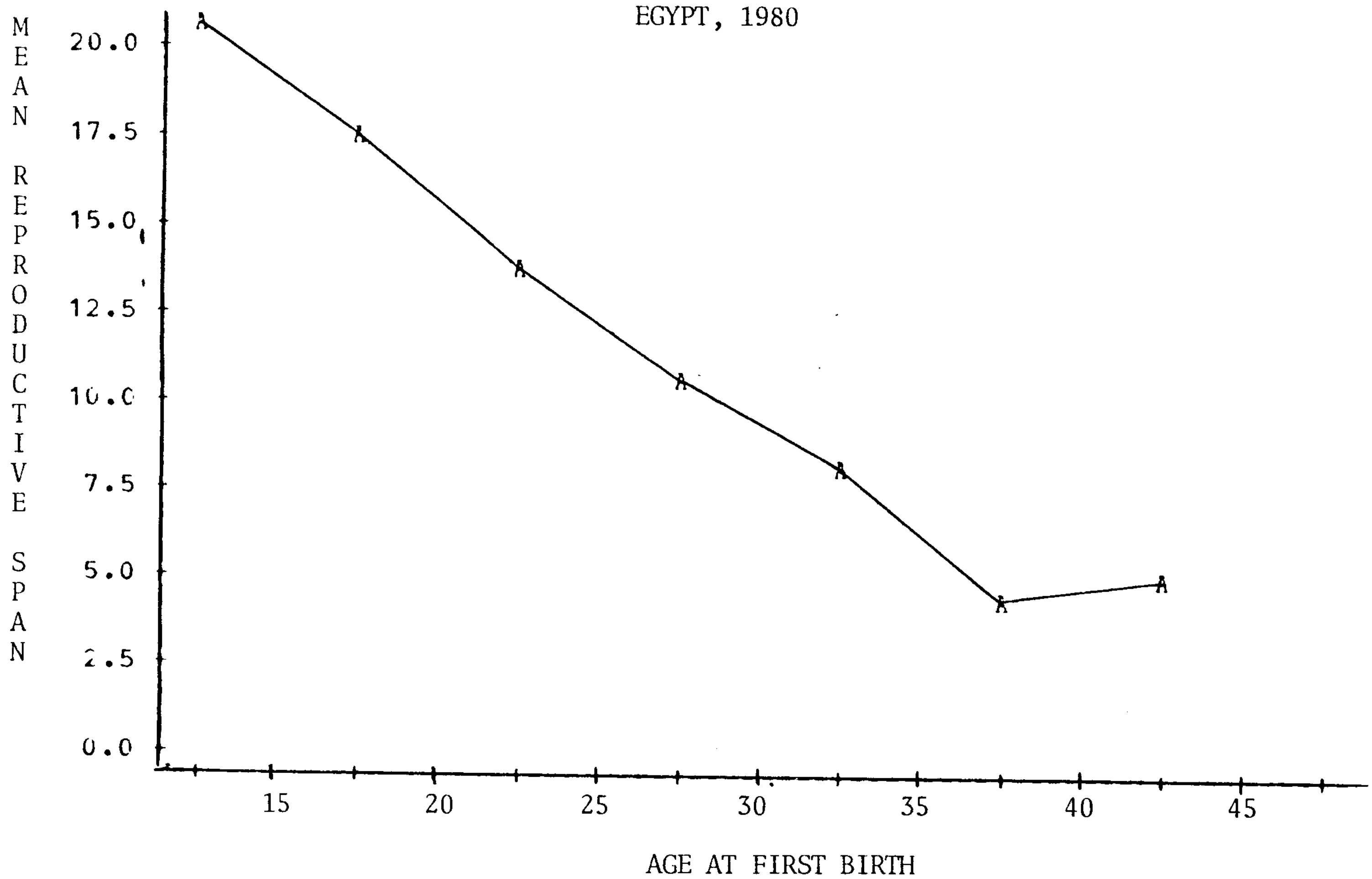


Source: 1980 Egyptian Fertility Survey.

FIGURE 2.9

MEAN REPRODUCTIVE SPAN BY MATERNAL AGE AT FIRST BIRTH, FOR WOMEN 45-49

EGYPT, 1980

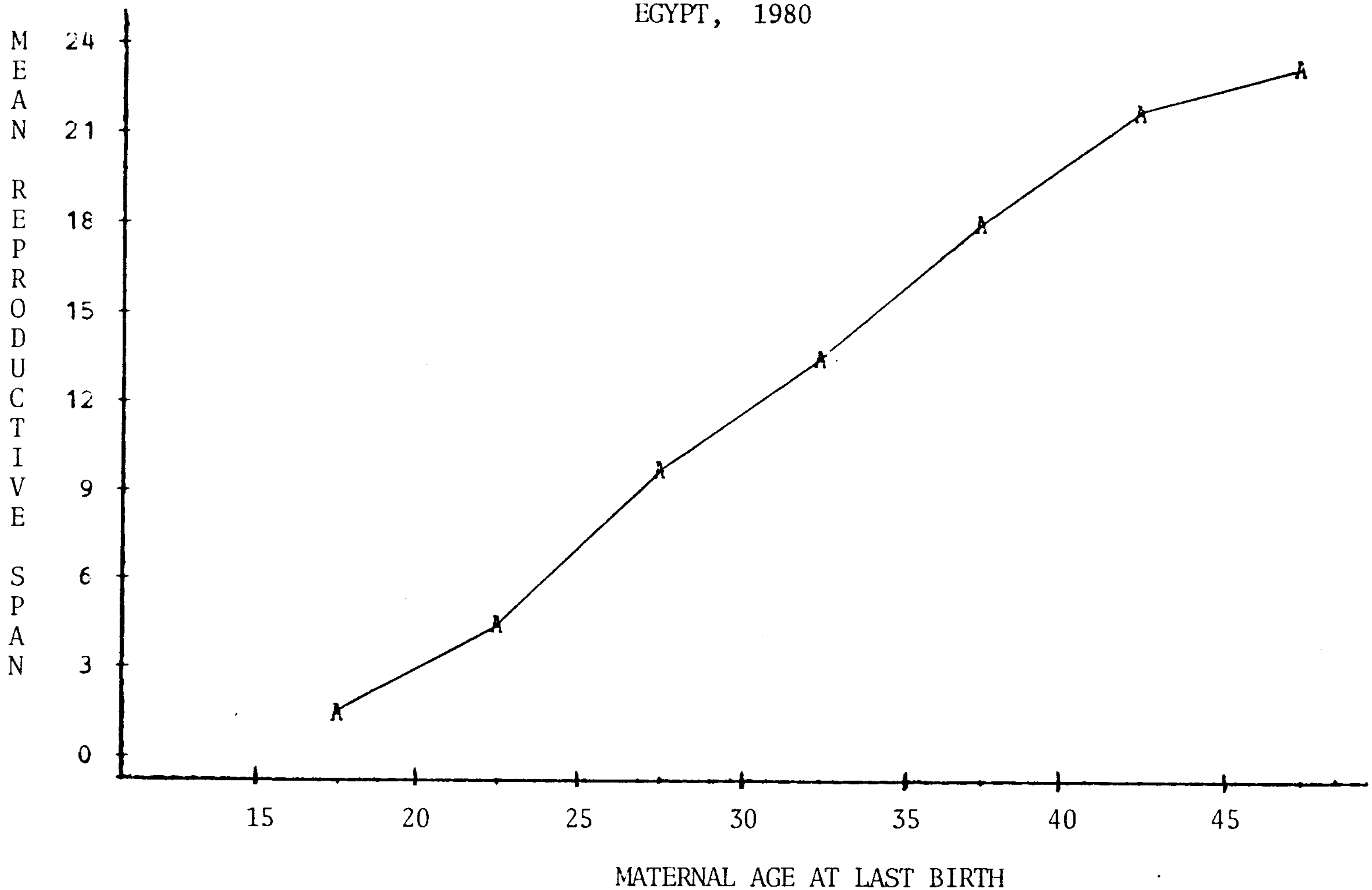


Source: 1980 Egyptian Fertility Survey.

FIGURE 2.10

MEAN REPRODUCTIVE SPAN BY MATERNAL AGE AT LAST BIRTH FOR WOMEN 45-49

EGYPT, 1980



Source: 1980 Egyptian Fertility Survey.

TABLE 2.9

MEAN REPRODUCTIVE SPAN IN YEARS BY CURRENT AGE AND CURRENT MARITAL STATUS FOR WOMEN AGED 35 AND OVER

EGYPT, 1980

Current Age	Current Marital Status			
	Married	Widowed	Separated	Divorced
35-39	13.121 (5.038)[a] N=1110	10.407 (5.241) N=96	11.129 (5.336) N=18	9.009 (5.839) N=11
40-44	14.943 (5.706) N=856	12.116 (5.506) N=107	9.861 (6.888) N=20	9.775 (5.132) N=9
45-49	16.509 (5.946) N=663	14.072 (5.599) N=141	12.208 (9.160) N=16	10.547 (7.606) N=10

[a]Figures in parentheses are standard deviations.

Source: 1980 Egyptian Fertility Survey.

TABLE 2.10

MEAN REPRODUCTIVE SPAN IN YEARS, BY CURRENT AGE AND YEARS SPENT IN THE MARRIED STATE FOR WOMEN AGED 35 AND OVER

EGYPT, 1980

Current Age	Years Spent in Married State							
	<5	5-9	10-14	15-19	20-24	25-29	30-34	35+
35-39	1.917	5.511	8.417	12.167	15.353	17.504		
	(1.165)[a]	(3.369)	(4.037)	(4.336)	(4.187)	(4.535)		
	N=4	N=45	N=174	N=482	N=471	N=59		
40-44	2.972	5.609	8.491	11.060	14.342	16.728	19.545	
	(0.966)	(3.943)	(4.375)	(5.014)	(4.709)	(5.326)	(6.032)	
	N=3	N=23	N=56	N=123	N=370	N=376	N=41	
45-49	7.167	6.972	9.025	11.012	13.252	15.828	18.477	17.963
	(8.014)	(7.025)	(5.354)	(5.014)	(5.389)	(5.770)	(5.193)	(5.807)
	N=2	N=12	N=17	N=49	N=111	N=315	N=297	N=27

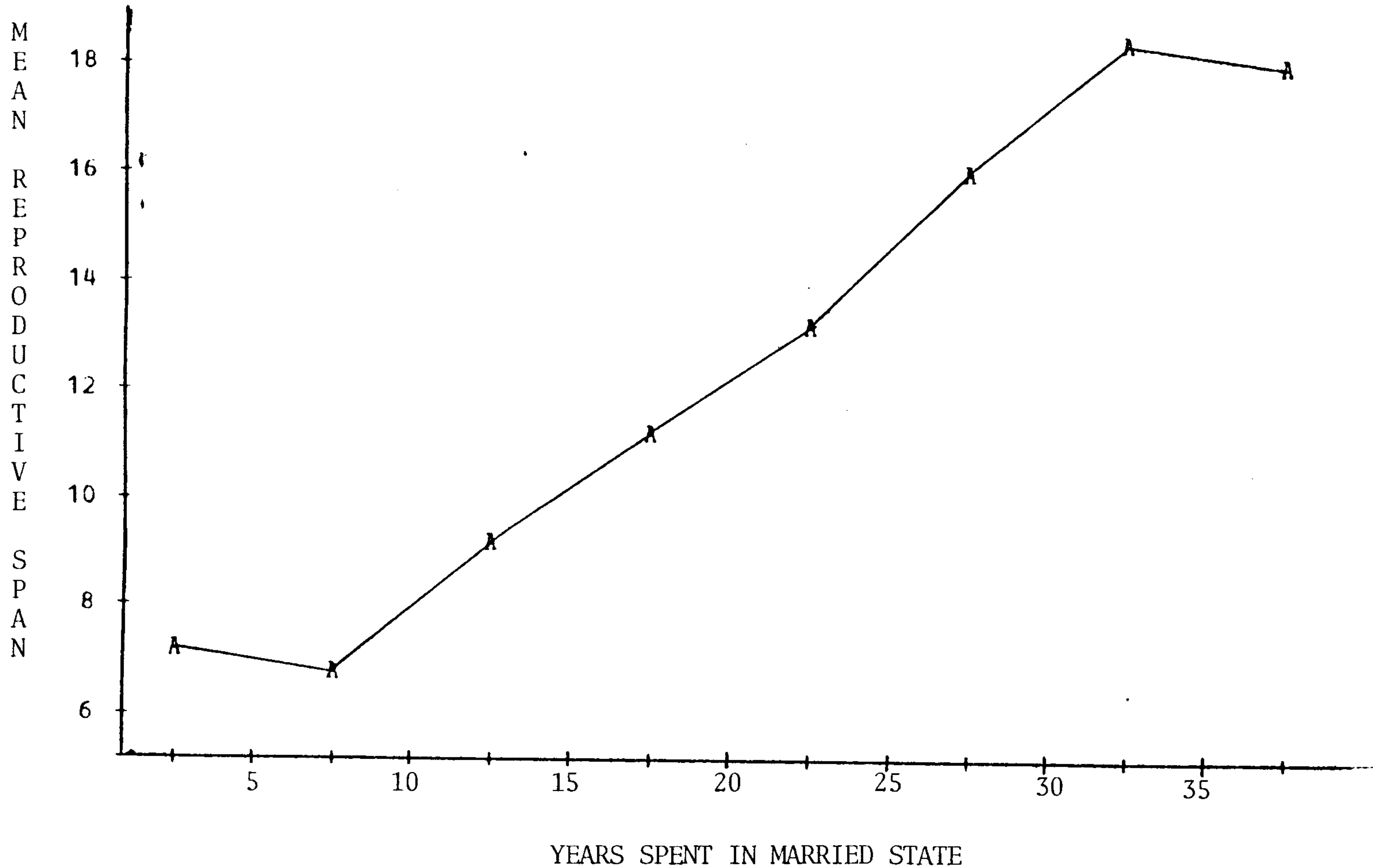
[a]Figures in parentheses are standard deviations.

Source: 1980 Egyptian Fertility Survey.

FIGURE 2.11

MEAN REPRODUCTIVE SPAN BY YEARS SPENT IN MARRIED STATE FOR WOMEN 45-49

EGYPT, 1980



Source: 1980 Egyptian Fertility Survey.

TABLE 2.11

MEAN REPRODUCTIVE SPAN BY CURRENT AGE AND STATUS OF FIRST MARRIAGE FOR WOMEN AGED 35 AND OVER

EGYPT, 1980

Current Age	Status of First Marriage		
	Intact	Dissolved & Remarried	Dissolved, No Marriage
35-39	13.10 (4.964)[a] N=997	13.03 (5.471) N=138	9.90 (5.493) N=100
40-44	15.01 (5.709) N=770	14.00 (5.767) N=106	11.48 (5.523) N=116
45-49	16.41 (6.021) N=575	16.73 (5.668) N=122	13.13 (6.021) N=133

[a]Figures in parentheses are standard deviations.

Source: 1980 Egyptian Fertility Survey.

of the first marriage, for women aged 35 and over. For the three groups of women, those whose first marriages were dissolved and who had not remarried had spent fewer years in childbearing, on average, than the other women. There appears to be very little difference between women with intact first unions and those who remarried. This result could mean that women who remarried tended to do so quickly, so that they lost little time of exposure to pregnancy. Or, this finding could be typical of countries such as Egypt which have high birth rates.

Table 2.12 gives mean reproductuve span by current and childhood residence, for women aged 35 and older. Current urban women experienced an average childbearing span of 13.091 years, compared to 15.091 years for current rural women. This difference of two years is statistically significant (the normal variate z is approximately 9.6). Childhood residential differences were slightly smaller, 13.225 versus 14.977, but also statistically significant (z equal to 8.4).

Mean reproductive span is shown in figure 2.12 by level of education for women aged 45 to 49. Advancement to each higher educational level tended to occur with shorter times spent in childbearing, with the briefest period occurring for those with secondary or higher education. Figure 2.13 gives the same information by years of education, again showing a general tendency for time spent in childbearing to decline with higher education.

2.4 Conclusion

The preceding descriptive statistics and graphical displays have provided a first glimpse at the EFS data on a mostly univariate basis. In cases where no statistical testing was done, it may be found that some apparent relationships prove to be statistically insignificant

TABLE 2.12

MEAN REPRODUCTIVE SPAN IN YEARS BY CURRENT AND CHILDHOOD RESIDENCE, FOR WOMEN AGED 35 AND OVER

EGYPT, 1980

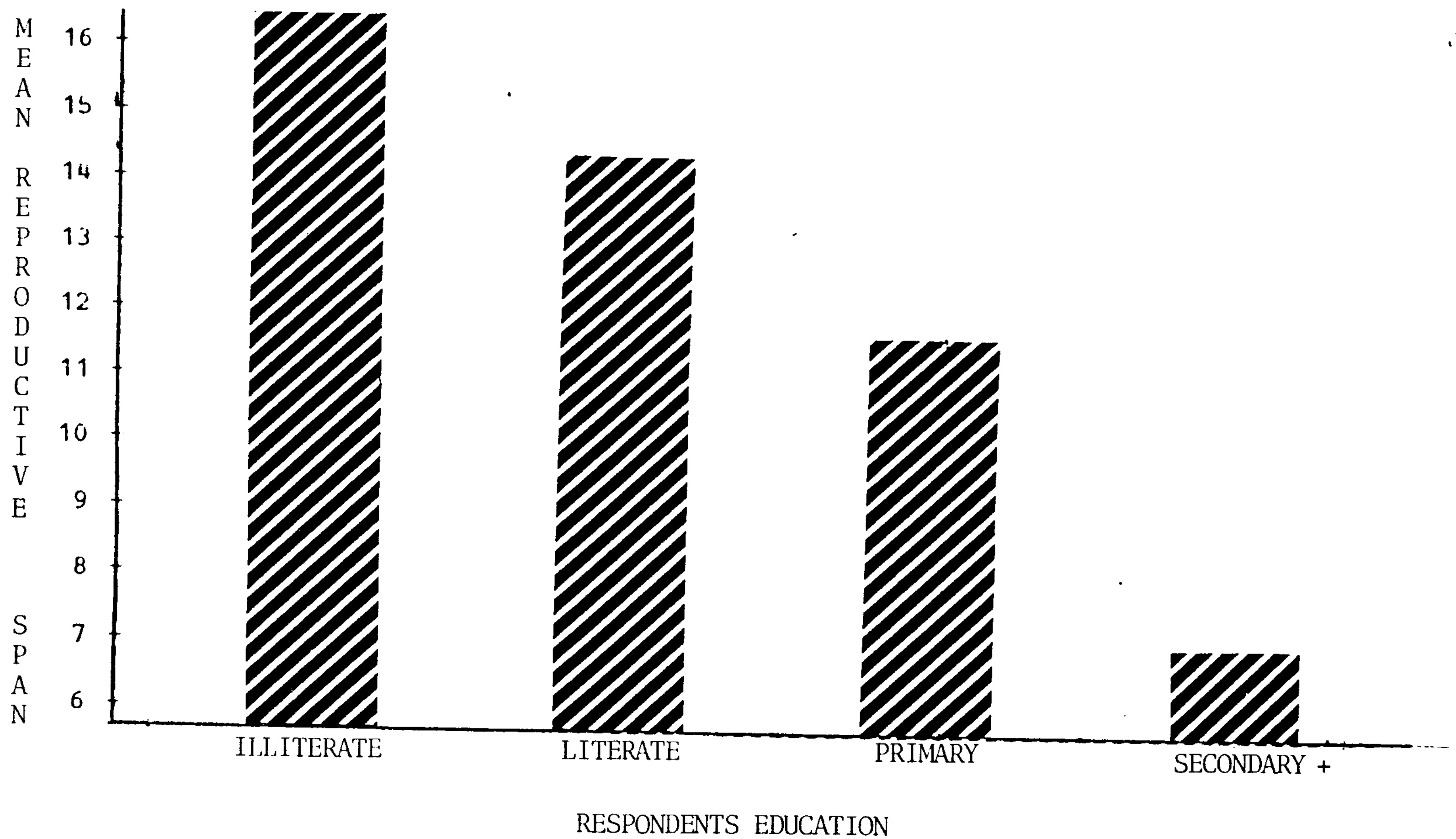
Residence	N	Mean	STD. DEV.
Current			
Urban	1346	13.091	5.8808
Rural	1711	15.091	5.5141
Childhood			
Urban	1338	13.225	5.9064
Rural	1719	14.977	5.5316

Source: 1980 Egyptian Fertility Survey.

FIGURE 2.12

MEAN REPRODUCTIVE SPAN BY EDUCATION, FOR WOMEN 45-49

EGYPT, 1980

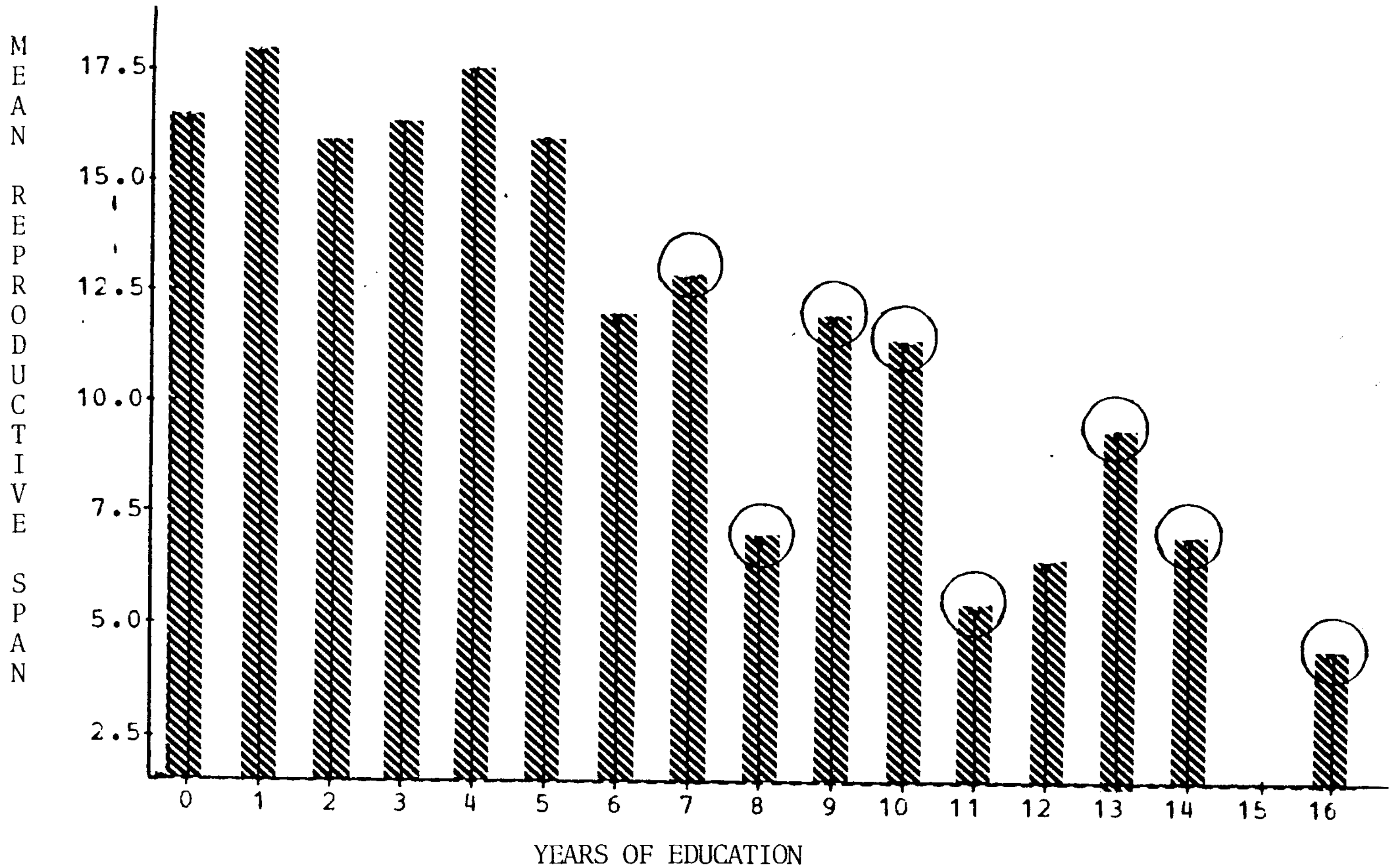


Source: Egyptian Fertility Survey.

FIGURE 2.13

MEAN REPRODUCTIVE SPAN BY WIFE'S YEARS OF EDUCATION, FOR WOMEN 45-49

EGYPT, 1980



Note: Circles represent frequencies less than 10.

Source: 1980 Egyptian Fertility Survey.

and vice versa. Formal determination of relationships, while other variables are controlled, and testing of hypotheses will be done with statistical models in chapters III and IV.

CHAPTER III

MATERNAL AGE AT LAST BIRTH

3.0 Introduction

Research on the timing of fertility events has concentrated on the beginning of the reproductive period. Numerous studies in the literature are concerned with analysis of age at marriage or maternal age at the birth of the first child (e.g., Bulatao (1984), El-Guindy (1971), Kafafi (1983), Kiernan and Diamond (1983), Population Reports (1979), Trussell and Bloom (1983)). Both of these variables have important social and demographic relevance, and the latter is particularly consequential regarding maternal and child health. Likewise, as indicated in chapter I, the age at which women tend to have their last child can be a material factor in social, demographic, and health studies--especially in developing countries, such as Egypt, which experience rapid population growth and high maternal and perinatal mortality rates at the older maternal ages.

In this chapter, two methods of study are applied to the topic of maternal age at last birth. In the first approach, which is termed direct modeling, hazards regression models are employed to assess the effects of independent variables on the hazard of the last birth. A regression-type method is appropriate if detailed micro-level data are available. This analysis is supplemented by hazard modeling without covariates. Afterward in what may be called indirect modeling, a procedure is shown for deriving an estimate of the mean age at last birth for the population of interest, when detailed information is not at hand.

3.1 Direct Modeling

The Egyptian Fertility Survey dataset contains the date of birth of each child for every woman interviewed, as well as each woman's date of birth and age at interview. Thus, a variable called age at last birth could be computed and modeled directly.

The hypotheses that were tested were the following:

1.) women who are well educated have a greater probability of ending childbearing early than women with less education;

2.) women in rural areas have a higher probability of having their last children at older ages than urban women;

3.) marital disruption without remarriage lowers the probability of older maternal age at last birth; and

4.) marital disruption with remarriage increases the probability that a woman stops reproducing at an older age.

For the sake of simplicity, marital disruption only with respect to the status of the first marriage was considered. A first union was considered to be dissolved if a woman said that she was divorced, widowed, or separated, the definition used by CAPMAS.

Since the date of age at last birth is more accurate for women in the 45-49 age cohort, as almost all of these women had finished childbearing, a model was developed for the 870 women in this group only. To model the effects of the independent variables on the hazard of last birth occurring at age t, a proportional hazards or Cox regression model (Cox, 1972) was applied. This model was originally developed for the analysis of life table or survival data in which time to failure may be either observed (complete), or not observed by the end of the follow-up (censored). In the present study, the birth of the last child may be thought of as the failure or the event being watched for, and the woman's age at this event as the "survival time." Follow-up, then, begins at birth. Since data only for women aged 45 to 49 were analyzed, and the assumption is made that this group of women had finished childbearing, all failure times, or ages at last birth, are treated as complete observations.

The hazard that a woman with given characteristics will have her last birth at age t is given in a proportional hazards form as

$$\lambda(t,\underset{\sim}{z}) = \lambda_0(t)e^{\underset{\sim}{z}\underset{\sim}{\beta}} \tag{3.1}$$

where t is the age at last birth, $\underset{\sim}{z}$ is the vector of the independent variables, $\underset{\sim}{\beta}$ is the vector of regression coefficients, and $\lambda_0(t)$ is an unknown baseline hazard function of t. Alternatively, model (3.1) may be written as

$$\ell n[\lambda(t,\underset{\sim}{z})/\lambda_0(t)] = \underset{\sim}{z}\underset{\sim}{\beta} , \tag{3.2}$$

which allows interpretation of the coefficients in the usual regression sense. Model (3.1) contains implicitly three assumptions:

1.) at each age t there is a risk of the last birth occurring that applies to all women in the analysis population with a given covariate vector $\underset{\sim}{z}$;

2.) the relationship between the underlying hazard function and the log-linear function of the covariates is multiplicative (the proportionality assumption); and

3.) the independent variables have a log-linear effect upon the hazard function.

The effects of age at first marriage and parity (number of children ever born), both of which are known to influence age at last birth, were controlled for in the analysis. Women's years of schooling was used to measure the effect of education.

Effect coding was used for the residential and marriage indicator variables. Rural residence was given a code of 1, while urban residence was given a value of -1. If the first marriage was still intact at the time of the interview, the variable MAR1 was coded 1 and MAR2 coded 0. If the first union was dissolved and the woman had remarried, then the variable MAR2 was coded 1 and MAR1, 0. The omitted category--those with dissolved first marriages who had not remarried--was given a value of -1 for both variables. This coding allows interpretation of the regression coefficients as deviations from the overall mean. All covariates were treated as baseline.

High correlation among independent variables introduces multicollinearity into a regression model, and thus instability of the parameter estimates. Spearman's correlation coefficients for the independent variables appear in table 3.1. None of these variables are highly correlated with any other, except for the indicator variables representing marital experience, which have a correlation coefficient of -0.615. Since this degree of correlation is borderline for exclusion from the model, and because the effects of these variables are measured to test the hypotheses about marital disruption, these variables were retained in the model.

Coefficients for model (3.2) were estimated by program P2L (Regression with Incomplete Survival Data) in the Biomedical Data Processing package (Dixon, 1983). The conditional probability that a woman with covariate vector $\underset{\sim}{z}_i$ has her last birth at time t_i, given that a single last birth occurs at t_i and given the set R_i of women at risk just before t_i, is given as

$$e^{\underset{\sim}{z}_i \underset{\sim}{\beta}} \Big/ \sum_{j \in R_i} e^{\underset{\sim}{z}_j \underset{\sim}{\beta}} \, . \qquad (3.3)$$

The partial likelihood function, due to Cox (1975), is obtained by mul-

TABLE 3.1

SPEARMAN CORRELATION COEFFICIENTS FOR INDEPENDENT VARIABLES

FOR WOMEN AGED 45 TO 49

EGYPT, 1980

Variables	AGEMAR	PARITY	WYRSED	RESIDENCE	MAR1	MAR2
AGEMAR	1.0000					
PARITY	-0.3656	1.0000				
WYRSED	0.1231	-0.1419	1.0000			
RESIDENCE	-0.0813	0.0839	-0.2869	1.0000		
MAR1	0.1287	0.2080	0.1124	-0.0334	1.0000	
MAR2	-0.1858	-0.1263	-0.0950	0.0177	-0.6154	1.0000

Source: 1980 Egyptian Fertility Survey.

tiplying (3.3) for each of K distinct ages at last birth, and is written as

$$L(\underset{\sim}{\beta}) = \prod_{i=1}^{K} \left(e^{\underset{\sim}{z}_i \underset{\sim}{\beta}} \Big/ \sum_{j \varepsilon R_i} e^{\underset{\sim}{z}_j \underset{\sim}{\beta}} \right) . \tag{3.4}$$

The parameter estimates obtained by maximization of (3.4) have properties similar to those of maximum likelihood, e.g., asymptotic normality.

When there are ties among the ages at last birth, P2L maximizes the approximate likelihood, according to Breslow (1974). This function is defined as

$$L(\underset{\sim}{\beta}) = \prod_{i=1}^{K} \left[e^{\underset{\sim}{s}_i \underset{\sim}{\beta}} \Big/ \left(\sum_{j \varepsilon R_i} e^{\underset{\sim}{z}_j \underset{\sim}{\beta}} \right)^{m_i} \right] , \tag{3.5}$$

where m_i is the number of last births occurring at age t_i, and $\underset{\sim}{s}_i$ is a vector of sums of the covariate values of m_i women. (Since measuring age at last birth in completed years for 870 women would result in a large number of ties, age as a continuous variable was used to minimize tied ages.)

The global chi-square statistic for testing the hypothesis that all coefficients are zero is defined as

$$U'(\underset{\sim}{0}) I^{-1}(\underset{\sim}{0}) U(\underset{\sim}{0}) \tag{3.6}$$

where $U(\underset{\sim}{0})$ is the vector of first derivatives of the likelihood function evaluated at $\underset{\sim}{\beta} = \underset{\sim}{0}$, and $I(\underset{\sim}{0})$ is the observed information matrix when $\underset{\sim}{\beta} = \underset{\sim}{0}$.

To check the proportionality assumption, strata for independent variables which are suspected to have nonproportional effects upon the hazard function can be defined, and the log minus log survival function $\ell n[-\ell n\hat{S}(t;\bar{z})]$, where $\bar{z}$ is the mean of the covariates, can be plotted. ($\hat{S}(t;\bar{z})$ is estimated as $[\hat{S}_0(t)]^{\exp(\bar{z}\hat{\beta})}$.) If the proportionality assumption is met, this plot should show constant differences between strata, i.e., roughly parallel curves[1] (Kalbfleisch and Prentice, 1980). Figure 3.1 igves this plot by current residence, since it was suspected that any nonproportional effect would come from this variable. There is no strong suggestion from this picture that the proportionality assumption does not hold.

To check the goodness of fit of the model, the cumulative hazard function of the residuals is plotted against the residuals themselves. If the model fits the data perfectly, the plot should be a straight line of unit slope (Kay, 1977). Figure 3.2 exhibits this plot, done as a matter of convenience by current residence. This graph attests that the Cox regression model is a reasonably good fit to the data.

Table 3.2 presents a summary of the results from the regression. The global chi-square statistic indicates that a regression relation exists between the set of covariates and the hazard of last birth--that is, all coefficients are not identically zero. P-values for testing the statistical significance of the coefficients were computed by the likelihood ratio, score, and Wald statistics (see Cox and Oakes (1984), Kalbfleisch and Prentice (1980), and Wald (1943)).

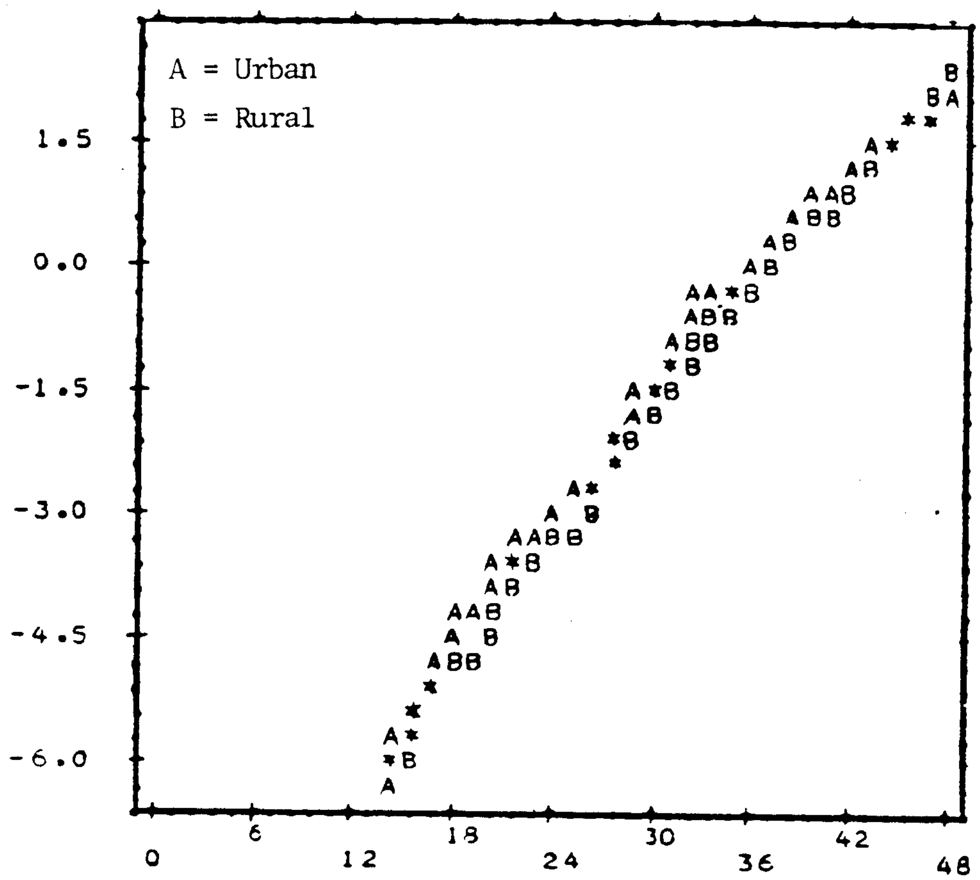
A word on interpretation of the parameter estimates may be in order here. The coefficients express the effects of the independent variables upon the hazard or probability of last birth occurring at age X. A positive coefficient increases the hazard of last birth, and

---

[1]This condition of parallelism is not strictly appropriate, since, in general terminology, $S(0) = 1$ and $S(\infty) = 0$ for both curves. That is, the curves should tend to converge toward the lower and upper age limits of reproduction.

FIGURE 3.1

PLOT OF LOG MINUS LOG SURVIVAL FUNCTION

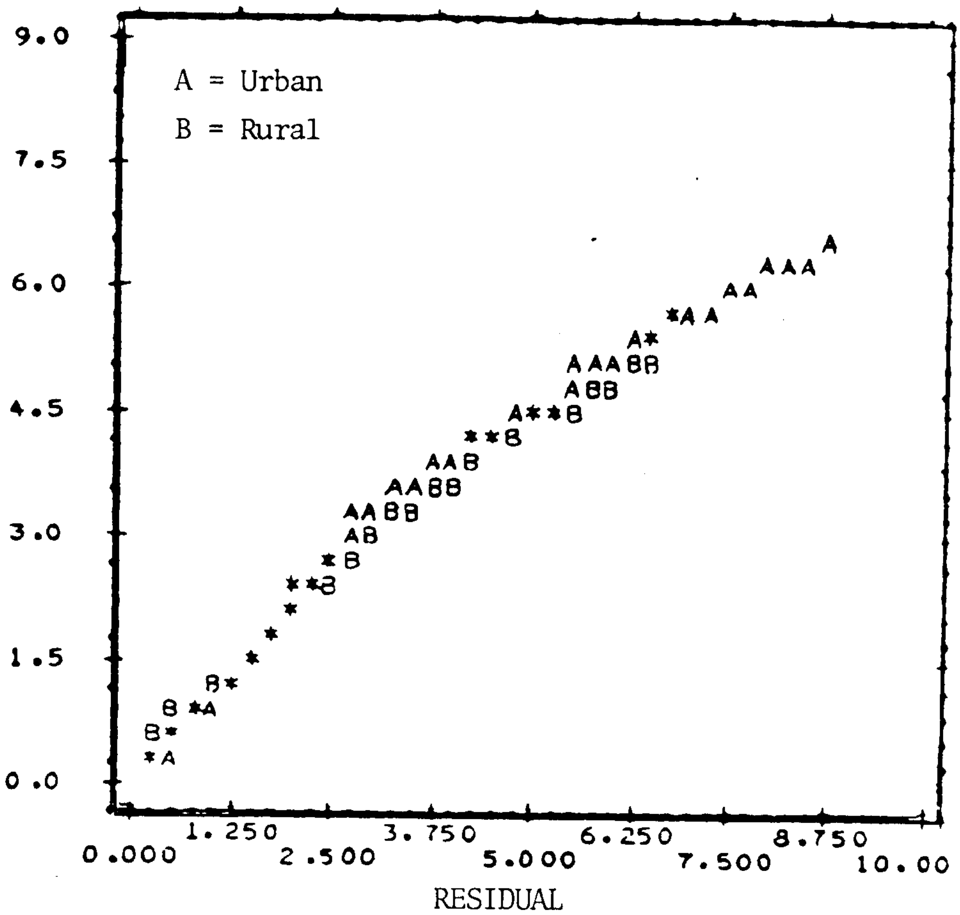


AGE AT LAST BIRTH

Source: 1980 Egyptian Fertility Survey.

FIGURE 3.2

PLOT OF CUMULATIVE HAZARD FUNCTION OF RESIDUALS



Source: 1980 Egyptian Fertility Survey.

TABLE 3.2

RESULTS FROM PROPORTIONAL HAZARDS REGRESSION MODEL 3.1 FOR WOMEN AGED 45 TO 49

EGYPT, 1980

Global X^2 = 355.19 D.F. = 6 P-value = 0.000				Chi-squares[a]			
Variable	Coefficient $\hat{\beta}$	Std. Error	Coeff./S.E.	L. Ratio	Score	Wald	P-value[b]
AGEMAR	$\hat{\beta}_1 = -0.1018$	0.0089	-11.4406	153.20	135.54	130.89	< 0.0001
PARITY	$\hat{\beta}_2 = -0.2347$	0.0158	-14.8861	225.87	222.37	221.60	< 0.0001
WYRSED	$\hat{\beta}_3 = 0.0553$	0.0128	4.3147	17.46	18.80	18.62	< 0.0001
RESIDENCE	$\hat{\beta}_4 = -0.1053$	0.0359	-2.9359	8.52	8.65	8.62	0.003
MAR1	$\hat{\beta}_5 = 0.0352$	0.0542	0.6493	0.42	0.42	0.42	0.515
MAR2	$\hat{\beta}_6 = -0.6205$	0.0728	-8.5235	81.35	76.54	72.65	< 0.0001
MAR3	$\hat{\beta}_7 = 0.5853$	0.0667	8.7751				

[a] All degrees of freedom are 1.

[b] P-values for the three chi-square statistics are equal to three decimal places.

Source: 1980 Egyptian Fertility Survey.

thus indicates that the last birth tends to occur earlier as the value of the covariate increases. Put another way, a positive coefficient signifies a negative relationship with "survival" time or age at last birth. A negative coefficient has the opposite meaning.

Table 3.2 shows that all coefficients are significant, except the coefficient for women with intact first marriages (MAR1). The positive coefficient for WYRSED indicates that women with more years of schooling have a greater probability of ending childbearing early than women with less education. In other words, women with little education tend to have their last children at older ages than other women. This result was expected.

The negative residential (RESIDENCE) coefficient indicates that current residence in rural areas (rural is coded 1) tends to reduce the hazard of last birth, meaning that rural women tend to have their last children at ages significantly older than the overall average age at last birth. Current residence in urban areas (coded -1) has the opposite effect. Again, these findings were expected.

As mentioned earlier, the coefficient for those with intact first marriages (MAR1 coded 1) is not significant, meaning that the mean age at last birth for this group of women is not much different from the overall mean. This finding is as anticipated, since the vast majority of the women analyzed belong to this category.

The negative coefficient for MAR2, or women with dissolved first unions who had remarried, indicates that this group of women tended to end childbearing at ages significantly older than the overall average age at last birth. This result suggests that these women had children by their new husbands.

For the omitted category, those with dissolved first marriages who had not remarried, the coefficient and its standard error, were calculated and are given in table 3.2. This parameter estimate indicates that this group of women had a higher probability of ending childbearing early than did other women. This result also is not surprising, and suggests that these women tended to end their marriages before passing through the childbearing period, thus cutting short their time spent in reproduction.

The variables controlled for, age at marriage (AGEMAR) and final parity (PARITY), had highly significant negative coefficients. That is, as age at marriage and number of children ever born increased, so did the "survival" time or the age at last birth.

It was mentioned before that all the covariates used in model (3.1) were considered to be baseline information. For the variable PARITY this treatment is equivalent to knowing each woman's final parity at the time that follow-up begins. Since the value of PARITY changes often during follow-up for women with large numbers of children, it was decided to modify the proportional hazards model by letting PARITY assume the values 0, 1, 2, 3, 4, and 5+ successively as women progressed from one parity to the next.

Model (3.1) then was revised as

$$\lambda[t,\underset{\sim}{z}(t)] = \lambda_0(t)e^{\underset{\sim}{z}(t)\underset{\sim}{\beta}} \tag{3.7}$$

where $\underset{\sim}{z}(t)$ denotes the covariate vector at time t. This revised model contains both the time-dependent covariable PARITY and the remaining fixed covariates of model (3.1). It will be noted that model (3.7) is not a proportional hazards model as (3.1), since the covariate vector $\underset{\sim}{z}$ now changes with time.

Table 3.3 presents the results from fitting model (3.7) to the data for women aged 45 to 49. The significance and the signs of the coefficients remained the same as those for the previous model, shown in table 3.2. Only slight changes in magnitude of coefficients are observed. Thus, treating PARITY as time-dependent did not change any conclusions made from fitting model (3.1).

Relative risks for the categorical variables may be determined from table 3.3. The relative risk of having the last birth for urban compared to rural women may be written as $\lambda_o(t)\exp(-\beta_4)/\lambda_0(t)\exp(\beta_4)$, or $\exp(-2\beta_4)$. That is, urban women were 1.31 times as likely to have the last birth by age x as rural women.

The relative risk of last birth occurring by age x for women with intact first marriages (MAR1), compared to those who had not remarried (MAR3), is $\exp(2\beta_5 + \beta_6) = .52$. The relative risk for those with intact first marriages (MAR1), compared to those who had remarried (MAR2), is $\exp(\beta_5 - \beta_6) = 2.38$. For women who had remarried (MAR2) compared to those who had not (MAR3), the relative risk is $\exp(\beta_5 + 2\beta_6) = 0.22$.

The preceding results from the hazards models indicate that the hypothesized effects were as anticipated. It is emphasized again that the definition of marital dissolution includes unions ending by widowhood and separation, so that inferences based upon these results will apply to the general state of marital dissolution and not just to the event of divorce. However, for those who experienced dissolution and remarriage, it can be concluded that their effect on the probability of last birth was due to either widowhood or divorce.

In conjunction with the above Cox regression analysis, hazard mod-

TABLE 3.3

RESULTS FROM HAZARD REGRESSION MODEL 3.7 WITH TIME-DEPENDENT COVARIATE FOR WOMEN AGED 45 TO 49

EGYPT, 1980

Global X^2 = 676.85 D.F. = 6 P-value = 0.000

Variable	Coefficient	Std. Error	Coeff./S.E.
AGEMAR	$\hat{\beta}_1 = -0.1355$	0.0086	-15.6746
PARITY (time-dependent)	$\hat{\beta}_2 = -0.5877$	0.0258	-22.7903
WYRSED	$\hat{\beta}_3 = 0.0718$	0.0127	5.6603
RESIDENCE	$\hat{\beta}_4 = -0.1361$	0.0361	-3.7714
MAR1	$\hat{\beta}_5 = 0.0719$	0.0542	1.3286
MAR2	$\hat{\beta}_6 = -0.7929$	0.0740	-10.7178
MAR3	$\hat{\beta}_7 = 0.7210$	0.0672	10.7292

Source: 1980 Egyptian Fertility Survey.

eling without covariates was done on one subgroup of the 45-49 cohort at a time. The survival distribution was estimated by program BMDP1L (Dixon, 1983) via the actuarial life table method. Again, all observations were assumed to be complete. From this analysis it was possible to obtain cumulative life table probabilities of last birth occurring by various age intervals. These probabilities and the median ages at last birth are given in table 3.4.

The median, or the age by which half of the women in the sample had had their last birth, is a convenient point of comparison. The median age at last birth was about two years older for rural compared to urban women (36.47 versus 34.29 years of age).

Illiterate women had the oldest median age at last birth of the education groups. Their median was 36.23 years of age, compared to median ages of 33.78 for literates, and 30.95 for women with at least a primary certificate.

There was very little difference between median ages at last birth for women with intact first marriages and those whose first unions were dissolved and who had remarried. Median ages for these groups were 36.31 and 36.08 years, respectively. Women with dissolved first marriages who had not remarried had the lowest median age at last birth of the three groups--32.34 years of age.

The median age at last birth increased with final parity. Women whose completed family size was 1-2 children had a median age at last birth of 28.83 years. From there the median increased to 30.74 years for women who ended with 3-4 children, to 36.47 years of age for those who had 5 or more children.

Women who were married at ages less than 25 years had a median

TABLE 3.4

CUMULATIVE LIFE TABLE PROBABILITIES OF LAST BIRTH OCCURRING BY BEGINNING OF AGE INTERVAL FOR WOMEN AGED 45 TO 49

EGYPT, 1980

Characteristics	<20	20-24	25-29	30-34	35-39	40-44	45-49	Median
Current Residence								
Urban	.0000 (.0000)[a]	.0126 (.0056)	.0505 (.0110)	.2121 (.0205)	.5480 (.0250)	.8333 (.0187)	.9798 (.0071)	34.29 (0.42)
Rural	.0000 (.0000)	.0190 (.0063)	.0527 (.0103)	.1477 (.0163)	.3966 (.0225)	.7489 (.0199)	.9684 (.0080)	36.47 (0.42)
Women's Education								
Illiterate	.0000 (.0000)	.0172 (.0049)	.0460 (.0079)	.1509 (.0136)	.4167 (.0187)	.7543 (.0163)	.9684 (.0066)	36.23 (0.37)
Literate	.0000 (.0000)	.0192 (.0135)	.0481 (.0210)	.1731 (.0371)	.6058 (.0479)			33.78 (0.62)
Primary +	.0000 (.0000)	.0000 (.0000)	.1143 (.0380)	.4429 (.0594)	.7429 (.0522)	.9286 (.0308)	.9857 (.0142)	30.95 (1.33)
Status of First Marriage								
Intact	.0000 (.0000)	.0117 (.0044)	.0336 (.0074)	.1443 (.0144)	.4111 (.0202)	.7517 (.0177)	.9732 (.0066)	36.31 (0.39)
Dissolved & Remarried	.0000 (.0000)	.0233 (.0133)	.0465 (.0185)	.1860 (.0343)	.4264 (.0435)	.7674 (.0372)	.9457 (.0199)	36.08 (0.85)
Dissolved & not Remarried	.0000 (.0000)	.0276 (.0136)	.1310 (.0280)	.3034 (.0382)	.7241 (.0371)			32.34 (0.59)

TABLE 3.4 (continued)

Characteristics	<20	20-24	25-29	30-34	35-39	40-44	45-49	Median
Parity								
1-2	.0000 (.0000)	.1867 (.0450)	.3467 (.0550)	.5467 (.0575)	.7733 (.0483)	.9333 (.0288)	.9733 (.0186)	28.83 (1.79)
3-4	.0000 (.0000)	.0000 (.0000)	.1429 (.0353)	.4592 (.0503)	.7347 (.0446)	.9184 (.0277)	.9796 (.0143)	30.74 (1.25)
5+	.0000 (.0000)	.0000 (.0000)	.0072 (.0032)	.0976 (.0112)	.3945 (.0185)	.7532 (.0163)	.9727 (.0062)	36.47 (0.34)
Age at Marriage								
<25	.0000 (.0000)	.0171 (.0045)	.0548 (.0079)	.1839 (.0135)	.4762 (.0174)	.7917 (.0142)	.9756 (.0054)	35.38 (0.38)
25+			.0000 (.0000)	.0612 (.0342)	.2857 (.0645)	.7143 (.0645)	.9388 (.0342)	37.50 (0.99)
Total	.0000 (.0000)	.0161 (.0043)	.0517 (.0075)	.1770 (.0129)	.4655 (.0169)	.7874 (.0139)	.9736 (.0054)	35.54 (0.36)

[a] Figures in parentheses are standard deviations.

Source: 1980 Egyptian Fertility Survey.

age at last birth of 35.38 years. For those who were married at age 25 or older, the median age was 37.50 years.

The observed median age at last birth for the total sample of women aged 45 to 49 was 35.54 years.

In this section the hazard of last birth was modeled directly because age at last birth was known for each woman in the dataset. In the following segment, a method is demonstrated for obtaining a summary estimate of age at last birth, when the only available information is a set of age-specific fertility rates and a life table for women.

3.2 Indirect Modeling

If data on the age at the birth of the last child for each woman under study is not available and cannot be derived, age at last birth can be modeled indirectly from age-specific fertility rates. This procedure yields an estimate of the mean age at last birth for the population of interest.

In the following derivations, it is assumed that birth and death processes are independent. Let $m(x)dx$ denote the probability that a woman of age x will have a birth in the age interval $(x,x+dx)$, and also assume that $m(x)$ is a continuous function. The symbols α and β represent the lower and upper ages, respectively, of childbearing, so that $m(x) = 0$ for $x < \alpha$ and $x > \beta$. Then $e^{-\int_x^\beta m(t)dt}$ is the probability of not having a birth after age x. Further let $\ell(X)$ denote the probability that a newborn girl survives the age x. The total fertility rate $TFR = \int_\alpha^\beta m(t)dt$, and the cumulative fertility up to age x $TFR(X) = \int_\alpha^x m(t)dt$.

Let $g_L(x)$ be the probability that a woman will have her last child at age x. Then

$$g_L(x) = \ell(x)m(x)\left[\int_0^{\beta-x} \underbrace{\frac{\ell(x+y)}{\ell(x)}\mu(x+y)}_{A}\underbrace{e^{-\int_x^{x+y} m(t)dt}}_{B} dy + \underbrace{\frac{\ell(\beta)}{\ell(x)} e^{-\int_x^{\beta} m(t)dt}}_{C}\right]$$

for $\alpha < x < \beta$

$$= 0 \text{ otherwise,} \qquad (3.8)$$

where $\mu(x+y)$ denotes the force of mortality; part A of (3.8) is the probability of dying at age x+y; B is the probability of no additional birth until death; and part C is the probability of surviving to age β without an additional birth. Expression (3.8) can be rewritten as

$$g_L(x) = m(x)\int_0^{\beta-x} e^{-\int_x^{x+y} m(t)dt} \ell(x+y)\mu(x+y)dy + m(x)\ell(\beta)e^{-\int_x^{\beta} m(t)dt} .$$

Since $\mu(x+y)dy = -d \log [\ell(x+y)] = \frac{-d\ell(x+y)}{\ell(x+y)}$,

$$g_L(x) = m(x)\int_0^{\beta-x} e^{-\int_x^{x+y} m(t)dt} (-d\ell(x+y)) + m(x)\ell(\beta)e^{-\int_x^{\beta} m(t)dt} .$$

Integration by parts gives

$$g_L(x) = m(x)\left\{\left[-e^{-\int_x^{x+y} m(t)dt}\ell(x+y)\right]_{y=0}^{\beta-x} - \int_0^{\beta-x} \ell(x+y)m(x+y)e^{-\int_x^{x+y} m(t)dt}\right\}$$

$$+ m(x)\ell(\beta)e^{-\int_x^\beta m(t)dt}$$

$$= \ell(x)m(x) - m(x)\int_0^{\beta-x} \ell(x+y)m(x+y)e^{-\int_x^{x+y} m(t)dt} dy\ .$$

A linear transformation of the integrating variable gives

$$g_L(x) = \ell(x)m(x)-m(x)\int_x^\beta \ell(y)m(y)e^{-\int_x^y m(t)dt} dy \quad \text{for } \alpha < x < \beta$$

$$= 0 \text{ otherwise.} \tag{3.9}$$

The probability that a newborn girl will ever have a last birth is also the probability that she will ever become a mother (if she has only one child, then her first child is also her last). This probability (≤ 1) is written as

$$S_1 = \int_{x=\alpha}^\beta \ell(x)e^{-\int_\alpha^x m(a)da} m(x)dx, \tag{3.10}$$

(see equation 4.3).

Then mean age at last birth is

$$\bar{A}_L = \frac{\int_{x=\alpha}^\beta x\ g_L(x)dx}{S_1} \tag{3.11}$$

$$= \frac{\int_{x=\alpha}^\beta x\ell(x)m(x)dx - \int_{x=\alpha}^\beta xm(x) \int_{y=x}^\beta \ell(y)m(y)e^{-\int_x^y m(t)dt} dydx}{S_1}\ .$$

Changing the order of integration gives

$$\bar{A}_L = \frac{\int\limits_{x=\alpha}^{\beta} x\ell(x)m(x)dx - \int\limits_{y=\alpha}^{\beta} \ell(y)m(y)e^{-\int_\alpha^y m(t)dt} dy \int\limits_{x=\alpha}^{y} xm(x)e^{\int_\alpha^x m(t)dt} dx}{S_1}$$

$$= \frac{\int\limits_{x=\alpha}^{\beta} x\ell(x)m(x)dx - \int\limits_{y=\alpha}^{\beta} \ell(y)m(y)e^{-\int_\alpha^y m(t)dt} dy \left[xe^{\int_\alpha^x m(t)dt} \Big|_\alpha^y - \int\limits_{x=\alpha}^{y} e^{\int_\alpha^x m(t)dt} dx \right]}{S_1}$$

$$= \frac{\int\limits_{x=\alpha}^{\beta} x\ell(x)m(x)dx - \int\limits_{y=\alpha}^{\beta} \ell(y)m(y)e^{-\int_\alpha^y m(t)dt} dy \left[\left(ye^{\int_\alpha^y m(t)dt} - \alpha \right) - \int\limits_{x=\alpha}^{y} e^{\int_\alpha^x m(t)dt} dx \right]}{S_1}$$

which simplifies to

$$= \frac{\int\limits_{x=\alpha}^{\beta} x\ell(x)m(x)dx - \int\limits_{y=\alpha}^{\beta} \ell(y)m(y)dy \left[y - \alpha e^{-\int_\alpha^y m(t)dt} - e^{-\int_\alpha^y m(t)dt} \int\limits_{\alpha}^{y} e^{\int_\alpha^x m(t)dt} dx \right]}{S_1}$$

$$= \frac{\alpha \int\limits_{y=\alpha}^{\beta} \ell(y)m(y)e^{-\int_\alpha^y m(t)dt} dy + \int\limits_{y=\alpha}^{\beta} \ell(y)m(y)e^{-\int_\alpha^y m(t)dt} dy \int\limits_{x=\alpha}^{y} e^{\int_\alpha^x m(t)dt} dx}{S_1}$$

$$= \frac{\alpha \int\limits_{y=\alpha}^{\beta} \ell(y)m(y)e^{-\int_\alpha^y m(t)dt} dy + \int\limits_{y=\alpha}^{\beta} \int\limits_{x=\alpha}^{y} \ell(y)m(y)e^{-\int_\alpha^y m(t)dt} e^{\int_\alpha^x m(t)dt} dxdy}{S_1}$$

$$= \frac{\alpha \int_{y=\alpha}^{\beta} \ell(y)m(y)e^{-\int_{\alpha}^{y} m(t)dt} dy + \int_{y=\alpha}^{\beta} \ell(y)m(y)e^{-\int_{\alpha}^{y} m(t)dt} \int_{x=\alpha}^{y} e^{\int_{\alpha}^{x} m(t)dt} dxdy}{S_1}$$

$$\bar{A}_L = \alpha + \frac{\int_{y=\alpha}^{\beta} \ell(y)m(y)e^{-\int_{\alpha}^{y} m(t)dt} \psi_1(y)dy}{S_1} \tag{3.12}$$

where $\psi_1(y) = \int_{x=\alpha}^{y} e^{\int_{\alpha}^{x} m(t)dt} dx$. Expression (3.12) may be viewed as a weighted average of $\psi_1(y)$, with weight being the number of women having first birth at age y.

By the usual definition of variance, the variance of age at last birth can be written as

$$V_L = \frac{\int_{x=\alpha}^{\beta} x^2 g_L(x)dx}{S_1} - (\bar{A}_L)^2 . \tag{3.13}$$

The numerator of the left part of this expression can be expanded as follows:

$$\text{numerator} = \int_{x=\alpha}^{\beta} x^2[\ell(x)m(x) - m(x) \int_{x}^{\beta} \ell(y)m(y)e^{-\int_{x}^{y} m(t)dt} dy]dx$$

$$= \int_{x=\alpha}^{\beta} x^2\ell(x)m(x)dx - \int_{x=\alpha}^{\beta} x^2m(x) \int_{y=x}^{\beta} \ell(y)m(y)e^{-\int_{x}^{y} m(t)dt} dydx .$$

Changing the order of integration gives

$$\text{Numerator} = \int_{x=\alpha}^{\beta} x^2 \ell(x)m(x)dx - \int_{y=\alpha}^{\beta} \ell(y)m(y)e^{-\int_\alpha^y m(t)dt} dy \int_{x=\alpha}^{y} x^2 m(x)e^{\int_\alpha^x m(t)dt} dx$$

and integration by parts yields

$$= \int_{x=\alpha}^{\beta} x^2 \ell(x)m(x)dx - \int_{y=\alpha}^{\beta} \ell(y)m(y)e^{-\int_\alpha^y m(t)dt} dy \left[x^2 e^{\int_\alpha^x m(t)dt} \Big|_\alpha^y - \int_{x=\alpha}^{y} 2xe^{\int_\alpha^x m(t)dt} dx \right]$$

$$= \int_{x=\alpha}^{\beta} x^2 \ell(x)m(x)dx - \int_{y=\alpha}^{\beta} \ell(y)m(y)e^{-\int_\alpha^y m(t)dt} dy \left[\left(y^2 e^{\int_\alpha^y m(t)dt} - \alpha^2 \right) \right.$$

$$\left. - 2\int_{x=\alpha}^{y} xe^{\int_\alpha^x m(t)dt} dx \right]$$

$$= \int_{x=\alpha}^{\beta} x^2 \ell(x)m(x)dx - \int_{y=\alpha}^{\beta} \ell(y)m(y)dy \left[y^2 - \alpha^2 e^{-\int_\alpha^y m(t)dt} - 2e^{-\int_\alpha^y m(t)dt} \int_{x=\alpha}^{y} xe^{\int_\alpha^x m(t)dt} dx \right]$$

$$= \alpha^2 \int_{y=\alpha}^{\beta} \ell(y)m(y)e^{-\int_\alpha^y m(t)dt} dy + 2\int_{y=\alpha}^{\beta} \int_{x=\alpha}^{y} x\ell(y)m(y)e^{-\int_\alpha^y m(t)dt + \int_\alpha^x m(t)dt} dxdy$$

$$= \alpha^2 \int_{y=\alpha}^{\beta} \ell(y) m(y) e^{-\int_{\alpha}^{y} m(t)dt} dy + 2 \int_{y=\alpha}^{\beta} \ell(y) m(y) e^{-\int_{\alpha}^{y} m(t)dt} \left[\int_{x=\alpha}^{y} x e^{\int_{\alpha}^{x} m(t)dt} dx \right] dy .$$

Then

$$V_L = \frac{\text{numerator}}{\int_{y=\alpha}^{\beta} \ell(y) m(y) e^{-\int_{\alpha}^{y} m(t)dt} dy} - (\bar{A}_L)^2$$

$$= \frac{\alpha^2 + 2 \int_{y=\alpha}^{\beta} \ell(y) m(y) e^{-\int_{\alpha}^{y} m(t)dt} \left[\int_{x=\alpha}^{y} x e^{TFR(X)} dx \right] dy}{S_1} - (\bar{A}_L)^2$$

$$V_L = \alpha^2 + 2 \frac{\int_{y=\alpha}^{\beta} g_1(y) \psi_2(y) dy}{S_1} - (\bar{A}_L)^2 \qquad (3.14)$$

where $\psi_2(y) = \int_{x=\alpha}^{y} x e^{TFR(X)} dx$.

Equations (3.12) and (3.14) give the general formulations when mortality is taken into account. When the effect of mortality is not considered (when $\ell(x) = 1$ and $\mu(x) = 0$), expression (3.9) becomes

$$g_{L_{NM}} = m(x) - m(x) \int_{x}^{\beta} m(y) e^{-\int_{x}^{y} m(t)dt} dy$$

$$= m(x) - m(x) \int_x^\beta - d\left(e^{-\int_x^y m(t)dt}\right) dy$$

$$= m(x) + m(x)\left[e^{-\int_x^\beta m(t)dt} - 1 \right]$$

$$g_{L_{NM}} = m(x)\ e^{-\int_x^\beta m(t)dt} \quad . \tag{3.15}$$

Then mean age at last birth without mortality considered can be written as

$$\bar{A}_{L_{NM}} = \frac{\int_\alpha^\beta x\, m(x) e^{-\int_x^\beta m(t)dt} dx}{\int_\alpha^\beta m(x) e^{-\int_x^\beta m(t)dt} dx}$$

$$= \frac{\int_\alpha^\beta x d\left(e^{-\int_x^\beta m(t)dt}\right)}{\int_\alpha^\beta m(x) e^{-\int_x^\beta m(t)dt} dx} \quad .$$

Integration by parts gives

$$\bar{A}_{L_{NM}} = \frac{x e^{-\int_x^\beta m(t)dt}\Big|_\alpha^\beta - \int_\alpha^\beta e^{-\int_x^\beta m(t)dt} dx}{\int_x^\beta d\left(e^{-\int_x^\beta m(t)dt}\right)}$$

$$= \frac{\beta - \alpha e^{-\int_\alpha^\beta m(t)dt} - \int_\alpha^\beta e^{-\int_x^\beta m(t)dt} dx}{e^{-\int_x^\beta m(t)dt}\Big|_\alpha^\beta}$$

$$= \frac{\beta - \alpha e^{-\int_\alpha^\beta m(t)dt} - \int_\alpha^\beta e^{-\int_x^\beta m(t)dt} dx}{1 - e^{-\int_\alpha^\beta m(t)dt}}$$

$$\bar{A}_{L_{NM}} = \frac{\beta - \alpha e^{-TFR} - \int_\alpha^\beta e^{-(TFR-TFR(X))} dx}{1 - e^{-TFR}} . \qquad (3.16)$$

The variance of age at last birth without mortality considered is

$$V_{L_{NM}} = \frac{\int_{x=\alpha}^\beta x^2 g_{L_{NM}} dx}{\int_\alpha^\beta m(x) e^{-\int_x^\beta m(t)dt} dx} - (\bar{A}_{L_{NM}})^2$$

$$= \frac{\int_{x=\alpha}^\beta x^2 m(x) e^{-\int_x^\beta m(t)dt} dx}{1 - e^{-TFR}} - (\bar{A}_{L_{NM}})^2$$

$$= \frac{\int_{x=\alpha}^\beta x^2 d\left(e^{-\int_x^\beta m(t)dt}\right)}{1 - e^{-TFR}} - (\bar{A}_{L_{NM}})^2$$

$$= \frac{x^2 e^{-\int_x^\beta m(t)dt}\Big|_\alpha^\beta - 2\int_\alpha^\beta x e^{-\int_x^\beta m(t)dt} dx}{1 - e^{-TFR}} - (\bar{A}_{L_{NM}})^2$$

$$= \frac{\beta^2 - \alpha^2 e^{-\int_\alpha^\beta m(t)dt} - 2\int_\alpha^\beta x e^{-\left[\int_\alpha^\beta m(t)dt - \int_\alpha^x m(t)dt\right]} dx}{1 - e^{-TFR}} - (\bar{A}_{L_{NM}})^2$$

$$V_{L_{NM}} = \frac{\beta^2 - \alpha^2 e^{-TFR} - 2\int_\alpha^\beta x e^{-(TFR - TFR(X))} dx}{1 - e^{-TFR}} - (\bar{A}_{L_{NM}})^2 . \qquad (3.17)$$

Computational formulas to approximate the general expressions for the mean and variance of age at last birth are as follows:

$$S_1 = \sum \left(n_i {}^L x_i / \Big|_0 \right) e^{-TFR(X_{i+})} m(x_{i+}) \qquad (3.18)$$

$$\bar{A}_L = \alpha + \frac{\sum \left(n_i {}^L x_i / \Big|_0 \right) e^{-TFR(X_{i+})} m(x_{i+}) \psi_1(x_{i+})}{S_1} \qquad (3.19)$$

and

$$V_L = \alpha^2 + 2\left[\frac{\sum \left(n_i {}^L x_i / \Big|_0 \right) e^{-TFR(X_{i+})} m(x_{i+}) \psi_2(x_{i+})}{S_1} \right] - (\bar{A}_L)^2 \qquad (3.20)$$

where x_{i+} is the midpoint of the age interval $(x, x+n_i)$; n_i is the width of the age interval; $m(x_{i+})$ is the age-speci c fertility rate of the age interval;

$$\psi_1(x_{i+}) = \int_{\alpha}^{x_{i+}} e^{TFR(z)}dz \,, \tag{3.21}$$

and

$$\psi_2(x_{i+}) = \int_{\alpha}^{x_{i+}} ze^{TFR(z)}dz \,. \tag{3.22}$$

TFR(X) is computed from the available age-specific fertility rates. Since the value of TFR(X) is usually computed to the beginning of the age interval, $e^{-TFR(X_{i+})}$ is computed as the average of $e^{-TFR(X_i)}$ and $e^{-TFR(X_{i+5})}$.

Table 3.5 gives essential calculations for estimating the mean and variance of age at last birth, with and without mortality taken into account. The age-specific fertility rates (ASFR) are for the twelve-month period 1979-1980, and were calculated directly from the EFS dataset and the household schedule (CAPMAS, 1983 c). The data on mortality, the ${}_5L_x/l_0$ column, were taken from a life table for Egyptian women for the year 1976 (El-Madani, 1981), which is the latest life table data available.[2]

In the presence of mortality, which can be seen from column (4) to be quite high at the older ages, mean age at last birth for Egypt is 36.96 years. (This figure compares to 35.06 as the observed mean age at last birth for the 45-49 cohort of women in the EFS.) When mortality is not considered, the estimated mean is 36.76 years--only slightly younger than above. This example demonstrates that even when the mortality level is high, taking mortality into consideration

---

[2]New life table information will not be available until the 1986 census is conducted and population size by age is determined. There have been no surveys to estimate this information since 1976.

TABLE 3.5

ILLUSTRATIVE CALCULATION OF MEAN AGE AT LAST BIRTH

EGYPT, 1980

(1)	(2)	(3)[a]	(4)[b]	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Age	X_{i+}	ASFR	$_5L_X/l_0$	TFR(X)	$e^{-TFR(X)}$	$e^{-TFR(X_{i+})}$	$g_1(X_{i+})$ (3)x(4)x(7)	$e^{TFR(X)}$	$e^{TFR(X_{i+})}$	$\psi_1(X_{i+})$	$\psi_2(X_{i+})$
15-19	17.5	.0785	4.640	.0000	1.0000	.8377	.3051	1.0000	1.2404	2.8004	46.6335
20-24	22.5	.2557	4.222	.3925	.6754	.4318	.4662	1.4807	3.3991	12.3014	250.1450
25-29	27.5	.2801	3.809	1.6710	.1881	.1178	.1251	5.3175	13.4459	46.6513	1171.6844
30-34	32.5	.2386	3.399	3.0715	.0464	.0303	.0246	21.5742	46.3518	175.3339	5247.8265
35-39	37.5	.1389	2.993	4.2645	.0141	.0106	.0044	71.1293	106.7903	544.5849	18539.3520
40-44	42.5	.0526	2.593	4.9590	.0070	.0062	.0008	142.4513	163.8779	1239.4860	46674.6843
45-49	47.5	.0124	2.199	5.2220	.0054	.0053	.0001	185.3044	191.2307	2146.6327	87667.2415
50	-	.0000	-	5.2840	.0051	-	-	197.1569	-	-	-

$\sum = .9263$

Case 1: With Mortality, (by formulas (3.18)-(3.20)) $S_1 = \sum g_1(X_{i+}) = .9263$

Mean Age at Last Birth $= 15 + \frac{\sum g_1(X_{i+})\psi_1(X_{i+})}{S_1} = 36.96$ years; Variance $= (15)^2 + \frac{2\sum g_1(X_{i+})\psi_2(X_{i+})}{S_1} - (\bar{A}_L)^2$

Stand. Dev. = 3.18 years.

Case 2: No Mortality, (when $_5L_X/l_0 = 5.000$)

Mean Age at Last Birth = 36.76 years

Stand. Dev. = 6.10 years

Source: [a]CAPMAS, 1983 c.

[b]El-Madani, 1981.

does not make much difference in the estimate of mean age at last birth. Thus, using the simpler approximation to formula (3.16) may be justified.

3.3 Conclusion

In section 3.1 hazards models were fitted to the data, to estimate the effects of the independent variables on the hazard of last birth. This could be done because direct observations on the age at last birth for each woman under analysis could be computed.

When such detailed information is not available, an alternative is to evolve a summary estimate of the mean age at last birth for the population of interest. In section 3.2 theoretical expressions for the mean and variance of age at last birth were derived, with and without mortality taken into account. With only age-specific fertility rates and a recent abridged life table for women assumed available, theoretical expressions to compute estimates of the means and variances of age at last birth for Egypt were approximated. An extension of this indirect modeling can be done if age-specific fertility rates, and perhaps life table information, are available by population subgroups. Then the formulas used in the indirect modeling can be used to compare subgroup means.

In fitting the hazards models, data only for the oldest cohort of women were used, in order to more accurately measure the ages at last birth. The results, then, apply only to women of that cohort. No inference can be made about women in the younger age groups, unless their fertility and mortality experiences resemble those of the older women.

In the indirect modeling of age at last birth, cross-sectional data on fertility and mortality were used for all cohorts of women in the childbearing ages to compute a summary measure for Egypt. The estimate of mean age at last birth thus obtained yields the expected age at the end of childbearing for a new cohort of women, under the assumption that they experience the current fertility and mortality conditions.

CHAPTER IV

REPRODUCTIVE SPAN

4.0 Introduction

As with maternal age at last birth, the time a woman spends in childbearing can have important social and health consequences. All that was said in chapter I regarding the implications of age at last birth also applies to the span of reproduction. Also, in a society such as Egypt, in which most women who use birth control methods do so only after they have had all the children they want, the length of the childbearing span can be an important component in studies of fertility.

In the following sections the study of reproductive span is approached in two ways. In the first, which is called direct modeling, a multiple regression model to determine the effects of the independent variables on the length of the childbearing span is applied. Afterward a method of obtaining a summary estimate of reproductive span, when direct observations on age at first and last birth are not available, is shown by indirect modeling.

4.1 Direct Modeling

Since the Egyptian Fertility Survey dataset contains the dates of the first and last births for each woman, the span of reproduction for each respondent is obtained by subtracting the former date from the latter. Thus, reproductive span can be modeled directly. Women who had had less than two children at the time of the survey were excluded from analysis, as were women who had had only two children which were twin births.

The hypotheses that were tested are the following:

1.) well educated women tend to have shorter childbearing spans than less educated women;

2.) women in rural areas tend to spend more time in childbearing than urban women;

3.) marital disruption without remarriage tends to shorten the reproductive span; and

4.) marital disruption with remarriage tends to lengthen the time spent in childbearing.

Women's years of schooling (WYRSED) was used to assess the effect of education on the length of the reproductive span. Current residence (RESIDENCE) was used to make urban/rural comparisons. As in chapter III, marital disruption was considered only with respect to the status of the first marriage, and a first union again was considered to be dissolved if a woman said that she was divorced, widowed, or separated. The effects of age at first marriage (AGEMAR) and final parity (PARITY) were controlled for by including these variables as covariates in the model.

Multiple regression was the method of analysis used to test the hypotheses. Since the measurement of reproductive span is assumed to be more accurate for women in the oldest cohort, a model for the 830 women in this group only was developed. The model fitted for the i-th woman in the dataset is

$$\begin{aligned} \text{RSPAN}_i = \beta_0 + \beta_1 \text{ AGEMAR}_i + \beta_2 \text{ PARITY}_i + \beta_3 \text{ WYRSED}_i \\ + \beta_4 \text{ RESIDENCE}_i + \beta_5 \text{ MAR1}_i + \beta_6 \text{MAR2}_i + \text{RESIDUAL}_i \;. \end{aligned} \quad (4.1)$$

As in chapter III, effect coding was used for the residential and marriage indicator variables. The coefficients for these variables then are interpreted as deviations from the overall mean.

Examination of plots of residuals from the fitted model found no reason to believe that the model violated any assumptions underlying regression analysis. Figure 4.1 shows a plot of the residuals from the fitted regression surface against the fitted values $\hat{RSPAN}_i$, and a normal probability plot of the residuals. The Kolmogorov D-statistic, equal to .027 and with an associated p-value greater than .15, suggests no reason to believe that the residuals are not normally distributed.

The estimated regression surface obtained is

$$\hat{RSPAN} = 11.255 = 0.306 \text{ AGEMAR} + 1.326 \text{ PARITY} - 0.107 \text{ WYRSED} + 0.389 \text{ RESIDENCE} + 0.243 \text{ MAR1} + 1.465 \text{ MAR2}.$$

Table 4.1 gives other results from the fitted regression. The F-value attests that there is a regression relation between the dependent and independent variables. The R^2 value indicates that 57% of the variation in reproductive span is accounted for by the set of independent variables in the model.

Since alternatives to the null hypotheses are one-sided, one-sided p-values, which appear in table 4.1, were computed for testing the individual hypothesis that $B_i = 0$.[1] When all other variables in the model were controlled, wife's years of education had a negative effect on the length of the reproductive span, significant at the .05 level of significance. An increase of one year of education was associated with a decrease of .11 of a year in mean reproductive span.

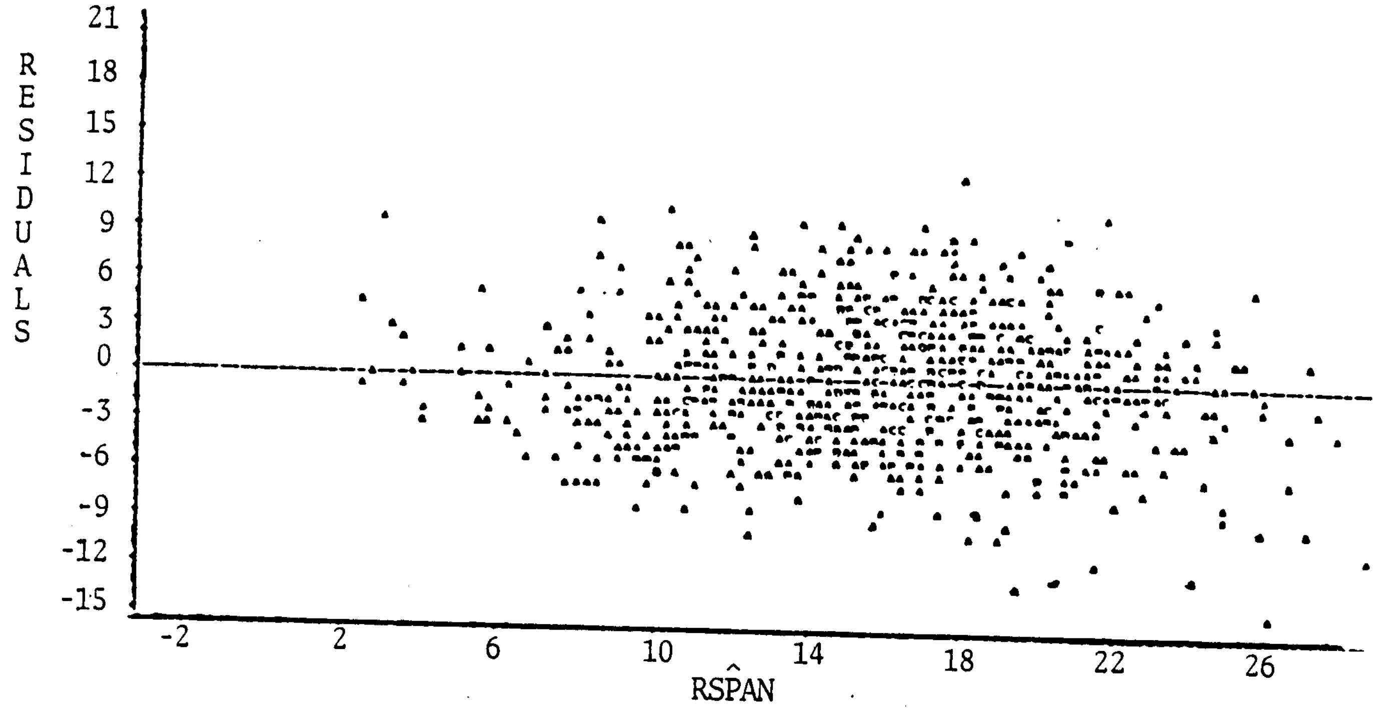
Current residence in rural areas (coded 1) was associated with longer spans of childbearing than the overall mean span, while residence in urban regions (coded -1) had the reverse effect. Rural residence was associated with a mean span of about .39 of a year longer

[1]These p-values, based on the t-statistic, $b_i/S(b_i)$, provide marginal tests. There are no inferences made about the joint significance of the coefficients.

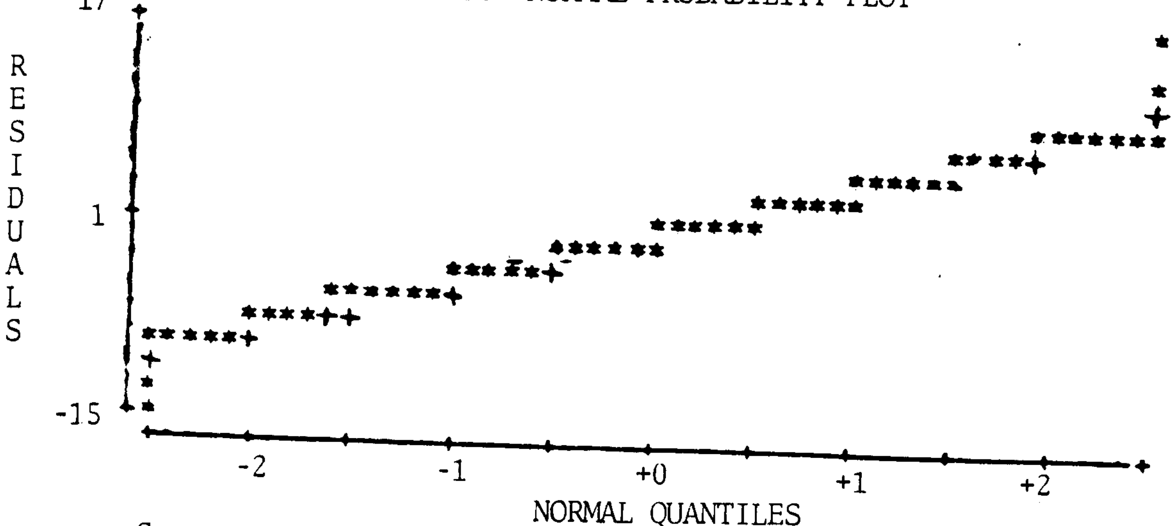
FIGURE 4.1

GRAPHICAL DIAGNOSTICS OF REGRESSION MODEL 4.1 FOR WOMEN AGED 45-49, EGYPT, 1980

a. RESIDUALS AGAINST PREDICTED VALUES



b. NORMAL PROBABILITY PLOT



Source: 1980 Egyptian Fertility Survey.

TABLE 4.1

RESULTS OF REGRESSION ANALYSIS FOR WOMEN AGED 45 TO 49

EGYPT, 1980

Variables in Model	Estimated Coefficients	Standard Deviation	One-sided p-value
Intercept	11.255	0.8871	.0001
AGEMAR	-0.306	0.0391	.0001
PARITY	1.326	0.0570	.0001
WYRSED	-0.107	0.0519	.0193
RESIDENCE	0.389	0.1470	.0042
MAR1 (intact first unions)	0.243	0.2140	.1282
MAR2 (dissolved & remarried)	1.465	0.2833	.0001
MAR3 (dissolved, no remarriage)	-1.708	0.2690	.0001

$F = 179.4$ $P > F = .0001$ $R^2 = 0.57$

Source: 1980 Egyptian Fertility Survey.

than the overall average, and urban residence, .39 of a year shorter.

Women whose first marriages were still intact tended to have childbearing spans not significantly different from the overall mean (p-value = .1282). This result is expected since the vast majority of the women belonged to this group. Those with dissolved marriages who had remarried had the longest spans of all--about 1.5 years longer than the overall mean span. The corresponding statistics were calculated for the omitted category--those with dissolved first unions who had not remarried by the time of the interview. These women had spans of about 1.7 years shorter than the overall average. These findings imply that the marriages which had dissolved without remarriage tended to do so before the women had completed the childbearing period, and that women who remarried were inclined to produce children from their new unions.

Span of reproduction tended to decrease as age at first marriage increased. An increase of one year in age at marriage was associated with a reduction of about .31 of a year in mean length of the reproductive span.

Longer spans were associated with higher parity. Each additional child extended the mean reproductive span an average of 1.33 years. Since a birth interval must be at least .75 of a year (for full-term births), this small change in length of the span manifests a predominant lack of birth spacing among Egyptian women.

4.2 Indirect Modeling

It was demonstrated in chapter III that if data are not directly available on age at last birth for individual women, age at last birth can be modeled indirectly from age-specific fertility rates. Likewise age at first birth can be derived, and, with both components, reproductive span can also be modeled indirectly.

Since the method of deriving mean age at last birth was described in chapter III, the procedure for modeling the other component, age at first birth, is now presented. Again it is assumed that birth and death processes are independent.

Let $m(x)dx$ denote the probability that a women of age x will have a birth in the age interval $(x,x+dx)$. The symbols α and β represent the lower and upper ages, respectively, of childbearing, so that $m(x)=0$ for $x < \alpha$ and $x > \beta$. Then $e^{-\int_\alpha^x m(t)dt}$ is the probability of not having a birth before age x. Further, let $\ell(x)$ denote the probability that a newborn girl survives to age x. The total fertility rate $TFR = \int_\alpha^\beta m(t)dt$, and the cumulative fertility rate $TFR(X) = \int_\alpha^X m(t)dt$.

Let $g_1(x)$ be the probability of having the first birth in the age interval $(x,x+dx)$. Then

$$g_1(x) = \ell(x)e^{-\int_\alpha^x m(a)da} m(x) \qquad \text{for } \alpha < x < \beta \qquad (4.2)$$

$$= 0 \text{ otherwise.}$$

The probability that a newborn girl will ever become a mother is

$$S_1 = \int_\alpha^\beta g_1(x)dx \qquad (\leq 1) \qquad (4.3)$$

$$= \int_\alpha^\beta \ell(x)e^{-\int_\alpha^x m(a)da} m(x)dx$$

$$= -\int_\alpha^\beta \ell(x)d\left(e^{-\int_\alpha^x m(a)da}\right)$$

$$= -\left[\ell(x)e^{-\int_\alpha^x m(a)da}\Big|_\alpha^\beta - \int_\alpha^\beta e^{-\int_\alpha^x m(a)da} d(\ell(x))\right]$$

$$= - \ell(\beta)e^{-TFR} + \ell(\alpha) - \int_\alpha^\beta e^{-\int_\alpha^x m(a)da}[-d\ell(x)]$$

$$S_1 = \ell(\alpha) - \ell(\beta)e^{-TFR} - \int_\alpha^\beta e^{-TFR(X)}\ell(x)\mu(x)dx \qquad (4.4)$$

where $\mu(x)$ is the force of mortality.

Then mean age at first birth is

$$\overline{A}_1 = \frac{\int_\alpha^\beta xg_1(x)dx}{S_1} \qquad (4.5)$$

$$= \frac{\int_\alpha^\beta x\ell(x)e^{-\int_\alpha^x m(a)da}m(x)dx}{S_1}$$

$$= \frac{-\int_\alpha^\beta x\ell(x)d\left(e^{-\int_\alpha^x m(a)da}\right)}{S_1}$$

$$= \frac{-\left[x\ell(x)e^{-\int_\alpha^x m(a)da}\Big|_\alpha^\beta - \int_\alpha^\beta e^{-\int_\alpha^x m(a)da}d[x\ell(x)]\right]}{S_1}$$

$$\overline{A}_1 = \frac{\alpha\ell(\alpha) - \beta\ell(\beta)e^{-TFR} + \int_\alpha^\beta e^{-TFR(X)}d[x\ell(x)]}{S_1} . \qquad (4.6)$$

By the usual definition of variance, the variance of age at first birth is given by

$$V_1 = \frac{\int_\alpha^\beta x^2 g_1(x)dx}{S_1} - \overline{A}_1^{\,2} \qquad (4.7)$$

$$= \frac{-\int_\alpha^\beta x^2\ell(x)d\left(e^{-\int_\alpha^x m(a)da}\right)}{S_1} - \overline{A}_1^{\,2}$$

$$= \frac{-[x^2 \ell(x) e^{-\int_\alpha^x m(a)da} \Big|_\alpha^\beta - \int_\alpha^\beta e^{-\int_\alpha^x m(a)da} d[x^2 \ell(x)]}{S_1} - \bar{A}_1^2$$

$$V_1 = \frac{\alpha^2 \ell(\alpha) - \beta^2 \ell(\beta) e^{-TFR} + \int_\alpha^\beta e^{-TFR(X)} d[x^2 \ell(x)]}{S_1} - \bar{A}_1^2. \qquad (4.8)$$

Equations (4.6) and (4.8) give the general formulations when mortality is considered. When mortality is not taken into account (when $\ell(x)=1$ and $\mu(x)=0$), then (4.6) reduces to

$$\bar{A}_{1_{NM}} = \frac{\alpha - \beta e^{-TFR} + \int_\alpha^\beta e^{-TFR(X)} dx}{1 - e^{-TFR}} \qquad (4.9)$$

and (4.8) simplifies to

$$V_{1_{NM}} = \frac{\alpha^2 - \beta^2 e^{-TFR} + 2\int_\alpha^\beta x e^{-TFR(X)} dx}{1 - e^{-TFR}} - \bar{A}_1^2 . \qquad (4.10)$$

By approximating expression (4.2) equation (4.3) can be computed as

$$S_1 = \sum_x \left(n_i {}^L x_i / |_0 \right) e^{-TFR(X_{i+})} m(X_{i+}) \qquad (4.11)$$

and

$$M_1 = S_1 \bar{A}_1 = \sum_x X_{i+} \left(n_i {}^L x_i / |_0 \right) e^{-TFR(X_{i+})} m(X_{i+}) \qquad (4.12)$$

and

$$M_2 = S_1 (V_1 + \bar{A}_1^2) = \sum_x X_{i+}^2 \left(n_i {}^L x_i / |_0 \right) e^{-TFR(X_{i+})} m(x_{i+}) \qquad (4.13)$$

where X_{i+} is the midpoint of the age interval $[x, x+n_i]$.

Table 4.2 gives essential calculations to estimate mean age at first birth for Egypt. Age- specific fertility rates were calculated directly from the EFS dataset (CAPMAS, 1983 c), in conjunction with the household schedule. ${}_5L_X/|_0$ values, however, were taken from a life table for Egyptian women for the year 1976 (El-Madani, 1981), which is the latest life table data available.[1] The data in column (6) of table 4.2 exhibits relatively high mortality at the older ages. In the presence of mortality, mean age at first birth is 21.89 years, while without taking mortality into account the mean age is 22.06. This small difference between the two estimates demonstrates that even when the mortality level is high, taking mortality into consideration does not make much difference in the estimate of mean age at first birth.

Now that the mean ages at first and last birth have been computed (the latter is done in chapter III), an estimate of mean reproductive span can be calculated.

The average span of childbearing can be computed simply as

$$E(RSPAN) = \bar{A}_L - \bar{A}_1 . \tag{4.14}$$

Since reproductive span is defined only for women with at least two births, the mean span conditional on having at least two births can be defined. The probability that a woman will have at least two births in her lifetime is given by

$$B_2 = \int_{y=\alpha}^{\beta} \ell(y)m(y) \int_{a=\alpha}^{y} m(a)da \; e^{-\int_{\alpha}^{y} m(a)da} dy$$

$$= \int_{y=\alpha}^{\beta} \ell(y)m(y)TFR(y) \; e^{-TFR(y)} dy. \tag{4.15}$$

When mortality is not considered, (4.15) simplifies to

$$B_2 = 1 - e^{-TFR} - TFR \; e^{-TFR} . \tag{4.16}$$

---

[1]New life table information will not be available until the 1986 census is conducted and population size by age is determined. There have been no surveys to estimate this information since 1976.

TABLE 4.2

ILLUSTRATIVE CALCULATION OF MEAN AGE AT FIRST BIRTH

EGYPT, 1980

(1) AGE	(2) X_{i+}	(3) ASFR	(4) $TFR(X_i)$	(5) $e^{-TFR(X_{i+})}$	(6) ${}_5L_{x_i}/l_0$	(7) $g_1(X_{i+})$ =(3)x(5)x(6)
15-19	17.5	.0785	.0000	.8377	4.640	.3051
20-24	22.5	.2557	.3925	.4318	4.222	.4662
25-29	27.5	.2801	1.6710	.1178	3.809	.1251
30-34	32.5	.2386	3.0715	.0303	3.399	.0246
35-39	37.5	.1389	4.2645	.0106	2.993	.0044
40-44	42.5	.0526	4.9590	.0062	2.593	.0008
45-49	47.5	.0124	5.2220	.0053	2.199	.0001
50	-	-	5.2840	-	-	
			TFR = 5.2840			$\sum$ = .9263

Case 1: With Mortality, based upon expression (4.5)

$S_1 = \sum g_1(X_{i+}) = .9263$, by formula (4.11)

$M_1 = \sum X_{i+}g_1(X_{i+}) = 20.2723$, by (4.12)

$M_2 = \sum X_{i+}^2 g_1(X_{i+})$ - 457.8944, by (4.13)

Mean Age at First Birth = M_1/S_1 = **21.89 years**

Standard Dev. = $\sqrt{M_2/S_1-\bar{A}_1^2}$ = **3.89 years**

Case 2: No Mortality (when ${}_5L_{X_i}/l_0$ =5)

Mean Age at First Birth = **22.06 years**

Standard Dev. = **4.82 years**

Reproductive Span

Case 1: With Mortality = **15.07 years**

by (4.15) B_2 = 0.8237, $E(RSPAN|B_2)$ = 18.77 years

Case 2: No Mortality = **14.70 years**

by (4.16) B_2 = 0.9681, $E(RSPAN|B_2)$ = 15.10 years

Conditional on having at least two births, the mean span of childbearing then is

$$E(RSPAN|B_2) = \frac{S_1}{B_2} (\bar{A}_L - \bar{A}_1). \tag{4.17}$$

With preceding results for age at first birth (from table 4.2) and age at last birth (chapter III),

$$E(RSPAN) = \bar{A}_L - \bar{A}_1 = 15.07 \text{ years}$$

when mortality is considered. Without mortality accounted for, the mean span is 14.70 years. Conditional on having at least two births, $E(RSPAN|B_2) = 18.77$ years when mortality is considered, and equal to 15.10 years with no mortality.

4.3 Conclusion

In section 4.1 multiple regression analysis was employed to model reproductive span. Since direct observations on ages at first and last births for each woman in the EFS dataset could be computed, the span of childbearing for each woman could be determined and the effects of the independent variables of interest estimated, while controlling for other variables known to have an effect on the response variable.

Such detailed information is not always available, and thus it is not always possible to estimate effects of other variables or even compute reproductive span directly. The only alternative in this case is to derive a summary estimate for the population of interest. To demonstrate this approach, only age-specific fertility rates and a recent abridged life table by sex were assumed available. From these data it was possible to compute summary measures, estimates of the population mean and its variance, for ages at first and last births (the latter in chapter III), from which the average span of childbearing was derived.

An extension of this indirect modeling can be done if age-specific fertility rates, and perhaps life table information, are available by population subgroups. Then the formulas used in the indirect modeling can be used for comparing subgroup means.

In the regression analysis, data only for the oldest cohort were used, in order to obtain more accurate direct measurements of the span of reproduction. The results, then, apply only to women in that cohort. Inferences can be made about younger groups of women only if their fertility and mortality experiences resemble that of the older women.

In the indirect modeling, since reproductive span cannot always be observed directly, cross-sectional data on fertility and mortality were used for all cohorts of women in the childbearing ages to compute a summary measure for the population of interest--in the present case, Egypt. This measure, the average span of reproduction, estimates the expected time spent in childbearing for a new cohort of women, if the current age-specific fertility and mortality rates apply to them.

CHAPTER V

PROJECTIONS OF MATERNAL AGE AT LAST BIRTH

5.0 Introduction

As it is important to know conditions as they currently exist, it is also valuable information to planners and policymakers to have some idea of what is likely to occur in the future under particular assumptions. In the present chapter, two methods are presented for projecting the expected age at completion of childbearing. The first procedure predicts the maternal age at last birth for different current parity groups, while the second method allows projections for young current age cohorts. Both procedures are applied to the Egyptian Fertility Survey data.

5.1 Projection Based on Parity

A simple method of projection is given by Nour (1984) in his development of parity-specific fertility tables. He presents a life table-type technique, based on the observed current parity distribution, which allows the computation of mean age at completion of childbearing. This method is based on the assumption that women at a given parity and age will have the same fertility behavior regardless of their previous experience.

Nour's fertility table represents a departure from the tradition of considering a woman's age, closely related to her fecundability, as the underlying variable for the study of reproduction. Chiang and Van Den Berg (1982) point out that the advent of modern contraceptive methods has made the reproductive process more a matter of a couple's choice than a function of a woman's fecundability. For this reason they propose the use of women's parity as the basis for summarizing the reproductive experience of a population.

Nour's statistical model for the construction of a parity-specific fertility table is given in the next section.

5.1.1 Nour's Model

Consider the childbearing age interval $X = [x_0, x_\omega]$ and let $\{I(x); x \varepsilon X\}$ be the parity of a woman at age $x \varepsilon X$. Assume that $\{I(x); x \varepsilon X\}$ is a Markov process where for every $x \varepsilon X$, $I(x)$ has a finite space $S = \{0, 1, \ldots, K\}$, with K being the maximum attainable parity. The Markovian assumption implies that all women who are found in a given parity at the same age will have the same probabilistic behavior regardless of their previous paths. The process is governed by a set of instantaneous fertility rates $\{r_0(x), r_1(x), \ldots, r_{K-1}(x)\}$ such that

$r_i(x)\Delta x + 0(\Delta x)$ = P{a woman of age x and parity i will have a live birth in the age interval $(x, x+\Delta x)$}, $x \varepsilon X$ and $i \varepsilon S$.

The rates $r_i(x)$ satisfy the relationship

$$r_i(x) = \lim_{\Delta x \to 0} \frac{1 - q_{i,i}(x, x+\Delta x)}{\Delta x}, \tag{5.1}$$

where $q_{i,i}(y,x) = P\{I(x) = i | I(y) = i\}$, $i \varepsilon S$, $y \leq x$. The unique solution to (5.1), subject to the initial condition $q_{i,i}(x,x) = 1$, is

$$q_{i,i}(y,x) = \exp\{-\int_y^x r_i(t)dt\} . \tag{5.2}$$

The function $p_i(y) = 1 - q_{i,i}(y, x_\omega)$, where x_ω is the maximum age of childbearing, may be termed the age-specific parity progression ratio. Let $g_i(y)$, $y \varepsilon X$ and $i \varepsilon S$, be the age distribution of women whose current parity is i. The conventional parity progression ratios, denoted by p_i, $i \varepsilon S$, are computed as

$$p_i = E\{p_i(y)\} = \int p_i(y) \cdot g_i(y)dy. \tag{5.3}$$

The synthetic cohort estimates of the distribution of final parity may be obtained from p_i upon observing that the parity progression ratio is defined in the context of cohort analysis, as

$$p_i = P\{N > i | N \geq i\}, \quad i \varepsilon S,$$

where N denotes final parity. Accordingly, the probability distribution of N is given by

$$P\{N = i\} = (1-p_i) \prod_{j=0}^{i-1} p_j, \quad i \varepsilon S. \tag{5.4}$$

Let V_i, $i \varepsilon S$, be the waiting time until the birth of the (i+1)st child and Z_i, $i \varepsilon S$, be the waiting time until completion of fertility, for a woman whose current parity is i. Also, define the indicator random variable δ_i, $i \varepsilon S$, by

$$\delta_i = \begin{cases} 1 & \text{if a woman whose current parity is } i \text{ goes on to have an } (i+1)\text{st child,} \\ 0 & \text{otherwise.} \end{cases}$$

It follows that

$$Z_i = \sum_{j=i}^{K-1} \beta_j^{\,i} V_j,$$

where

$$\beta_j^{\,i} = \prod_{t=i}^{j} \delta_t, \qquad i, j \varepsilon S. \tag{5.5}$$

Also,

$$E(Z_i) = \sum_{j=i}^{K-1} P\{\beta_j^{\,i} = 1\} \cdot E\{V_j | \beta_j^{\,i} = 1\}$$

$$= \sum_{j=i}^{K-1} P\{\beta_j^{\,i} = 1\} \cdot E\{V_j | \delta_j = 1\},$$

$$= \sum_{j=i}^{K-1} P\{\beta_j^{\,i} = 1\} \cdot E\{U_j\}, \; i \in S, \tag{5.6}$$

where U_j, $j \in S$, is the waiting time until the birth of the (j+1)st child for a woman whose current parity is j and who goes on to have a (j+1)st child. In addition

$$P\{\beta_j^{\,i} = 1\} = P\{\delta_t = 1 \text{ for all } i \le t \le j\}$$

$$= \prod_{t=i}^{j} p_t = A_j^i \text{ , say,} \tag{5.7}$$

where p_t, $t \in S$, is the parity progression ratio defined in (5.3). Therefore

$$E(Z_i) = \sum_{j=i}^{K-1} A_j^i E(U_j), \; i \in S. \tag{5.8}$$

To evaluate $E(U_i)$, it is noted that

$$E(U_i) = E_{Y_i}(E_{X_{i+1}}(X_{i+1} - Y_i | Y_i = y, \; I(y) = i, \; X_{i+1} < x_\omega)), \; i \varepsilon S, \tag{5.9}$$

where X_{i+1} is the age at which a woman has her (i+1)st birth and Y_i is the current age at parity i. Noting that,

$$P\{X_{i+1} > x | Y_i = y, \; I(y) = i\} = q_{i,i}(y,x)$$

where $q_{i,i}(y,x)$ is given by (5.2), it follows that

$$P\{X_{i+1} > x | Y_i = y, \; I(y) = i, \; X_{i+1} < x_\omega\}$$

$$= \begin{cases} 1, & x \leq y, \\ \dfrac{\exp\{-\int_y^x r_i(t)dt\} - \exp\{-\int_y^{x_\omega} r_i(t)dt\}}{1 - \exp\{-\int_y^{x_\omega} r_i(t)dt\}}, & y < x \leq x_\omega \\ 0, & x > x_\omega. \end{cases} \tag{5.10}$$

Therefore,

$$E_{X_{i+1}}(X_{i+1} - Y_i | Y_i = y,\ I(y) = i,\ X_{i+1} < x_\omega)$$

$$= \int_0^\infty P\{X_{i+1} > x | Y_i = y,\ I(y) = i,\ X_{i+1} < x_\omega\}dx - y$$

$$= \frac{\int_y^{x_\omega} \exp\{-\int_y^x r_i(t)dt\}dx - (x_\omega - y)\exp\{-\int_y^{x_\omega} r_i(t)dt\}dx - y}{1 - \exp\{-\int_y^{x_\omega} r_i(t)dt\}} . \tag{5.11}$$

By taking expectation with respect to Y_i on both sides of (5.11), the required $E(U_i)$ as defined by (5.9), is obtained.

Computational approximations for p_i and $E(U_i)$ based on a finite number of age groups are desirable. The functions p_i and $E(U_i)$ may be computed by approximating $r_i(x)$, $x \varepsilon X$, by a step function. The reproductive interval $X = [x_0, x_\omega]$ is divided into a set of n disjoint age groups of the form $(a_j, a_{j+1}]$ where $a_1 = x_0$ and $a_{n+1} = x_\omega$ such that $r_i(x)$ may be considered approximately constant within each of these age groups. That is

$$r_i(x) = r_{ij} , \qquad a_j < x \leq a_{j+1} . \tag{5.12}$$

Then

$$P_i = 1 - \sum_{j=1}^{n} B_{ij} \cdot q_i(j) , \tag{5.13}$$

and

$$E(U_i) = \sum_{j=1}^{n} \frac{C_{ij} - (x_\omega - y_j^*)B_{ij}}{1 - B_{ij}} \cdot g_i(j), \quad i \varepsilon S, \tag{5.14}$$

where $g_i(j) = P\{a_j < Y_i \le a_{j+1}\}$, $j = 1,\ldots,n$; y_j^* is an intermediate value for the age at i-th parity, Y_i, in the age interval $a_j < y \le a_{j+1}$; and

$$B_{ij} = \exp\{-(a_{j+1} - y_j^*)r_{ij} - \sum_{K=j+1}^{n} (a_{K+1} - a_K)r_{iK}\} \tag{5.15}$$

and

$$C_{ij} = \frac{1}{r_{ij}} \{1 - \exp\{-r_{ij}(a_{j+1} - y_j^*)\}\}$$

$$+ \exp\{-r_{ij}(a_{j+1} - y_j^*)\}\left\{ \sum_{K=j+1}^{n} \left[\frac{1}{r_{iK}} \{1 - \exp\{-r_{iK}(a_{K+1} - a_K)\}\}\right.\right.$$

$$\left.\left.\cdot \exp\{-\sum_{u=1}^{K-j-1} r_{i(K-1)}(a_{j+u+1} - a_{j+u})\}\right]\right\} . \tag{5.16}$$

5.1.2 Application of Nour's Model to the EFS Data

The fertility table prescribed by Nour's model is completely specified in terms of the parameters p_i and $E(U_i)$. Thus, it is helpful to see first how these intermediate figures are obtained.

Table 5.1 gives illustrative calculations to obtain the parity progression ratio, p_i, for parity 0. These computations were repeated for each parity group.

The fertility rates for parity i and age group j, the r_{ij}'s in column (3), are defined as

$$r_{ij\,(\text{year } t)} = \frac{\text{number of births of order (i+1) to women of parity i and age group j during t}}{\text{number of women of parity i and age j during t}} . \tag{5.17}$$

TABLE 5.1

ILLUSTRATIVE CALCULATION OF PARITY PROGRESSION RATIO FOR PARITY 0

EGYPT, 1980

(1) Parity i	(2) Age Group j	(3) Fertility Rate[a] r_{ij}	(4) y_j^*	(5) $(a_{j+1}-y_j^*)$	(6) $-(3)x(5)$	(7) $\sum_{K=j+1}^{n} 5xr_{iK}$	(8) $(6)-(7)$	(9) B_{ij}[b] $=e^{(8)}$	(10) $g_i(j)$[c]	(11) $(9)x(10)$	Parity Progression Ratio[d] $1-\sum(11)$
0	1	.331224	17.5	2.5	-.82806	5.8126	-6.64066	.001306	.3666	.000479	
	2	.495726	22.5	2.5	-1.239315	3.33397	-4.573285	.010324	.2976	.003072	
	3	.375796	27.5	2.5	-.93949	1.45499	-2.39448	.091220	.1493	.013619	
	4	.184615	32.5	2.5	-.461538	0.531915	-.993453	.370296	.0639	.023662	
	5	.106383	37.5	2.5	-.265958	0	-.265958	.766472	.0443	.033955	
	6	0	42.5	2.5		0	0	1	.0443	.0443	
	7	0	47.5	2.5		0	0	1	.0340	.0340	
										$\sum$=.153087	.846913

[a]Calculated by definition (5.17).
[b]Calculated by formula (5.15).
[c]Calculated from table (5.2).
[d]Calculated by formula (5.13).

Source: 1980 Egyptian Fertility Survey .

The numerator is given as the number of births in the preceding twelve months, and the denominator as the number of women at exactly six months prior to the survey date.

The B_{ij}'s in column (9) are the probabilities of not having a birth in the remaining age intervals. These probabilities are computed according to formula 5.15.

The $g_i(j)$'s in column (10) give the age distribution of women for parity 0. Alternatively, $g_i(j)$ may be viewed as the probability that a woman's age at the i-th parity is greater than a_j and less than or equal to a_{j+1}. These probabilities were obtained from the percentage distribution of women by current parity and age, presented in table 5.2.

The parity progression ratio, calculated by formula (5.13), is 1 minus the summation of column (11). This ratio, p_i, gives the probability that final parity is greater than i, given that it is at least i.

The other essential quantity of the fertility table is the $E(U_i)$, or the expected waiting time until the (i+1)st birth for a woman with current parity i. $E(U_i)$ is easily attained by formula (5.14) once the intermediate figure C_{ij} is obtained. The calculation of the C_{ij}'s was done by formula (5.16), and is illustrated for parity 0 in table 5.3. These calculations were repeated for each parity group.

Table 5.4 gives the final computations for obtaining the projected mean age at last birth, based on current parity. Because of relatively small numbers of births of order i+1 in the parity groups of seven or more children, these groups were combined.

The quantity $\overline{y}_i$ in table 5.4 is the observed mean age of the women who are currently at parity i.

The column of ℓ_i's are the numbers of women out of ℓ_0 (10,000) who

TABLE 5.2

PERCENTAGE DISTRIBUTION OF WOMEN BY CURRENT AGE AND PARITY

EGYPT, 1980

Current Age	Parity								
	0	1	2	3	4	5	6	7+	Total
<20	36.66	22.69	4.92	1.50	0.53	0.12	0.00	0.00	7.72
20-24	29.76	38.80	37.00	28.25	14.66	3.02	0.66	0.25	18.18
25-29	14.93	20.19	28.69	32.46	31.22	21.71	15.66	3.48	19.30
30-34	6.39	7.78	14.30	18.05	24.58	30.88	27.76	16.10	17.33
35-39	4.43	4.72	7.06	10.85	14.03	19.42	25.26	27.53	15.12
40-44	4.43	2.41	4.65	5.80	8.12	14.60	18.68	26.74	12.07
45-49	3.40	3.43	3.40	3.09	6.86	10.25	11.97	25.89	10.28
Total	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

Source: Egyptain Fertility Survey, 1980.

TABLE 5.3

ILLUSTRATIVE CALCULATION OF C_{ij} FOR PARITY 0

EGYPT, 1980

(1) Parity i	(2) Current Age Group j	(3) $-r_{ij} x 2.5$	(4) $e^{(3)}$	(5) 1-(4)	(6) $^{(5)}/_{r_{ij}}$	$(7)=\sum_{K=j+1}^{n}\left[\frac{1}{r_{iK}}\left\{1-\exp\left\{-r_{iK}(a_{K+1}-a_K)\right\}\right\}\exp\left\{-\sum_{u=1}^{K-j-1} r_{i(K-1)}(a_{j+u+1}-a_{j+u})\right\}\right]$ K = 2	3	4	5	6	7	(7) $\sum_k$	(8) C_{ij} [a] =(6)+(4)x(7)
0	1	-.82806	.4369	.5631	1.7001	1.8481	.1891	.0762	.2432	-	-	2.3565	2.7296
	2	-1.23932	.2896	.7104	1.4331	-	2.2546	.4987	.6121	-	-	3.3653	2.4076
	3	-.93949	.3908	.6092	1.6210	-	-	3.2647	1.5406	-	-	4.8052	3.4990
	4	-.46154	.6303	.3697	2.0025	-	-	-	3.8777	-	-	3.8777	4.4466
	5	-.26596	.7665	.2335	2.1952	-	-	-	-	-	-	-	-
	6	0	1	0	-	-	-	-	-	-	-	-	-
	7	0	1	0	-	-	-	-	-	-	-	-	-

[a]Calculated by formula (5.16).

Source: Egyptian Fertility Survey data.

TABLE 5.4

PROJECTIONS OF MEAN AGE AT LAST BIRTH BY CURRENT PARITY

EGYPT, 1980

Parity i	Mean age of Women at Parity i $\bar{y}_i$	Parity Progression Ratio p_i	ℓ_i	$E(U_i)$ in Years	L_i	T_i	e_i in years	Mean Age at Last Birth in Years $\bar{A}_L$
0	24.06	.84691	10,000	1.9086	16,164	75,910	7.59	31.65
1	24.96	.82579	8,469	1.5185	10,620	59,746	7.05	32.01
2	27.51	.81166	6,994	1.6898	9,593	49,126	7.02	34.53
3	29.19	.81821	5,677	4.8651	22,598	39,533	6.96	36.15
4	31.62	.71537	4,645	2.0205	6,714	16,935	3.65	35.27
5	34.69	.59183	3.323	3.5475	6,978	10,221	3.08	37.77
6	36.13	.56392	1,967	2.9243	3,243	3,243	1.65	37.78
7+	39.87	-	1,109	-	-	-	-	-

Source: 1980 Egyptian Fertility Survey data.

are expected to have a final parity equal to or greater than i, and

$$\ell_{i+1} = p_i \ell_i \ . \qquad (5.18)$$

The total exposure time to childbearing of women of parity i is given by L_i, which is calculated as

$$L_i = \ell_{i+1} E(U_i). \qquad (5.19)$$

The column of T_i's give the total reproductive span remaining after the i-th birth, and is computed as

$$T_i = \sum_{i=j}^{K-1} L_i \ . \qquad (5.20)$$

The quantity e_i is the expected waiting time until the last birth for a woman of current parity i. It is computed as

$$e_i = T_i / \ell_i \ . \qquad (5.21)$$

Finally, the mean age at the birth of the last child is obtained as

$$\overline{A}_L = \overline{y}_i + e_i \ . \qquad (5.22)$$

The projected age at last birth for Egypt exhibits a general tendency to increase with current parity, a trend which is consistent with results in previous chapters. The exception is that women with currently three children are expected to end childbearing about one year later than women who presently have four children. This difference is because of the relatively long waiting time (4.8651 years) expected between the third and fourth births. A possible explanation is that many Egyptian women use contraception after the third child. Previous published results from the EFS data indicate that "three living children [and] age 28 . . . are the effective points at which more than 50 per cent of [Egyptian] women wish to limit their family size" (CAPMAS, 1983 c, p. 93).

Projections in table 5.4 indicate that Egyptian women who currently have no children are expected to end childbearing at just under 32 years of age. Women currently with six children are expected to have their last child at almost 38 years of age. Such are the projections for a new cohort of Egyptian women should they experience the family building process as indicated by the EFS data.

5.2 Projection Based on Age

Another method of projecting the mean age at last birth is based upon a woman's current age. This procedure uses information on all cohorts of women to model the distribution of the number of children ever born by a given age.

5.2.1 The Model

Denote the force of fertility by m(t)dt. Then the distribution of the number of children ever born is given by the time-dependent Poisson process

$$P(N=r) = \frac{\emptyset^r e^{-\emptyset}}{r!}, \quad r = 0, 1, 2, \ldots \tag{5.23}$$

where N is the number of children ever born, $\emptyset = \int_\alpha^\beta m(t)dt$, and α and β are the lower and upper age limits, respectively, of reproduction.

Specificly

$$m(t,\underset{\sim}{z}) = m_0(t)e^{\underset{\sim}{z}\underset{\sim}{\beta}}, \tag{5.24}$$

where $m_0(t)$ is a baseline force of fertility function, $\underset{\sim}{z}$ is a vector of covariates, and $\underset{\sim}{\beta}$ is the vector of regression coefficients. Under the assumption of (5.24), the distribution of the number of children ever born is given by

$$P(N=r) = \frac{(\emptyset_0 e^{\underset{\sim}{z}\underset{\sim}{\beta}})^r e^{-\emptyset_0 e^{\underset{\sim}{z}\underset{\sim}{\beta}}}}{r!}, \quad r = 0, 1, 2, \ldots \tag{5.25}$$

where $\emptyset_0 = \int_{t=\alpha}^{\beta} m_0(t)dt$. If r_i denotes the number of children born to the i-th woman in the sample, the log likelihood can be written as

$$\log L = \log \emptyset_0 \sum r_i + \sum r_i(z_{i1}\beta_1 + \ldots + z_{ip}\beta_p) - \emptyset_0 \sum e^{\underset{\sim}{z}\underset{\sim}{\beta}} . \tag{5.26}$$

The likelihood equations are given by

$$\emptyset_0 = \frac{\sum_i r_i}{\sum_i e^{\underset{\sim}{z}_i \underset{\sim}{\beta}}} \tag{5.27}$$

and

$$\frac{\partial \log L}{\partial \beta_j} = \sum_i r_i z_{ij} - \emptyset_0 \sum_i e^{\underset{\sim}{z}_i \underset{\sim}{\beta}} z_{ij} . \tag{5.28}$$

5.2.2 Application of the Model to the EFS Data

A maximum likelihood procedure, BMDP3R (Dixon, 1983), was used to estimate the parameters of model (5.25) for all women in the EFS dataset of age 25 and older.

The covariate vector $\underset{\sim}{z}$ contained the variables wife's years of education (WYRSED), current residence (RESIDENCE), indicator variables representing the status of the first marriage, and indicator variables to denote the five five-year cohorts from age 25 to 49.

Wife's years of schooling (WYRSED) was given a value of 0 for no formal education and 1 for one or more years of schooling. Current residence (RESIDENCE) was coded 0 if rural and 1 if urban. If a woman's first marriage had been dissolved and she had remarried, then MAR2 was given a value of 1 and 0 otherwise. If she experienced dissolution without remarriage, MAR3 was coded 1 and 0 otherwise. The omitted category consisted of those with intact first marriages. All parameters were initialized at zero, except for ϕ_0, which was assigned an initial value equal to the mean number of children ever born for the sample of women of age 25 and over.

Estimates and their standard deviations are given in table 5.5. All variables were highly significant.

To determine the appropriateness of the model, Kolmogorov's goodness-of-fit test was done. The D-statistic for the 25-29 age cohort of women was .037, with an associated p-value of .20, indicating that the Poisson model provides a reasonable fit to the data.

Model (5.25) was then used to project the mean age at last birth for Egyptian women aged 25 to 29 in 1980. In order to do the projection, it was necessary to obtain the mean covariate vector $\bar{\underset{\sim}{z}}$ for each age cohort. Then $\exp(\bar{\underset{\sim}{z}}\hat{\underset{\sim}{\beta}})$ could be computed for each of the age groups. This information appears in table 5.6 along with the age-specific fertility rates observed in Egypt in 1980.

By assumption (5.24), the fertility rate $m(t,\underset{\sim}{z})$ is composed of the two factors $m_0(t)$ and $e^{\underset{\sim}{z}\underset{\sim}{\beta}}$. For the i-th age group, $m(t,\underset{\sim}{z}_i)$ is approximated by the observed age-specific fertility rates in column (3) of table 5.6. Then by assumption (5.24), $m_0(t)$ can be estimated by dividing the observed m(i) by $\exp(\bar{\underset{\sim}{z}}_i\hat{\underset{\sim}{\beta}}_i)$. The $\hat{M}_0(i)$'s thus obtained estimate the baseline age-specific fertility rates when cohort characteristics are not considered.

Column (6) of table 5.6 contains the observed and the projected age-specific fertility rates for Egyptian women aged 25 to 29 in 1980. The projected rates, the bottom four figures in the column, were ob-

TABLE 5.5

MAXIMUM LIKELIHOOD PARAMETER ESTIMATES AND STANDARD DEVIATIONS FOR EVER-MARRIED WOMEN AGED 25 AND OVER

EGYPT, 1980

Variable	Estimate $\hat{\beta}$	Standard Deviation	$\|t\|$
ϕ_0	8.3303	0.1029	80.96
WYRSED	-0.1344	0.0106	12.68
RESIDENCE	-0.1603	0.0104	15.41
MAR2	-0.2260	0.0153	14.77
MAR3	-0.2803	0.0165	16.99
AGE 25-29	-0.8414	0.0172	48.92
AGE 30-34	-0.4345	0.0156	27.85
AGE 35-39	-0.2085	0.0149	13.99
AGE 40-44	-0.0816	0.0148	5.51
AGE 45-49	omitted		

Source: Egyptian Fertility Survey data.

TABLE 5.6

CALCULATION OF PROJECTED AGE-SPECIFIC FERTILITY RATES

25-29 COHORT

EGYPT, 1980

(1) Age	(2) Age Group i	(3) Observed M(i)[a]	(4) $e^{\bar{z}_i \hat{\beta}_i}$	(5) $\hat{M}_0(i)$ (3)/(4)	(6) Observed and Projected M(3) (5) x $e^{\bar{z}_3 \hat{\beta}_3}$
15-19	1	.0785			.1432[b]
20-24	2	.2557			.2616[c]
25-29	3	.2801	.3655	.7663	.2801
30-34	4	.2386	.5480	.4354	.1591
35-39	5	.1389	.6875	.2020	.0738
40-44	6	.0526	.7665	.0686	.0251
45-49	7	.0124	.8141	.0152	.0056

[a] CAPMAS, 1983 c

[b] Computed as the average of the observed rates for women aged 15-19 in 1965-70 and 1970-75, from CAPMAS, 1983 c, p. 45.

[c] Computed as the average of the observed rates for women aged 20-24 in 1970-75 and 1975-80, from CAPMAS, 1983 c, p. 45.

Source: Egyptian Fertility Survey data.

tained by multiplying the estimated baseline age-specific fertility rates by the cohort characteristics factor $\exp(\bar{z}_3\hat{\beta}_3)$, or .3655.

Comparison of these projected rates with the current observed ones in column (3) indicates that age-specific fertility rates in Egypt are expected to decline. These projections are based not only on assumption (5.24), but also on the assumption that the cohort characteristics factor, or the mean covariate vector $\bar{z}$, does not change at least until the end of the reproductive period. The forecasted trend is reasonable and suggests a continuation of the decline in age-specific fertility rates already observed in Egypt in recent years.

Table 5.7 shows the application of the observed and projected age-specific fertility rates to project the mean age at last birth for the 25-29 cohort (see chapter III, section 3.2). Under the assumptions of the model, the expected age at last birth for this group of women is 33.77 years. This age is a three-year reduction from the hypothetical 36.76 years obtained by the corresponding method in chapter III, based on the age-specific fertility rates in Egypt in 1980. This result gives the reduction in age at last birth that will occur if the age-specific fertility rates of the 25-29 cohort are decreased to roughly one-half of the rates observed in 1980. Table 5.7 also shows the reduction in completed fertility from the total fertility rate of 5.284 in 1980 to a completed family size of 4.7425 children, if the 25-29 cohort experiences the projected fertility rates. This number also indicates a decrease of over two children from the 6.87 mean number of children ever born to the 45-49 cohort in the EFS.

The projected age at last birth represents a decline also from the observed age at last birth of 35.06 years for the 45-49 cohort inter-

TABLE 5.7

PROJECTED AGE AT LAST BIRTH FOR WOMEN AGED 25 TO 29

EGYPT, 1980

(1) Age	(2) X_{i+}	(3) Observed and Projected Fertility M(X)	(4) $TFR(X)^{b} = \sum_{15}^{X} m(t)\cdot 5$	(5) $e^{TFR(X)}$	(6) $\left[\frac{e^{TFR(X)}+e^{TFR(X+5)}}{2}\right]\cdot 5$	(7) (2)x(6)
15-19	17.5	.1432[a]	0.0000	1.0000	7.6155	133.27
20-24	22.5	.2616[a]	0.7160	2.0462	24.0368	540.83
25-29	27.5	.2801	2.0240	7.5685	95.6895	2631.46
30-34	32.5	.1591	3.4245	30.7073	246.8520	8022.69
35-39	37.5	.0738	4.2200	68.0335	416.0738	15602.77
40-44	42.5	.0251	4.5890	98.3960	524.8725	22307.08
45-49	47.5	.0056	4.7145	111.5530	565.6840	26869.99
50	-	-	4.7425	114.7206	-	-
			TFR = 4.7425		$\sum$ = 1880.8241	$\sum$ = 76108.09

By formula (3.15):

$$\bar{A}_{L_{NM}} = \frac{50-15(e^{-4.7425})-(e^{-4.7425})(1880.8241)}{1-e^{-4.7425}} = 33.77 \text{ years of age}$$

By formula (3.16):

$$V_{L_{NM}} = \frac{(50)^2-(15)^2e^{-4.7425}-2(e^{-4.7425})(76108.09)}{1-e^{-4.7425}} - (33.77)^2 = 41.09$$

STD. DEV.= 6.41

[a] See footnotes to table 5.6.

[b] TFR(X) is computed at the beginning of the age interval.

Source: Egyptian Fertility Survey data.

viewed in the EFS. This comparison points to a reduction of less than two years in actual age at last birth in the next twenty years. This small decline suggests that perhaps not all the women aged 45 to 49 in 1980 had ended childbearing by the time of the EFS, and that the actual mean age at last birth for this cohort could be older than 35.06 years.

In the same manner as above, the mean age at last birth was projected for the 30-34 age group of women. The Kolmogorov D-statistic of .041, with an approximate p-value of .16, indicates that model (5.25) provides an adequate fit to the data for this cohort. The projected mean age of 35.60 years signifies the decline of slightly more than one year in age at last birth that will occur by 1995, if the assumptions of the model hold.

5.3 Conclusion

Two procedures in the preceding sections gave projections of the expected age at completion of childbearing. One method is based on current parity and the other on current age.

The first method, proposed by Nour (1984), provides a means of computing a current fertility table which completely describes the reproductive experience of the study population. In addition to projecting the mean ages at last birth for current parity groups, this table also may be used to compare fertility experiences of different countries, and, as Chiang and Van Den Berg (1982) suggest, to evaluate the effectiveness of family planning programs.

The projections produced by applying Nour's model to the EFS data indicate that in Egypt the expected age at last birth has a general tendency to increase with current parity. Projected ages at last birth for women of various parities in Egypt in 1980 range from 31.65 to 37.78, the latter being for women with six children. Women with more than six children can be assumed to end childbearing, on average, some time after their 38th birthday.

The second procedure, based on a Poisson model of children ever born, indicates that women aged 25 to 29 in Egypt in 1980 are expected to end childbearing, on average, at about 33.77 years of age. This age represents a three-year reduction in age at last birth from that indicated by Egyptian fertility rates in 1980.

CHAPTER VI

SUMMARY AND POLICY IMPLICATIONS

6.0 Introduction

The 1980 Egyptian Fertility Survey has provided a comprehensive dataset for studying the demographic situation in Egypt. In particular, it has allowed in the present study for a diverse examination of two variables indicative of the end of the childbearing period--maternal age at last birth and reproductive span--and on a national basis. In the sections that follow, methodologies and results of this study are summarized and policy implications considered.

6.1 Summary

6.1.1 Maternal Age at Last Birth

Analysis of maternal age at last birth was done in chapter III. Since the EFS dataset contains detailed micro-level information, it was possible to model age at last birth directly.

In this type of analysis, hazards models were used to assess the effects of women's education, current residence in rural or urban areas, and dissolution of the first marriage, on the hazard of last birth occurring. A first marriage was considered to be dissolved if it had ended by divorce, separation, or widowhood. Age at first marriage and parity (number of children ever born) were additional covariates included in the model to reduce variation in the dependent variable. The analysis was restricted to women currently aged 45 to 49, and it was assumed that all these women had had their last births. Thus, all observations were assumed to be complete. Women with no children by the time of the survey were excluded from the analysis.

The results indicated that women with more years of schooling had a greater probability of ending childbearing early than women with less education. That is, women with little education tended to have their last children at older ages than better educated women.

Current residence in rural areas reduced the hazard of last birth, meaning that rural women tended to have their last children at ages significantly older than the overall average age at last birth. Residence in urban areas, on the other hand, encouraged ending childbearing at ages younger than the overall average. Urban women were 1.31 times as likely to have the last birth by age x as rural women.

Egyptian women whose first marriages remained intact through the reproductive period, the majority of the women, ended childbearing at ages close to the overall average age. They were a little more than half as likely to have their last birth by age x as those with dissolved unions who had not remarried, and more than twice as likely as those who had remarried. Women whose first marriages dissolved--through divorce or widowhood--and who remarried before the end of the reproductive period, ended childbearing at ages significantly older than the overall average age at last birth. This result implies that these women tended to have children by their new husbands. On the other hand, women whose first marriages dissolved--through divorce, separation, or widowhood--before the end of the childbearing period and who did not remarry, tended to have their last children earlier than other women.

Age at marriage and final parity of Egyptian women had negative effects on the hazard of last birth. That is, as age at marriage and number of children ever born increased, so did the age at last birth.

In conjunction with the above Cox regression analysis, hazard modeling without covariates was done on one subgroup of the 45-49 cohort at a time. Again, all observations were assumed to be complete. From this analysis it was possible to obtain cumulative life table probabilities of last birth occurring by various age intervals.

The median, or the age by which half of the women in the sample had had their last birth, is a convenient point of comparison. The median age at last birth was about two years older for rural compared to urban women (36.47 versus 34.29 years of age).

Illiterate women had the oldest median age at last birth of the education groups. Their median was 36.23 years of age, compared to median ages of 33.78 for literates, and 30.95 for women with at least a primary certificate.

There was very little difference between median ages at last birth for women with intact first marriages and those whose first unions were dissolved and who had remarried. Median ages for these groups were 36.31 and 36.08 years, respectively. Women with dissolved first marriages who had not remarried had the lowest median age at last birth of the three groups--32.34 years of age.

The median age at last birth increased with final parity. Women whose completed family size was 1-2 children had a median age at last birth of 28.83 years. From there the median increased to 30.74 years for women who ended with 3-4 children, to 36.47 years of age for those who had 5 or more children.

Women who were married at ages less than 25 years had a median age at last birth of 35.38 years. For those who were married at age 25 or older the median age was 37.50 years.

The observed median age at last birth for the total sample of women aged 45 to 49 was 35.54 years. This figure compares to 35.06 as the observed mean age at last birth for women aged 45 to 49, which suggests a lack of skewness in the distribution of age at last birth.

As an alternative to the methods of direct modeling described above, which required direct observations of ages at last birth, an indirect method of modeling age at last birth was developed, to be used when only age-specific fertility rates, and perhaps a life table for women, are available. Theoretical expressions were derived for mean age at last birth and its variance for a population of interest, with and without mortality being considered. By numerical approximations, it was shown that taking mortality into account, which requires more computational effort, makes little difference in the estimates of mean age at last birth.

The estimate of mean age at last birth without mortality considered was 36.76 years. This figure indicates the expected age at the end of childbearing for a new cohort of women, should they reproduce according to the age-specific fertility rates observed in Egypt in 1980.

6.1.2 Reproductive Span

The analysis of reproductive span was done in chapter IV. Least squares multiple regression was the method of direct modeling used.

The purpose of the analysis was to assess the effects of women's education, current residence in rural or urban areas, and dissolution of the first marriage, on the length of the reproductive span. These variables were defined in the same manner as in the analysis of age at last birth. Again, age at first marriage and parity were included in the model to reduce variation in the dependent variable. The analysis was restricted to women aged 45 to 49 who had had at least two live births that were not twins.

Results of the regression analysis indicated that, on average, the more years of formal schooling a woman had, the shorter her reproductive span tended to be. An increase of one year of education reduced the length of the reproductive span an average of .11 of a year.

Current residence in rural areas was associated with longer spans of childbearing than the overall mean span for Egyptian women aged 45 to 49. Rural women tended to have reproductive spans an average of .4 of a year longer than the overall mean span, while urban women had spans an average of .4 of a year shorter.

Women whose first marriages were still intact, the majority of women, had a mean span close to the overall average. Women with dissolved first marriages who remarried had a mean span of 1.5 years longer than average, while those who did not remarry had a mean span of 1.7 years shorter than average.

Age at marriage and final parity had negative and positive effects, respectively, on the span of reproduction. An increase of one year in age at marriage reduced the length of the mean reproductive span by .3 of a year. This means that a raise in age at marriage from 15 to 25 reduced the mean reproductive span by 3 years, for women aged 45 to 49 in 1980. As final parity increased by one child, the mean span of childbearing increased by 1.3 years. Since a full-term birth requires .75 of a year, this result attests to the lack of birth spacing among Egyptian women.

In addition to the direct modeling described above, a method of indirect modeling was derived, to obtain an estimate of the mean span when direct observations on reproductive span are not available. This method is an extension of that used to model age at last birth indirectly.

First, theoretical expressions were developed for estimating the mean age at first birth, based on the age-specific fertility rates observed in Egypt in 1980. Accordingly, the mean age at the birth of the first child was estimated to be about 22 years. This mean is a hypothetical figure, indicating the mean age at first childbirth for a new cohort of women who reproduce according to the 1980 age-specific fertility rates. This figure then was subtracted from the corresponding mean age at last birth to obtain a mean reproductive span of about 15 years. This estimate indicates then the expected length of the childbearing span for a new cohort of women, if they experience the age-specific fertility rates observed in the EFS data in 1980.

6.1.3 Projections of Maternal Age at Last Birth

Two methods of projecting mean maternal ages at last birth were described in chapter V. The first method is the construction of a fertility table based on women's current parity, developed by Nour (1984).

Nour's table represents a departure from the tradition of considering women's age, or their age jointly with their parity, as the basis for describing their reproductive experience. Chiang and Van Den Berg (1982) posit that the advent of modern contraceptive methods has made the reproductive process more a matter of a couple's choice than a function of a woman's fecundability. For this reason they propose the use of women's parity as the basis for summarizing the reproductive experience of a population.

Among other things, Nour's fertility table allows the computation of the expected waiting time until the end of childbearing for women of current parity i. When this waiting time is added to the mean age of the women who are currently at parity i, the mean age at last birth for these women is obtained.

From Nour's fertility table, projected ages at last birth in Egypt, based on women's current parity, show a general tendency to increase with the number of children a woman currently has.. The exception is that women with presently three children are expected to end childbearing about one year later (36.15 versus 35.27 years) than women who currently have four children. This difference is because of the long average waiting time between the third and fourth births, which may be indicative of an inclination of Egyptian women to use contraception after the third birth. Published results from the Egyptian Fertility Survey data indicate that "three living children [and] age 28 . . . are the effective points at which more than 50 percent of [Egyptian] women wish to limit their family size" (CAPMAS, 1983 c, p. 93).

Women in Egypt who presently have no children are predicted to end childbearing at almost 32 years of age. Those with currently six children are expected to have their last birth at about age 38, while women who currently have more than six children can be expected to stop reproducing later than that. These projections are for a new cohort of women if they experience the family building process as depicted by the EFS data.

The second method of projecting the mean age at last birth presented in chapter V involved the modeling of the number of children ever born via a Poisson model. This model was fitted to the EFS data for women aged 25 and older.

Women's years of education, and indicator variables for rural or urban residence, status of the first marriage, and age cohort were covariates in the analysis. From this modeling it was possible to estimate a quantity, $e^{\bar{z}\beta}$, for each age cohort, which may be considered to distinguish each cohort from the others. The $e^{\bar{z}\beta}$'s were then used to project the age-specific fertility rates to the end of the childbearing period for the 25-29 and 30-34 age cohorts. From these age-specific rates it was possible to compute the expected ages at last birth for these two groups of women.

The projected rates indicated that age-specific fertility rates in Egypt are expected to continue the declining trend observed in recent years.

For women aged 25 to 29 in Egypt in 1980, the projected mean age at last birth is 33.77 years. This age is a three-year reduction from the hypothetical 36.76 years, obtained by the corresponding method applied to the 1980 age-specific fertility rates. This three-year decline in mean age at last birth is what will occur, if the age-specific fertility rates of the 25-29 cohort are reduced to roughly one-half of the 1980 rates.

The corresponding completed number of children born for this cohort is expected to be 4.74 children (by the year 2000), a decline of about half a child from the total fertility rate of 5.28 estimated for Egypt in 1980. This number represents a decline of over two children, from the actual 6.87 mean number of children ever born to the 45-49 cohort of ever-married women in the 1980 EFS.

In the same manner, the mean age at last birth was projected for the 30-34 age group of women. The projected mean age for this cohort of 35.60 years signifies the decline of slightly more than one year in age at last birth that will occur by 1995, if the assumptions of the model hold.

6.2 Policy Implications

The chief goals of the 1980 Egyptian Fertility Survey were to provide a set of data appropriate for identifying the determinants of the rapid population growth in Egypt and alternative approaches to dealing with Egypt's current demographic problems. Egypt's present annual population growth rate of 2.7% implies that the population of 45 million in 1982 will double in 25 years. This problem is compounded by the fact that Egypt has only a small percentage of arable land, the narrow strip bordering the Nile River.

In addition to the demographic dilemma in Egypt, the health status of the population is also of concern. Younis et al. (1979) and Fortney et al. (1982) found that maternal and infant mortality in Egypt increased significantly at maternal ages over 35 years. Omran and Johnston (1984) further documented for African countries in general, the deleterious effects to the health of the entire family of having babies after age 35.

In 1979, 23.3% of all live births in Egypt were born to women aged 35 and older, and of these births the vast majority were birth orders over 4. In rural areas the percentage of births to mothers 35 and older was 26.6%, and in urban areas, 18.3% (CAPMAS, 1982). These large figures suggest that reducing the percentage of births to women over age 35 can have a significant impact in reducing population growth and in lowering maternal and infant mortality rates.

Results of the present study showed old maternal ages at last birth and long reproductive spans to be associated with low levels of women's education, residence in rural areas, and remarriage after being divorced or widowed. Later age at marriage was associated with later age at last birth, which suggests a possible "catch-up" effect described in Population Reports (1979). However, later age at marriage was accompanied by a shortening of the reproductive span.

Effects of these variables were statistically significant, although the magnitudes of some effects appeared to be small demographically. It will be remembered, however, that the covariate analyses were done only on women aged 45 to 49 in 1980. These women began reproducing at a time when birth rates were relatively high because of the post-World War II Baby Boom, and differences in fertility between urban and rural areas were small. The housing shortage, increased status of women, etc., in urban areas of Egypt in recent years can be expected to manifest themselves by increasing the effects of the independent variables on age at last birth and reproductive span for younger cohorts of women.

This widening of the effect of residence, for example, was observed in chapter II in regard to age at first birth. While percentage distributions of age at first birth between urban and rural women were not significantly different among women aged 45 to 49, for women aged 25 to 29 the percentage having first birth at an early age was significantly lower among urban women.

An increase in the magnitude of the effect of education also can be expected for younger cohorts of women, as similar mechanisms causing the divergence by residence also should widen the differences by education.

Bulatao (1984) reports that on a worldwide basis, and in Egypt in particular, "the differentials in fertility between education groups increase . . . as [the] fertility transition progresses" (p. 16).

Thus results of the present analyses on Egyptian women aged 45 to 49, which were statistically significant, are expected to become even more so for younger groups of Egyptian women. Thus, implementation of policies to reduce maternal age at last birth in Egypt may be appropriate now. Lowering age at last birth will shorten the length of the reproductive span, and, since few Egyptian women space their births, this reduction in span can be expected to reduce final parity.

Legal minimum ages for marriage exist in most countries of the world (16 for women and 18 for men, in Egypt), and, depending on the degree of enforcement, this measure helps determine the beginning of the reproductive span through the timing of the first birth. The present study has shown that later age at marriage is associated with shorter reproductive spans. Results of the regression analysis described in chapter IV suggested that raising age at marriage from 15 to 25, among Egyptian women aged 45 to 49, reduced the mean reproductive span by three years. Thus efforts to raise age at marriage in Egypt should have some effect in reducing final parity. This negative association between age at marriage and final parity on a worldwide basis is documented by Bulatao (1984) and Population Reports (1979).

Later age at marriage can be encouraged by both direct and indirect measures. Direct measures include enforcement of current legal minimum ages at marriage and raising the legal ages for men and women to marry. Indirect measures include the provision of increased employment opportunities and the encouragement of educational advancement, for women.

Education was found to have a small but statistically significant effect in lowering age at last birth and shortening the childbearing span, for Egyptian women aged 45 to 49 in 1980. Increasing education from none to ten years of schooling reduced the mean reproductive span by a little more than a year for this group of women. Since the magnitude of this effect is expected to increase among younger cohorts of women, policies designed to increase educational opportunities for women are expected to reduce age at last birth and the length of the reproductive span. But, as the results for the 45 - 49 cohort show, and as Bulatao (1984) suggested, increasing women's education to at least the secondary level may be required to have significant impacts. Ultimately such a policy would effect lower maternal and infant mortality rates and reduce population growth rates. However, if increased educational opportunities are offered only to women in urban areas, such a policy will have no impact on the majority of the population that is rural.

The residential effect on age at last birth and reproductive span, and the expected widening of the disparity, makes it imperative that efforts to lower maternal age at last birth be intensified in rural areas. These regions are where most of the births to women over age 35 occur, and thus are where reductions in age at last birth can have large impacts demographically and healthwise.

Information/education campaigns can effectively broadcast the health, social, and demographic disadvantages of bearing children at old maternal ages. Dissemination of such information should be intensive in rural regions and accompanied by availability of family planning services.

In the dissemination of information and in the promotion of policy, the health factor should be emphasized. This point is vital, not only because of the high level of maternal and infant mortality in Egypt, but also because the health factor can provide strong motivation to end childbearing earlier, and thereby reduce final parity, apart from any concern about overpopulation. While it may be difficult for a couple, especially in rural areas, to appreciate how one or two additional births to themselves can affect the national population problem, they can more readily grasp the risks to the health of mother and child of an old-age pregnancy. That is, the health implications are more immediate and personal. As Omran and Johnston (1984) pointed out, the health rationale also provides a more palatable basis for policies, such as family planning programs, among most African leaders of government.

Most demographic and health policies are aimed at specific "target" populations, so that defining this population becomes an issue. The fertility table developed by Nour, table 5.4 in chapter V, may be of some assistance. According to this table, intensifying the dissemination of information and family planning services to women with at least four or five live births could reduce the number of births to women over age 35.

General aims of a health/demographic oriented policy should be the following:

1.) reduce maternal and infant mortality rates; and

2.) reduce population growth rates.

Specific policy goals to accomplish these aims could be to

1.) encourage the increase in age at marriage to shorten the span of reproduction from the lower end;

2.) reduce the percentage of births to women over age 35;

3.) encourage an increase in women's education to at least the secondary level;

4.) provide information regarding high risk ages for childbirths and make family planning services available to all married women;

5.) make the provision of family planning services an integral part of general family health services;

6.) intensify policy implementation efforts among rural women and women with at least four or five live births.

Program evaluation is an issue with the implementation of any policy. Needed are simple, sensitive means of measuring the impact of specific policy activities. A fertility table such as Nour's, based on women's current pairty, was suggested by Chiang and Van Den Berg (1982) for the evaluation of family planning programs. The table would be constructed before and then some time after policy implementation. Such use of Nour's fertility table would be more meaningful, however, with the development of a measure of sampling variability for the mean age at last birth.

An even simpler and perhaps more sensitive evaluation technique is the hazard modeling, with or without covariates, demonstrated in chapter III. Age at last birth "survival" distributions before and after the inaction of a policy could be compared and tested for equality.

Whatever policies are implemented, it is hoped that both family planning and development measures, such as increasing education of women, can be continued simultaneously to help alleviate the problem of rapid population growth and improve the overall standard of living of the Egyptian people. Perhaps some attention given to the variables age at last birth and reproductive span can aid in reducing population growth rates and levels of maternal and infant mortality.

CHAPTER VII

SUGGESTIONS FOR FURTHER RESEARCH

7.1 Theory

In the projections of mean maternal age at last birth done in chapter V, the covariate model used was the Poisson. The distribution of the number of children ever born was given as

$$P(N = r) = \frac{(\emptyset_0 e^{\underset{\sim}{z}\underset{\sim}{\beta}})^r \, e^{-\emptyset_0 e^{\underset{\sim}{z}\underset{\sim}{\beta}}}}{r!} , \qquad r = 0, 1, 2, \ldots$$

where $\emptyset_0 = \int_{t=\alpha}^{\beta} m_0(t)dt$. This model happened to provide a good fit for the Egyptian Fertility Survey data. The use of other models, however, might be explored, not only on Egyptian data but data for other countries as well. A Poisson model may not be appropriate for data from other countries, especially if the variance of children ever born greatly exceeds the mean. Boyle (1983) reviews other types of models of human reproduction, including modified Poisson, binomial, and negative binomial densities.

7.2 Applications

7.2.1 Data Collection

The analyses of age at last birth and reproductive span were confined to women currently aged 45 to 49 in 1980, in order to obtain the most accurate measures possible of the two variables. The EFS did not collect data on women over age 49, so there is no information on fertility of these women.

Since there may be some births to women over age 49, and also to women in the lower ages of the 45-49 interval, it would be desirable to extend data collection on fertility to women through age 54. Then perhaps more accurate analyses of age at last birth and reproductive span could be done on women aged 50 to 54.

7.2.2 Replication

It would have been useful to repeat some of the analyses done on the EFS data on another national sample of Egyptian women, in order to assess the reliability of the findings of this study. Such replication was not possible here because of the lack of any such data collected in Egypt during the last ten years. Forthcoming national surveys, however, may provide the opportunity for replication of the analyses done on the EFS. In particular, it would be interesting to see how well projected age-specific fertility rates, done via the Poisson model, compare with future observed rates.

7.2.3 Analyses by Different Geographic Regions

In the analyses done in this study, geographic comparison was by type of place of current residence. The effects of current residence in rural regions was compared to urban residence. Other regional comparisons could be between Upper (Southern) Egypt and Lower (Northern) Egypt.

Analysis also could be limited to Cairo, where the highest population density is. If there has been enough migration from rural areas into Cairo, then comparisons could be made between urban and rural childhood place of residence. This method would assess the effect of women's upbringing in rural or urban regions. In the case of significant migration, one could compare women who spent their childhoods in rural areas and later migrated to Cairo, with women who remained in rural regions. This comparison would assess the effect of urbanization on age at last birth and/or length of the reproductive span.

7.2.4 Use of Indirect Modeling for Comparative Purposes

The indirect modeling in chapters III and IV developed formulas for obtaining summary measures of mean age at last birth and mean reproductive span for a population of interest. Since it was shown that accounting for mortality does not make much difference in the mean estimates obtained, this method requires only that a set of age-specific fertility rates be available for the population of interest. For the present research, these rates were available only for all Egyptian women, so that the estimates of the means obtained here were for all of Egypt.

An extension of this indirect modeling is the application of the same formulas to obtain estimates of mean age at last birth and reproductive span, and their variances, for subgroups of a population--rural and urban subgroups, for example. The only requirement is that age-specific fertility rates be available or obtainable for each subgroup. Group comparisons then can be made by comparing the group means.

7.2.5 Projection of Age at First Birth

Chapter V of this dissertation was concerned with the projection of maternal age at last birth. When projected age-specific fertility rates were obtained by use of a covariate model of children ever born, the projected mean age at last birth was obtained, by using the formulas of the indirect modeling described in chapter III. The same projected rates can be used to project the mean age at first birth by applying the method presented in table 4.2 of chapter IV. These projections can be of use to people desiring to study the beginning of the childbearing period.

7.2.6 Use of Additional Time-dependent Covariates in Hazard Modeling

In the hazard model (3.7) presented in chapter III, the variable PARITY was treated as a time-dependent covariate, since its value changes as women progress through the reproductive ages. Model (3.7) then may be viewed as a refinement of model (3.1), in which all covariates were fixed, although results of both fitted models were similar. Further refining of model (3.7) could be done by allowing age at first marriage and the marital status variables also to change with time. However, it should be cautioned that the use of several time-dependent covariates in a hazards model can incur unusually high computer costs.

REFERENCES

Boyle, Kerrie Eileen. 1983. "Survival Model for Fertility Evaluation." Dr. P. H. dissertation, University of North Carolina, Chapel Hill.

Breslow, N. 1974. "Covariance Analysis of Censored Survival Data." Biometrics 30: 89-99.

Bulatao, Rodolfo A. 1984. Reducing Fertility in Developing Countries: A Review of Determinants and Policy Levers. World Bank Staff Working Papers, Number 680. Washington, D.C.: The World Bank.

CAPMAS. 1982. Birth and Death Statistics, 1979. Cairo: Central Agency for Public Mobilization and Statistics (Arabic).

----------. 1983 a. Egyptian Fertility Survey, 1980. Standard Recode, version 3. Cairo: Central Agency for Public Mobilization and Statistics.

----------. 1983 b. Egyptian Fertility Survey, 1980. Volume 1: Survey Design. Cairo: Central Agency for Public Mobilization and Statistics.

----------. 1983 c. Egyptian Fertility Survey, 1980. Volume 2: Fertility and Family Planning. Cairo: Central Agency for Public Mobilization and Statistics.

----------. 1983 d. Statistical Yearbook, 1952-1982. Cairo: Central Agency for Public Mobilization and Statistics.

Chiang, Chin Long and Bea J. Van Den Berg. 1982. "A Fertility Table for the Analysis of Human Reproduction." Mathematical Biosciences 62: 237-251.

Cox, D. R. 1972. "Regression Models and Life Tables." Journal of the Royal Statistical Society, Series B34: 187-220.

----------. 1975. "Partial Likelihood." Biometrika 62: 269-276.

Cox, D. R. and D. Oakes. 1984. Analysis of Survival Data. New York: Chapman and Hall.

Dixon, W. J., ed. 1983. BMDP Statistical Software. Berkeley: University of California Press.

El-Deeb, Bothina and John Casterline. 1983. Contraceptive Use in Egypt. Paper presented at the International Conference on Fertility in Egypt, 20-22 December 1983, Cairo, Egypt.

Elder, Glen H. 1975. "Age Differentiation and the Life Course." Annual Review of Sociology, Volume 1. Alex Inkeles, ed. Palo Alto: Annual Reviews.

El-Guindy, M. H. 1971. "Age at Marriage in Relation to Fertility in Egypt." In Fertility Trends and Differentials in Arab Countries. Research Monograph Series, No. 2. Cairo: Cairo Demographic Center.

El-Madani, Daoud Soliman. 1981. "Abridged Life Table for the Population of Arab Republic of Egypt in the Year 1976." L'Egypte Contemporaine 72 (383): 5-30 (Arabic).

El-Sayeh, M. A. 1971. "Fertility Studies and Population Programmes in Egypt." In Fertility Trends and Differentials in Arab Countries. Research Monograph Series, No. 2. Cairo: Cairo Demographic Center.

Fallo-Mitchell, L. and Carol D. Ruff. 1982. "Preferred Timing of Female Life Events." Research on Aging 4(2): 249-267.

Featherman, D. L. 1982. "The Life-span Perspective in Social Science Research." The 5-Year Outlook on Science and Technology 1981, Source Materials, Vol. 2. Washington, D.C.: National Science Foundation.

Fortney, Judith A.; J. E. Higgins; A. Diaz-Infante; F. Hefnawi; L. G. Lampe; and I. Batar. 1982. "Childbearing after Age 35: Its Effect on Early Perinatal Outcomes." Journal of Biosocial Science 14 (1): 69-80.

Hanna, B. 1971. "The Effect of Divorce on the Level of Fertility in Egypt." In Fertility Trends and Differentials in Arab Countries. Research Monograph Series, No. 2. Cairo: Cairo Demographic Center.

Ibrahim, Madlain Mitry. 1981. Some Aspects of Nuptiality in Egypt. Paper submitted in partial fulfillment of the requirements for the General Diploma in Demography. Cairo: Cairo Demographic Center.

Kafafi, Laila H. 1983. "Age at Marriage and Cumulative Fertility in Rural Egypt." Ph.D. dissertation. Duke University.

Kalbfleisch, J. D. and R. L. Prentice. 1980. The Statistical Analysis of Failure Time Data. New York: John Wiley.

Kay, R. 1977. "Proportional Hazard Regression Models and the Analysis of Censored Survival Data." Applied Statistics 26: 227-237.

Khalifa, A. M. 1974. "The Population of the Arabic Republic of Egypt." Egyptian Population and Family Planning Review 7 (2): 53-78.

Khalifa, A. M.; H. A. Sayed; M. N. El-Khorazaty; and A. A. Way. 1982. Family Planning in Rural Egypt 1980. Cairo: Population and Family Planning Board and Columbia, Md.: Westinghouse Health Systems.

Kiernan, K. and I. Diamond. 1983. "The Age at Which Childbearing Starts--A Longitudinal Study." Population Studies 37 (3): 363-380.

Krishnamoorthy, S. 1979. "Family Formation and the Life Cycle." Demography 16 (1): 121-129.

Loza, Sarah F. 1982. Social Science Research for Population Policy Design: A Case Study of Egypt. I.U.S.S.P. Papers, No. 22. Belgium: International Union for the Scientific Study of Population.

Neugarten, B. L.; J. W. Moore; and J. C. Lowe. 1965. "Age Norms, Age Constraints, and Adult Socialization." American Journal of Sociology 70: 710-717.

Neugarten, B. and G. Hagestad. 1976. "Age and the Life Course." In Handbook of Aging and the Social Sciences. R. Binstock and E. Shanus, eds. New York: Van Nostrand.

Nortman, D. 1974. Parental Age as a Factor in Pregnancy Outcome and Child Development. Reports on Population/Family Planning, No. 16. New York: The Population Council.

Nour, El-Sayed. 1984. "Parity-specific Fertility Tables." Proceedings of the American Statistical Association, Social Statistics Section, pp. 187-192.

Omran, Abdel R. and Alan G. Johnston, eds. 1984. Family Planning for Health in Africa. Chapel Hill: Carolina Population Center.

People. 1979. "Muslim Congress Urges Birth Spacing." People 6 (2): 36.

Population Reports. 1979. Age at Marriage and Fertility. Special Topic Monograph, Series M, No. 4. 7 (6): 105-159.

Rindfuss, R. R. and L. L. Bumpass. 1976. "Hold Old is Too Old? Age and the Sociology of Fertility." Family Planning Perspectives 8 (5): 226-230.

Trussell, J. and D. E. Bloom. 1983. "Estimating the Co-variates of Age at Marriage and First Birth." Population Studies 37 (3): 403-416.

Wald, A. 1943. "Tests of Statistical Hypotheses Concerning Several Parameters When the Number of Observations is Large." Transactions of the American Mathematical Society 54: 426-482.

Younis, N. et al. 1979. "Analysis of Maternal Deaths in Two Egyptian Maternity Hospitals." Population Sciences. Cairo: International Islamic Center for Population Studies and Research, Al-Azhar University.