Simplified Dynamic Response Calculations for the Design of Complex Structures and Components

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Abstract

This paper offers a technique to evaluate detailed dynamic stress response of a sub component within a very complex structure without the expense of having to set up a complicated F.E. model on the whole structure.

Detailed F.E. idealisation is needed only on the component(s) of interest, and its/their modal relationship with the overall structure is computed via a series of simplified models.

An example is given here to the application of this technique on a 'J' type steam generator with external constraints. The final dynamic response is evaluated using a shell model, while the modal coupling is done via a series of simplified beam models.

The example is extended to illustrate this technique to the seismic analysis.

1. Introduction

The dynamic response of complex structures or components may be calculated, usually with great accuracy, using the finite element method. There are numerous programmes and commercial Finite Element packages that one can choose from for doing this. However, during the design stage, when the structure is undergoing geometry or other changes, the repeated use of finite element models, which may contain plates or 3D solid elements, can be prohibitively expensive if the degree of complexity in the model cannot be reduced.

Simplified models using beams and lumped masses are commonly used in this situation. Not only are these computationally economical to use, they generally produce adequate results as far as obtaining the modal response is concerned. However, when it comes to predicting the stress response in the true structure these beam models are sometimes proved either unreliable or inadequate.

This paper presents a technique for generating simplified models which include the essential dynamic characteristic of a complex model, and a procedure for the use of the technique in the dynamic design and verification of complex structures*.

An example is given illustrating this technique. In the sections that follow, we examine the earthquake response of a steam generator which is essentially an axisymmetrical vessel shell but with asymmetrical components and non-axisymmetrical external restraints such as pipework and seismic snubbers etc. connected on to the main vessel shell.

* As this technique utilises the coupling of the modal characteristics between the complex model, which represents the real model, and the generated simplified models, it is referred to hereafter in this paper as the modal coupling technique.

2. Outline of the technique

Consider a complëx structure in which we are only interested in the detailed stress response on a certain part of it. In static calculation, one would, under such circumstances, choose to use the substructuring technique to solve the problem. This technique is somewhat similar to the substructuring modelling, but in a dynamic application environment.

To begin with, a detailed model is set up on the structure only where the in-depth stress responses are required. Having obtained its eigen values and eigen modes, we will shelve this model away and set up a similar model with a much simplified idealisation, e.g. a spring and lumped mass system. We require this model to be able to reproduce the modal characters of the first model, i.e. the detailed model. Having obtained the necessary information, this model is then extended to include the parts/components of the complex structure which had not been previously covered. Upon evaluation of a relationship between the mode shapes of these two simplified models, we aim to apply it to simulate the modal characteristics of the detailed model, with the effects of its connection to the rest of the structure properly accounted for.

To illustrate the evaluation of this technique we consider the following example:

Consider a cantilever beam with one end fully fixed and the other end free (Fig 1). The modal characteristic of this beam can be represented by a set of eigen values and eigen vectors $\underset{\sim}{\lambda}$ & $\underset{\sim}{\psi}$ respectively. Let the set of eigen vectors be $\underset{\sim}{\psi}_1$, $\underset{\sim}{\psi}_2$, $\underset{\sim}{\psi}_3$, $\underset{\sim}{\psi}_4$, ... $\underset{\sim}{\psi}_i$ and Fig 2. shows the first 4 modes.

Let us now consider another beam system, again a beam with one end fully fixed, but the other end hinged to a rigid link (Fig. 3). The modal characteristic of this beam can be represented by another set of eigen values and eign vectors $\underset{\sim}{\lambda}'$ & $\underset{\sim}{\psi}'$ respectively. Again, let the set of eigen vectors be $\underset{\sim}{\psi}'_1$, $\underset{\sim}{\psi}'_2$, $\underset{\sim}{\psi}'_3$, $\underset{\sim}{\psi}'_4$ .... $\underset{\sim}{\psi}'_i$ and Fig. 4 shows the first 4 modes.

We now seek a procedure to represent the eigen vectors $\underset{\sim}{\psi}'$ by $\underset{\sim}{\psi}$. That is:

$$\begin{aligned}
\underset{\sim}{\psi}'_1 &= \alpha_{11}\underset{\sim}{\psi}_1 + \alpha_{12}\underset{\sim}{\psi}_2 + \alpha_{13}\underset{\sim}{\psi}_3 + \alpha_{14}\underset{\sim}{\psi}_4 + \ldots \\
\underset{\sim}{\psi}'_2 &= \alpha_{21}\underset{\sim}{\psi}_1 + \alpha_{22}\underset{\sim}{\psi}_2 + \alpha_{23}\underset{\sim}{\psi}_3 + \alpha_{24}\underset{\sim}{\psi}_4 + \ldots \\
\underset{\sim}{\psi}'_3 &= \alpha_{31}\underset{\sim}{\psi}_1 + \alpha_{32}\underset{\sim}{\psi}_2 + \alpha_{33}\underset{\sim}{\psi}_3 + \alpha_{34}\underset{\sim}{\psi}_4 + \ldots \\
\underset{\sim}{\psi}'_4 &= \alpha_{41}\underset{\sim}{\psi}_1 + \alpha_{42}\underset{\sim}{\psi}_2 + \alpha_{43}\underset{\sim}{\psi}_3 + \alpha_{44}\underset{\sim}{\psi}_4 + \ldots
\end{aligned} \qquad \text{(1a)}$$

i.e.

$$\underset{\sim}{\psi}'_{n^* \times m} = \underset{\sim}{\psi}_{n\times m} \cdot \underset{\sim}{\alpha}_{m\times m} \qquad \text{(1b)}$$

Where suffices n and m are the total degrees of freedom in the system and the minimum number of modes considered necessary to approximate $\underset{\sim}{\psi}'$ from $\underset{\sim}{\psi}$ respectively.

It is noted that $n^* \geqslant n$ because of the extra element(s) from the rigid link. However, if we can make the assumption that these extra nodes can be made redundant and hence be partitioned out in the eigen vectors, then $n^* = n$. This reason will become apparent when we demonstrate the worked example further along the chapter.

Equation (1b) shows that provided the modal coupling matrix $\underset{\sim}{\alpha}$ can be found, the dynamic behaviour of the second beam model can be evaluated by operating purely on the modal behaviours of the first beam model.

The advantage of this $\underset{\sim}{\alpha}$ matrix is now apparent. Imagine we have a very complex structure where detailed stress response is required at only a section of it. We only need to set up a detailed F.E. model on this section for the purpose of further dynamic calculation. The modal coupling of this section of the structure and the rest of the structure can be obtained as $\underset{\sim}{\alpha}$ by using a set of much simplified models for instance, simple beam or spring-mass system.

3. Evaluation of the modal coupling matrix $\underset{\sim}{\alpha}$

The procedures involved in deriving the modal coupling matrix are as follows:

From equation (1b), i.e.

$$\underset{n\times m}{\underset{\sim}{\psi'}} = \underset{n\times m}{\underset{\sim}{\psi}} \cdot \underset{m\times m}{\underset{\sim}{\alpha}}$$

pre-multiplying both sides of the equation by $\underset{\sim}{\psi}^t.\underset{\sim}{M}$ where $\underset{\sim}{M}$ is the mass matrix of the simplified system (referred to the first simplified beam model described in Section 2), equation (1b) becomes:

$$\underset{m\times n}{\underset{\sim}{\psi}^t} . \underset{n\times n}{\underset{\sim}{M}} . \underset{n\times m}{\underset{\sim}{\psi'}} = \underset{m\times n}{\underset{\sim}{\psi}^t} . \underset{n\times n}{\underset{\sim}{M}} . \underset{n\times m}{\underset{\sim}{\psi}} . \underset{m\times m}{\underset{\sim}{\alpha}} \qquad (2)$$

The product of the matrices $\underset{\sim}{\psi}^t.\underset{\sim}{M}.\underset{\sim}{\psi}$ on the R.H.S. of equation (2) is, because of the orthogonality property of the eigen matrix, a unit vector. Hence equation (2) can be written as:

$$\underset{m\times m}{\underset{\sim}{\alpha}} = \underset{m\times n}{\underset{\sim}{\psi}^t} . \underset{n\times n}{\underset{\sim}{M}} . \underset{n\times m}{\underset{\sim}{\psi'}} \qquad (3)$$

It should be noted that the order of $\underset{\sim}{\psi'}$ is strictly speaking $n^* \times m$ rather than $n \times m$, thus making the multiplication of the matrices a little awkward. But if we recall the comments made towards the end of Section 2, $\underset{\sim}{\psi'}$ can indeed be partitioned to an order of $n \times m$.

4. Application of the modal coupling technique

The example we wish to use for demonstrating the effectiveness of this technique is a steam generator in the shape of the letter 'J' with various external piping and seismic snubber connections. Such an arrangement can be found in Fig. 5.

For structures like this, it is of course not uncommon to use beam elements to represent the model, and, after all, if it is for the purpose of estimating the dynamic responses which are resulted from an earthquake motion, the beam model would be fine to do just that. However, we often find that the results of a dynamic response cannot be isolated and be assessed on its own and therefore they must be in such a format that they can be directly integrated with the other results, such as those due to, say, pressure or thermal loads which often require the use of very detailed F.E. models. To this extent, one would find it necessary to use, to say the least, axisymmetrical shell elements which offer stress components in at least 4 directions. But then the immediate stumbling block of this is what are we going to do with the asymmetric components such as the 'J' bend, the piping and seismic snubber constraints and so on. We can not simply ignore them! To overcome this, the procedures described below are followed:

Firstly, the axisymmetrical part of the vessel shell is modelled using axisymmetrical shell elements. Fig 6. shows such F.E. idealisation. From this model, we obtain the eigen values and eigen vectors $\underset{\sim}{\eta}$ & $\underset{\sim}{\phi}$ respectively.

Secondly, we set up another model similar to the arrangement of the shell model, but using, instead, the beam elements (Fig. 7). With a small amount of tuning, if required, we should expect this beam model to produce the same frequencies and mode shapes to the axisymmetric shell model. We name the eigen values and eigen vectors from this model $\underset{\sim}{\lambda}$ and $\underset{\sim}{\psi}$, so that

$$\underset{\sim}{\lambda} = \underset{\sim}{\eta}$$

$$\text{and} \quad \underset{\sim}{\psi} \equiv \underset{\sim}{\phi}$$

Our third step is to add on to this beam models with component not already included, i.e. the 'J' bend, piping and seismic snubber contraints etc, using, of course, beam elements with appropriate boundary fixings. Fig. 8 shows such an arrangement. Let us call the calculated eigen values and eigen vectors λ' and ψ' respectively. This second beam model should give all the modal behaviour that can be expected from the full model of the steam generator which, had we idealised it with 3D shell elements, would produce eigen values and eigen vectors η' and ϕ' respectively. From this second beam model, we would expect

$$\lambda' = \eta'$$

$$\text{and} \quad \psi' \equiv \phi'$$

The relationship between the mode shapes of the two beam models, i.e. α, can be obtained via equations (1b) and (3). Following our argument in the last paragraph of Section 2, we can show that:

$$\phi' = \phi \cdot \alpha \quad \text{and} \quad \eta' = \lambda' \tag{4}$$

5. Application of the modal coupling technique in seismic analysis

The chosen example was set up with the application of a seismic analysis in mind. Thus, making the use of the beam models for the evaluation of the matrix particularly favourable.

In seismic analysis, the maximum modal displacement response of the full model in the j th mode is expressed as (ref. 1).

$$q'_{j\,max.} = \frac{\phi'^{t}_{j} . M . \{1\} \; . \; S_{a} \; (\mu'_j \, . \, \sqrt{\eta'_j})}{\eta'_j} \tag{5}$$

where suffix j indicates the j th mode

μ'_j is the damping coefficient

{1} is the unit vector with a unit value at the freedom coincident with the direction of the applied earthquake motion

$S_a (\mu'_j . \sqrt{\eta'_j})$ is taken directly from the specified floor response spectra

If we rewrite equation (5) as:

$$q'_{j\,max} = \frac{\phi'^{t}_{j} . \Omega . S_{a} (\mu'_j . \sqrt{\eta'_j})}{\eta'_j} \tag{6}$$

where

$$\Omega = M . \{1\}$$

By applying equation (4) into equation (5) the modal displacement of the full model can be expressed entirely by the mode shapes obtained from the axisymmetrical shell model, i.e.

$$q'_{j\,max.} = \frac{\left(\sum_{K=1}^{n} \sum_{i=1}^{m} \alpha^{t}_{ij} \, . \, \phi^{t}_{i,k} \, . \, \Omega_{K} \right) S_{a} \; (\mu'_j \, . \, \sqrt{\eta'_j} \;)}{\eta_j} \tag{7}$$

Where m is the number of modes required to approximate $\underset{\sim}{\phi}'$ from $\underset{\sim}{\phi}$ and n is the total degrees of freedom

the displacement vector due to mode j is

$$r_{,j} = \underset{\sim}{\phi}'_{,j} \cdot q'_j$$
$$= \left(\sum_{i=1}^{m} \phi_{,i} \cdot \alpha_{i,j} \right) \cdot q'_j \qquad (8)$$

Following the usual procedures applicable to seismic analysis, one can evaluate the total response, for instance, without closely spaced frequencies, as:

$$r_{,T} = \left(\sum_{p=1}^{3} r^2_{,p} \right)^{1/2}$$

and $r_{,p} = \left(\sum_{g=1}^{\ell} r^2_{gp} \right)^{1/2}$

where $r_{,T}$ is the total combined response at a point

$r_{,p}$ is the value of combined response of earthquake component p

r_{gp} is the absolute value of response for earthquake component p, mode g

ℓ is the total number of modes considered significant for the seismic excitation

For closely spaced frequencies, one could apply the same principal following the procedure shown for example in reference (2)

6. Conclusion

The method has been applied to a number of analyses and it has proved to be very successful. The example shown herewith highlights the advantages offered by this technique. Because of the limited space one is allowed in this conference paper, we have been unable to show the actual results. However, it is our intention to publish them in a separate journal.

The scope offered by this technique is not limited to axisymmetrical shell nor the application be confined to seismic analysis. Indeed, it could be used in any 2D or 3D F.E. modelling and be applied in any dynamic field. Of course, the efficiency of it depends very much on the vibrational characteristic of the structure in consideration.

7. References

1. Newmark, N.M., Earthquake Engineering, Ed. R.L. Weigel, Prentice-Hall. N.J., 1970

2. Wilson, E.R. et al "Replacement of SRSS Method in Seismic Analysis" Earthquake Engineering Structural Dynamic Vol. 9

Fig 1

Fig. 3

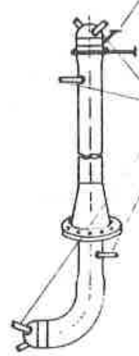

Fig. 5

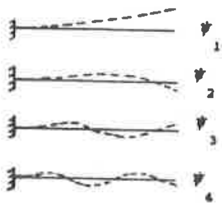

Fig 2

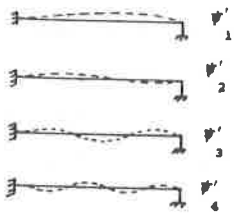

Fig. 4

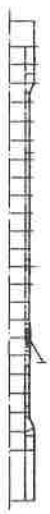

Fig 6

Fig. 7

Fig. 8