

Detailed Representation of Fluid Structure Interaction in Seismic Analysis

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Abstract

In the seismic analysis of large sodium cooled fast breeder reactors, the fluid description is usually based on the so-called acoustic approximation. The fluid is divided into several independent volumes separated by impervious boundaries. However the thermo-hydraulic equilibrium of the primary circuit requires connection between the fluid volumes and in some cases the utilisation of perforated shells.

This paper describes the methods available to modelize communications between fluid volumes, with a free surface in a component penetration, or through perforated plates.

Comparison is made between the finite element and analytical solutions in the case of simple geometry, and the effect of mesh size is discussed.

1. Introduction

The seismic analysis of the reactor vessel of a FBR pool-type must take into account fluid-structure interaction. Simplified hypotheses where a diagonal added mass of fluid is used are not acceptable for the design of the vessel and internal shell of the reactor.

However, the low velocities of the fluid involved in seismic motions allow to use a fluid description, based on the so-called acoustic approximation, neglecting viscosity and transport terms in the equations of the fluid. This approach where only inertia effects are taken into account, has proved to be accurate. It has been used in the finite element program NOVAX which performs dynamic and seismic analysis of axisymmetric structures in fluid using the Fourier series decomposition technique [1].

The dynamic equation of the fluid structure system can then be written :

$$(M + \tilde{M}) \ddot{U} + K U = 0$$

Where M , K , U are respectively the mass, the stiffness matrix for shells, and the displacement of structures, and where \tilde{M} is a full mass matrix, calculated from fluid properties and geometry, coupling the degrees of freedom of all structural nodes on the boundary of a same fluid superelement.

The sodium contained in the vessel can be divided in separate volumes : hot plenum, cold plenum... But the thermohydraulic equilibrium of the primary circuit implies fluid communications between these volumes, and between these volumes and free surface of sodium.

The three types of fluid communications encountered are :

- between two fluid volumes,
- with a free surface in a components penetrations,
- through perforated shells.

2. Communications between two fluid volumes

When two fluid volumes are connected by an area of size equivalent to or smaller than the mesh, a model with one large superelement is feasible. In that case a large mass matrix M is obtained and its bandwidth is large.

For a communication created by a series of circular holes, an equivalent axisymmetric passage can be established. When the opening is small a refined mesh is required, creating a source of numerical difficulties in the solution.

To reduce the matrix size the following method has been devised :

- define the dynamic equivalence of the communications as a relationship between pressure difference and flow rate,
- use an artificial boundary to connect two independant fluid superelements.

Dynamic equivalence

If we consider two fluid volumes connected by a small area S , the solution of the acoustic equation assuming rigid walls can be written

$$\Delta P = \rho L \ddot{U}_n = \rho \frac{L}{S} \frac{dQ}{dt}$$

where p , ρ , U_n are respectively pressure, fluid density and mean acceleration normal to the boundary.

L is the equivalent length and Q the flow rate.

Therefore $\frac{\rho L}{S}$ must be kept constant between equivalent models.

Artificial boundary

A fictitious membrane separating the two fluid superelements is used. The structural nodes follow the fluid motion and are attached to a very flexible spring.

The computed added mass on this boundary must be equal to the communication inertia.

This membrane does not modify the response of the system if its frequency is low. The rigidity is associated with the oscillatory motion of the fluid in the opening and has a physical meaning ; it introduces a diagonal term in the stiffness matrix which can be small but not zero.

In a NOVAX model a special 4 nodes elastic element is used. The equivalence of calculations with a unique superelement and with two superelements separated by a membrane has been tested in several cases. Figure 1 gives comparative results of frequencies and participation factors for the two methods.

Calculation of inertia

In the case of a row of holes the following method is used.

1. Compute inertia and area of holes and determine L/S ratio.
This can be done analytically or with a finite element model based on acoustic approximation using a refined mesh in the vicinity of openings. In the finite element approach an artificial boundary can be used to compute inertia.
2. Adjust the axisymmetric model.
The size of the mesh is modified to obtain the required L/S ratio.
The inertia must include the length of the flow path. If this term is not negligible it may be necessary to add a mass on the nodes of the artificial boundary.

Comparison with analytical solution

A comparison has been made with an analytical solution in the case of a circular or annular orifice at the boundary of an infinite fluid half space.

For a circular orifice of radius a : $L = \frac{8a}{3\pi}$

For an annular orifice of internal radius R_1 , and external radius R_2

$$L = \frac{3}{4\pi} \frac{1}{(R_2^2 - R_1^2)} \left\{ 2 (R_2^3 + R_1^3) - (R_1 + R_2) \left[(R_1^2 + R_2^2) E(k) - (R_1 - R_2)^2 K(k) \right] \right\}$$

$$k = \frac{2(R_1 R_2)^{\frac{1}{2}}}{R_1 + R_2} \quad E, K, \text{Hankel functions}$$

$$E = \int_0^{\frac{\pi}{2}} (1 - k^2 \sin^2 x)^{\frac{1}{2}} dx$$

$$K = \int_0^{\frac{\pi}{2}} (1 - k^2 \sin^2 x)^{-\frac{1}{2}} dx$$

These simple cases have been studied with the program NOVAX using different mesh sizes with square fluid elements in the vicinity of the opening of width a.

This study has shown that :

- if the size of the mesh is smaller or equal to the size of the orifice, the error on inertia can be neglected,
- if the orifice is smaller than the length of a fluid element, the shape of the mesh can be chosen to minimize the error on inertia.

With the shape given by figure 3 a parametric study has been performed giving the inertia error ratio $\frac{m}{m_{th}}$, versus $\lambda = L/l$ see figure 2.

l diameter or width of orifice, L size of mesh
m computed inertia, m_{th} theoretical inertia

The error can be corrected easily by the method described above.

3. Communications between internal volumes and free surface

In a pool type reactor internal fluid volumes may be connected to the free surface, for example through components penetrations.

These long communications can be considered as vertical fluid volumes with rigid walls. By integrating the Euler equation along vertical axis, and using continuity, we obtain :

$$p_i = \rho L_i \ddot{U}_i + K_i U_i$$

$$L_i = \int_0^h \frac{S_i}{S} dz \quad K_i = \rho g \frac{S_i}{S_0}$$

- S_i, S, S_0 : lower, current and upper section of column, see figure 3
 h : length of column
 ρ : fluid density
 p_i : pressure at lower end of column

Such a column can be modelled by an artificial boundary for the fluid superelement. The vertical stiffness per unit area is K_i and the vertical mass $\rho L S_i$.

When several communications are in parallel, one equivalent artificial boundary can be determined.

The area S^* , vertical stiffness K^* , mass m^* can be obtained from the conservation of kinetic and elastic energy.

$$\text{Then : } \frac{S^*}{L^*} = \sum_i \frac{S_i}{L_i} ; \frac{K^* S^*}{L^*} = \sum_i \frac{K_i S_i}{L_i} ; m^* = \rho L^* S^*$$

S^* can be chosen arbitrarily

4. Perforated plates

For a thin plate of radius R , perforated in a regular pattern of n holes of radius r the equivalent length of a hole is :

$$L^* = \frac{16}{3\pi} r \left(1 + n \frac{r}{R} - \sqrt{n} \frac{r}{R} \right)$$

The opening can be represented by one or more axisymmetric openings, depending on desired model accuracy.

According to the principles exposed above an artificial boundary element playing the role of a fictitious membrane allows to compute the inertia of the communications in the mesh and to adjust it to the inertia of the perforated plate.

In the case where the two sides of the plates are in the same fluid superelement, the fictitious membranes can then be removed from the model [2].

Conclusions

In the dynamic analysis of a large sodium cooled fast reactor, independant volumes are used to compute fluid structure interaction. Communication between these volume, can be introduced in the model through artificial boundaries.

The same technique can be used to describe a communication with a free surface.

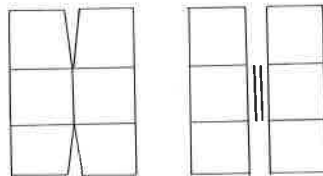
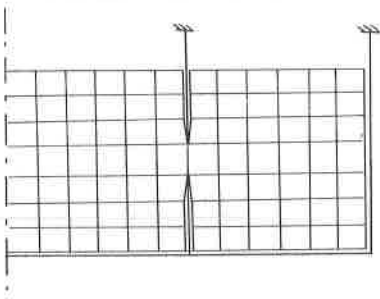
A locally reduced mesh size can be used to model a perforated shell. Accuracy can be achieved by a slight correction of the geometry.

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FIGURE 1 : HORIZONTAL PASSAGE
COMPARISON OF MODELS WITH A OR 2 FLUID SUPERELEMENT

DIRECTION	N° MODES	1 SUPERELEMENT		2 SUPERELEMENT	
		f	Γ	f	Γ
VERTICAL	1	1.57	190.2	1.55	187.5
	2	5.96	55.9	5.95	57.1
	3	15.1	44.4	14.5	52.0
	4	25.1	25.6	24.9	28.5
	5	32.2	5.4	27.9	9.7
	6	40.2	18.7	37.7	16.5
	7	62.3	2.4	62.3	2.4
	8	85.5	1.	85.4	0.9
	9	90.1	0.	90.1	0.
HORIZONTAL	1	5.89	115.6	5.88	116.8
	2	14.6	9.1	14.1	13.9
	3	24.2	201.3	24.0	200.9
	4	30.3	115.3	29.5	50.
	5	37.3	21.2	37.0	106.
	6	72.8	41.3	72.8	41.3
	7	78.3	9.4	78.3	9.4
	8	93.7	7.6	93.7	7.6



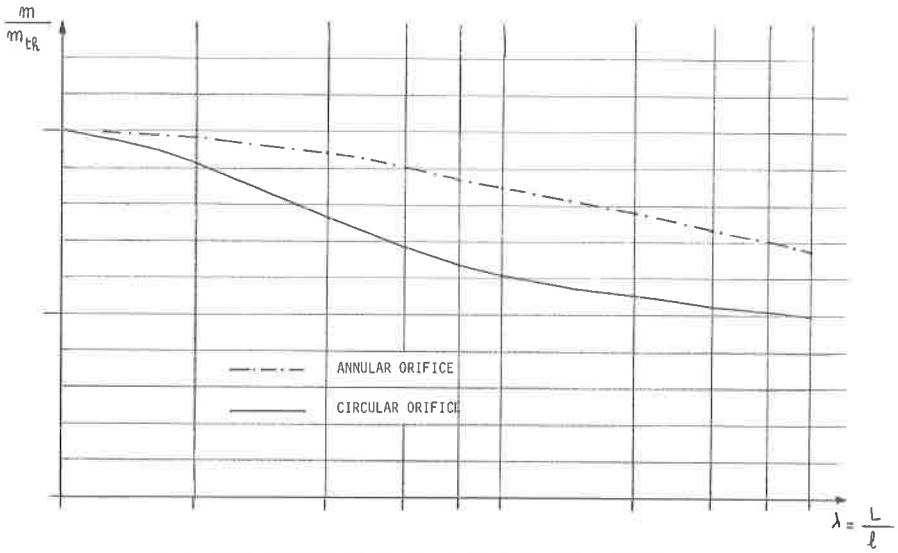


FIGURE 2 : ERROR INTRODUCED BY THE MESH IN THE CALCULATION OF INERTIA

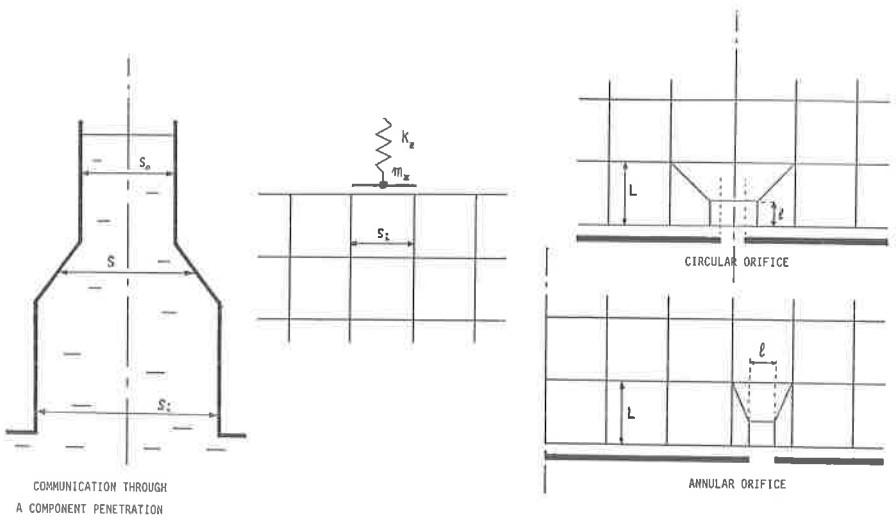


FIGURE 3 : MODELLISATION OF COMMUNICATIONS AND ORIFICES