Stochastic nonlinear dynamic analysis of piping systems on yielding supports

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1 INTRODUCTION

Energy absorbing yielding plates have been investigated (EPRI, 1983) as an alternative supporting elements for piping systems in nuclear power plant structures. The significance of this type of support is in its ability to absorb a large amount of displacement at or beyond the yield level. This allows pipe components to relieve themselves from substantial stresses and thus to withstand seismically-induced forces with a sufficient level of safety. The yielding supports are, however, not intended to substitute the entire support system of the pipes. They are, in fact, used along with rigid or other types of supports.

An experimental investigation of pipes on yielding supports has been previously conducted on a prototype system (EPRI, 1983) where yielding supports in the form of X-shaped plates were used. Each plate yields as the system vibration enhances and subsequently experiences a series of loading-unloading conditions. Thus a significant amount of energy is dissipated by the support. As a result, the pipe elements experience a reduced response.

The behavior of a piping system with yielding supports in a seismic load environment is generally complicated. The theoretical investigation of such system thus requires a comprehensive nonlinear dynamic analysis. Moreover, because of randomness in the nature of seismic forces, the problem, more appropriately, calls for a probabilistic formulation with a random vibration analysis. In this paper, a nonlinear finite element random vibration analysis is developed. The formulations are presented and some numerical results are given. Comparison of the results with deterministic nonlinear dynamic analysis models (such as ADINA) are also made and presented.

2 BASIC ASSUMPTIONS

The piping system considered is a general three-dimensional system with a combination of several yielding and rigid supports. Pipes are modeled as three-dimensional beam elements and yielding supports as isolated spring elements.

Only spring elements yield during a seismically-induced vibration and pipes are assumed to remain elastic. Figure 1 shows the finite element idealization of the problem and the nodal degrees of freedom involved.

The ground acceleration is idealized as a nonstationary zero-mean Gaussian process made of a stationary random process X(t) multiplied by an envelop function f(t). Suggested forms for f(t) are given by Amin and Ang (1968). The stationary process X(t) follows the Kanai spectrum (Kanai, 1967) as given by Eq. 1.

$$(1)\quad S(\omega)=S_0[1+(2\xi_g\omega/\omega_g]^2/\{[1-(\omega/\omega_g)^2]^2+[2\xi_g\omega/\omega_g]^2\}$$

in which S_0 is the intensity of the ground acceleration and ω_g and ξ_g are, respectively, the ground frequency and damping.

3 GENERAL FORMULATION

Following the proposed hysteresis model of Wen(1980), the restoring force, f_s, in the spring (yielding element) may be written as:

$$(2)\quad f_s=\alpha_s k_s v_s+k_s(1-\alpha_s)z_s \qquad s=1, 2,\ldots,l_s$$

in which l_s= total number of springs, α_s=parameter equal to the ratio of the post- to pre-buckling stiffness, k_s=initial stiffness, v_s=axial displacement in the spring s and z_s=an auxiliary variable obtained from the following differential equation.

$$(3)\quad dz_s/dt=A_s dv_s/dt-\beta_s|dv_s/dt||z_s|^{n-1}\, z_s-\tau_s dv_s/dt|z_s|^n$$

in which A_s, τ_s, β_s and n are parameters describing the hysteresis behavior of spring element s (Wen, 1980).

The method of equivalent stochastic linearization is used for the nonlinear random vibration analysis. The linearized form of Eq. 3 is

$$(4)\quad dz_s/dt = c_{es}\, dv_s/dt + k_{es}\, z_s \qquad s=1, 2, \ldots, l_s$$

where c_{es} and k_{es} are linearized parameters which are obtained by minimizing the square of difference between the nonlinear and the linearized equations using the process described by Atalik and Utku (1976), Wen(1980) and Simulescu, et al. (1985). Parameters c_{es} and k_{es} are response dependent. Thus a trial and error approach is generally required for the calculation of response, v_s.

The equation of motion for the spring elements is (Wen, 1980)

$$(5)\quad m_s(d^2v_s/dt^2)+c_s(dv_s/dt)+\alpha_s k_s v_s+(1-\alpha_s)k_s z_s = p_s$$

where m_s, c_s and k_s are, respectively, the mass, damping and the initial stiffness of spring s and p_s is the nodal force in the direction of the spring axial deformation v_s. The equation of motion for a pipe element is

(6) $m_p(d^2v_p/dt^2)+c_p(dv_p/dt)+k_pv_p = p_p$

in which v_p is a vector containing the pipe local degrees of freedom, and m_p, c_p, k_p and p_p are, respectively, the mass, damping, stiffness and nodal force matrices.

After transferring Eqs. 5 and 6 into the global system, the following sets of equations are obtained for the dynamic analysis of the system.

(7) $M(d^2U/dt^2)+C(dU/dt)+KU+GZ = P$

(8) $dZ_s/dt = H_e(dU_s/dt)+K_zZ_s$

in which, M=system NxN mass matrix (N=total degrees of freedom), P=system nodal loads vector, C=system NxN damping matrix, K=system NxN stiffness matrix, U=system vector of degrees of freedom, U_s=subvector of U containing nodal degrees of freedom associated with spring elements, Z_s=vector containing auxiliary parameters z_s, Z=vector Z_s augmented to satisfy the size of Eq. 7, G_s=system global matrix containing the $(1-\alpha_s)k_s$ part of the springs restoring forces, G=matrix G_s augmented to satisfy the size of Eq. 7, H_e=system global matrix containing parameters c_{es} and K_z=system global matrix containing k_{es} parameters.

Equations similar to these are also reported in Simulescu, et al. (1985) for nonlinear dynamic analysis of plane systems.

Equations 7 and 8 must be solved simultaneously. Since matrices H_e and K_z depend on the system response, a trial and error approach will be needed to determine the response. Introducing a vector Y as

(9) $Y = [U^T \quad dU^T/dt \quad Z_s^T]^T$

Eqs. 7 and 8 may be combined to form the following sets of equations.

(10) $A(dY/dt) + JY = P_0$

or

(11) $dY/dt + BY = F$

in which matrices A, J, B, and F are assembled from M, C, K, G_s, K_z and H_e and vector P_0 is the augmented form of P. The covariance response matrix S_{YY} is then obtained from:

(12) $dS_{YY}/dt + BS_{YY} + S_{YY}B^T + R = 0$

in which

(13) $R = -E(FY^T + YF^T)$

where E denotes expectation. Equation 12 may be solved using a numerical differentiation technique.

An alternative solution model is presented by Simulescu, et al. (1985). In this formulation, a complex modal analysis

similar to the conventional modal analysis is proposed. This solution model requires calculation of complex eigenvalues λ and eigenvectors Φ_R and Φ_L from homogeneous part of Eq. (10). Then modal responses in the following form are written in terms of Q, the generalized coordinate vector.

$$(14) \quad dQ/dt - \lambda Q = \Phi_L P_0$$

$$(15) \quad Y = \Phi_R Q$$

Statistics of the response are then evaluated from Eqs. 14 and 15 using the contributions from the individual modal responses. A detailed description of this method may be found in Simulescu, et al.(1985).

4 NUMERICAL ILLUSTRATIONS

The method presented in the previous sections was used for the nonlinear random vibration analysis of several multi-degree-of-freedom piping systems each with several yielding supports. The solution model based on the complex modal analysis was adopted in this study. The damping matrix C was taken as a linear combination of matrices M and K. The equations for evaluating linearization parameters c_{es} and k_{es} for each hysteretic spring element were obtained from (Wen, 1980).

In this paper, however, the results for a simple system made of two pipe elements, one yielding element and two rigid supports are presented. This is because, for this simple system a comprehensive investigation was performed and the results were compared with the results from the program ADINA (ADINA is a deterministic nonlinear dynamic analysis program). The system and its configuration are shown in Fig. 2. The spring hysteresis parameters needed in Eqs. 2 and 3 were then calculated based on relationships reported in Baber and Wen (1979). The values for these parameters are: k_s = 300 kips/ft (4375 KN/M), A=1.0, α = 0.05, $\beta = \tau$ =18.8 1/ft (3.28 1/M).

The results for S_0 = 0.1 and 117 ft^2/sec^3 (.0093 and 10.9 M^2/sec^3) are shown in Table 1. For each S_0 this table lists the standard deviation of the displacement, velocity and the correlation between the displacement and the velocity for the first 20 second of the response. This is the time that the response reaches its stationary phase for 2% of system critical damping. The correlation coefficient, however, does not approach to zero. This is because of the non-Gaussian input motion used in the problem. The statistics of the response in the form of the probability density function are presented in Fig. 3. The probability density function was obtained using a Poisson distribution for the number of times the response crosses different levels of displacements (Lin, 1967). Using the same type of force deformation behavior for the yielding element, the program ADINA was used for the deterministic response calculation of 0.3g base excitation. This value is equal to the standard deviation of a zero-mean acceleration process presented with Eq. 1 for S_0=0.1 ft^2/sec^3. The results obtained by ADINA in

several runs are also shown in Fig. 3. This figure shows that the deterministic calculated peak response can well be predicted by c.(mean of peak value displacement), where c is a nearly constant which is about 1.5 in Fig. 3.

The probability density function of maximum response and the corresponding ADINA result for S_0=117 ft^2/sec^3 are shown in Fig.4. This value of S_0, which is consistent with a maximum base acceleration of 10g, causes much more severe yielding system than the response in Fig. 3. Again, it is seen that the deterministic information can be approximated by a fixed multiple of the mean peak response value.

5 SUMMARY AND CONCLUSIONS

A stochastic finite element approach is discussed for the nonlinear random vibration analysis of a piping system with yielding supports. The solution is obtained by stochastic linearization which uses complex modes of the linearized system. Results from limited simulation studies using the program ADINA were compared to the distribution of the peak displacement derived through the random vibration analysis. The comparison shows that the deterministic peak response can reasonably be predicted by the statistics of the peak response obtained from random vibration analysis. Additionally, since random vibration analysis provides statistics for the response and its time derivative, this information can readily be synthesized to formulate a solution to the fatigue damage analysis including low cycle fatigue which has been a consideration to be addressed in connection with steel systems and components responding into the inelastic range.

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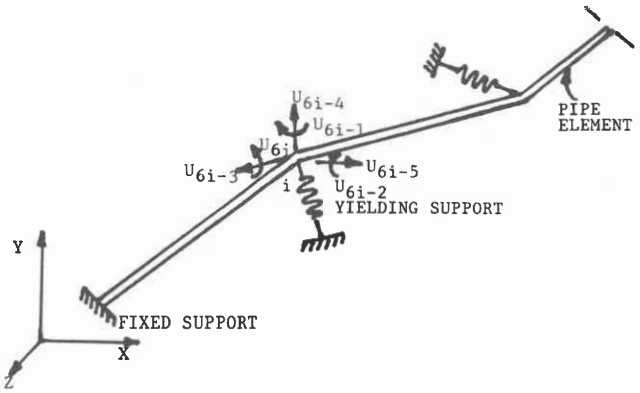

FIG. 1. Finite element idealization of a piping system on yielding supports

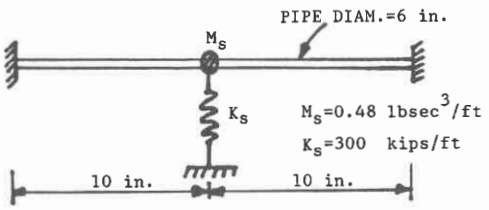

FIG. 2. Example used in numerical illustration

TABLE 1. Standard deviation of response for system in Fig. 2

Time (sec)	S_0=0.1			S_0=117		
	disp.	vel.	correlation	disp.	vel.	correlation
1.0	.4E-03	.0013	-.119	.026	.047	-.164
2.5	.7E-03	.0013	-.078	.037	.048	-.148
5.0	.9E-03	.0014	-.571	.047	.048	-.151
7.5	.0011	.0014	-.477	.054	.048	-.157
10.0	.0012	.0014	-.421	.057	.048	-.162
12.5	.0013	.0014	-.383	.060	.049	-.165
15.0	.0015	.0014	-.355	.061	.049	-.166
17.5	.0015	.0014	-.340	.061	.049	-.166
20.0	.0015	.0014	-.340	.061	.049	-.166

Note: Time interval used in response calculation=0.1 sec.

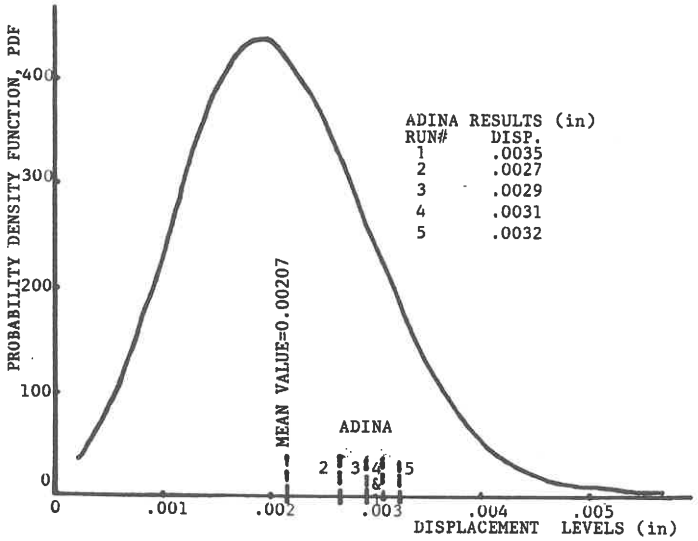

FIG. 3 PDF of response for S_0=0.1 ft^2/sec^3

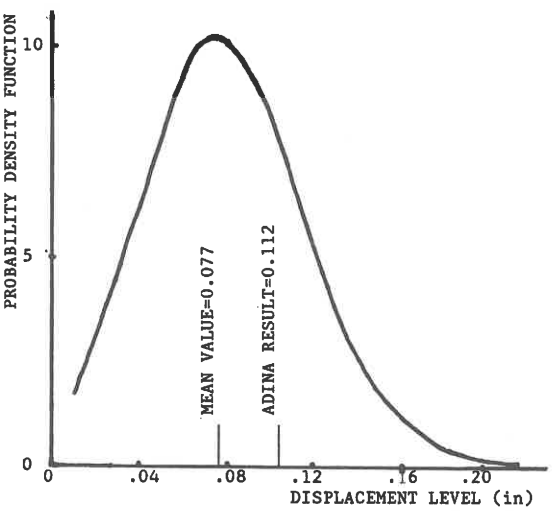

FIG. 4. PDF of response for S_0=117 ft^2/sec^3