Multimode Response Factors for the Seismic Analysis of Component-Support Systems

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SUMMARY

A common approach to the seismic response analysis of reactor component-support systems is to include the higher mode responses by applying a "conservative" multimode factor to the fundamental mode response. The purpose of this investigation is to relax this conservatism, where appropriate, by developing multimode factors dependent on system parameters. Response factors are developed for several types of system configurations. It is shown that the current industry practice is conservative when multimode factors are properly used.

1. INTRODUCTION

Nuclear power plant component-support systems essential in preventing the release of radioactive material to the environment must be designed to withstand the seismic loading from a safe shutdown earthquake in addition to normal operating loads. These systems include HVAC ducts, electrical conduit, cable trays, piping and their supporting structures. To determine their seismic responses, the response spectrum method can be applied to a structural model consisting of lumped masses and beam elements. This approach is sometimes bypassed in favor of a simpler and more approximate method which models a support and its tributary component mass as either a one or two degree-of-freedom system. After calculation of the fundamental mode frequency, a spectral acceleration, S_a, is obtained from the response spectrum curve, and stresses and displacements are computed from an equivalent static load, proportional to the mass distribution of the system and equal to (S_a/g) times the total weight. A so-called "multimode factor" is then applied to the response to account for higher mode effects. Current industry standards [1, 2, 3] and Stevenson [4] recommend a value of 1.5. Whether any such factor is a conservative analysis tool will depend on the component-support span configuration, component stiffness, support stiffness, mass distribution, boundary conditions, and response spectrum shape. The purpose of this investigation is to develop multimode factors which are not too conservative and yet have wide applicability.

To develop these factors, three types of systems will be considered:

1. Systems with flexible massless supports, stiff components, and unequal span lengths.
2. Systems with flexible supports, flexible components and equal spans.
3. Systems with stiff supports and unequal spans.

This categorization, though simple, is general enough to represent a wide variety of support-component systems. Under seismic inputs defined by a uniform acceleration response spectrum, each type is readily tractable by closed form solutions. The purpose of type 3 is to assess multimode effects for irregular span configurations. To this end, a 3-span flexural member (4 supports) will be evaluated for various locations of the interior supports. For each type the system will be analyzed for support motion defined by a uniform acceleration spectrum, and the sensitivity of higher mode responses to parameter adjustment will be investigated. The uniform spectrum assumption provides multimode factors which bound those for all spectrum shapes which decrease monotonically over the range of system natural frequencies. Typically, support-component systems and seismic floor response spectra satisfy this requirement. In other cases, multimode factors may be adjusted by the actual spectral acceleration ratios. The equivalent static load factors in [1, 2, 3] correspond to multimode factors for inputs defined by uniform spectra.

2. SEISMIC RESPONSE ANALYSIS

2.1 Case 1: Rigid Commodity and Flexible Supports

Some HVAC commodities and their supports may be modeled as essentially rigid beams of uniform mass distribution resting on flexible massless supports (Fig. 1). The frequencies, ω_i, and mode shapes, $\{\phi_1, \phi_2\}$, for the free vibration response of this coupled 2DOF system are [5] :

$$\omega^2_{1,2} = (1/M)\left[\frac{1}{2}\left(\frac{c}{r^2} + a\right) \mp \sqrt{\frac{1}{4}\left(\frac{c}{r^2} - a\right)^2 + \frac{b^2}{r^2}}\right] \tag{1}$$

$$\begin{Bmatrix}\phi_1\\ \phi_2\end{Bmatrix}_{1,2} = \begin{Bmatrix}y\\ \theta\end{Bmatrix} = \begin{Bmatrix} b\Big/\left[\frac{1}{2}\left(\frac{c}{r^2} - a\right) \mp \sqrt{\frac{1}{4}\left(\frac{c}{r^2} - a\right)^2 + \frac{b^2}{r^2}}\right] \\ 1 \end{Bmatrix} \tag{2}$$

where a, b, c, r, M, y and θ are defined in Fig. 1.

For support motions defined by a uniform acceleration response spectrum, the ratio of modal responses in normal coordinates is

$$\left|\frac{q_2}{q_1}\right| = \left|\frac{\Gamma_2 (S_a)_2/\omega_2^2}{\Gamma_1 (S_a)_1/\omega_1^2}\right| = \left|\frac{\Gamma_2\omega_1^2}{\Gamma_1\omega_2^2}\right|$$

where

Γ_i = i^{th} modal participation factor

$(S_a)_i$ = spectral acceleration at frequency ω_i

At the right end of the beam the corresponding modal displacement ratio is

$$R = \left|\frac{(z+\sqrt{3}r\theta)_2}{(z+\sqrt{3}r\theta)_1}\right| = \left|\frac{q_2(\phi_{12} + \sqrt{3}\ r\phi_{22})}{q_1(\phi_{11} + \sqrt{3}\ r\phi_{21})}\right| = \left|\frac{\Gamma_2(\phi_{12} + \sqrt{3}\ r\phi_{22})}{\Gamma_1(\phi_{11} + \sqrt{3}\ r\phi_{21})}\right| \left(\frac{\omega_1^2}{\omega_2^2}\right) \tag{3}$$

The participation factors Γ_i are related by

$$\Gamma_1\begin{Bmatrix}\phi_{11}\\ \phi_{21}\end{Bmatrix} + \Gamma_2\begin{Bmatrix}\phi_{12}\\ \phi_{22}\end{Bmatrix} = \begin{Bmatrix}1\\ 0\end{Bmatrix}$$

so that

$$\frac{\Gamma_2}{\Gamma_1} = -\frac{\phi_{21}}{\phi_{22}} = -1$$

and

$$R = \left|\frac{(\phi_{12}+\sqrt{3}r\phi_{22})}{(\phi_{11}+\sqrt{3}r\phi_{21})}\right|\left(\frac{\omega_1^2}{\omega_2^2}\right) = \left|\frac{\phi_{12}+r\sqrt{3}}{\phi_{11}+r\sqrt{3}}\right|\left(\frac{\omega_1^2}{\omega_2^2}\right) \tag{4}$$

Eliminating the radical term in eq's (1) and (2) leads to the relations

$$M(\omega_2^2 - \omega_1^2) = \frac{(\phi_{11}-\phi_{12})}{(\phi_{11}-\phi_{12})}\left(\frac{c}{r^2} - a\right) \tag{5}$$

$$M\omega_1^2 = \frac{1}{2}\left(\frac{c}{r^2} + a\right) - \frac{1}{2}\left(\frac{c}{r^2} - a\right)\left(\frac{\phi_{11}-\phi_{12}}{\phi_{11}-\phi_{12}}\right) \tag{6}$$

Expressing the frequency ratio as

$$\frac{\omega_1^2}{\omega_2^2} = \frac{\omega_1^2}{\omega_1^2 + (\omega_2^2-\omega_1^2)}$$

and using the relation $r = \sqrt{-\phi_{11}\ \phi_{12}}$ along with (5) and (6), (4) becomes after simplifying

$$R = \left|\frac{1 + \sqrt{-3\left(\frac{\phi_{11}}{\phi_{12}}\right)}}{\left(\frac{\phi_{11}}{\phi_{12}}\right) + \sqrt{-3\left(\frac{\phi_{11}}{\phi_{12}}\right)}}\right| \left(\frac{1 - \gamma\left[\left(\frac{\phi_{11}}{\phi_{12}} - 1\right)\Big/\left(\frac{\phi_{11}}{\phi_{12}} + 1\right)\right]}{1 - \gamma\left[\left(\frac{\phi_{11}}{\phi_{12}} - 1\right)\Big/\left(\frac{\phi_{11}}{\phi_{12}} + 1\right)\right]}\right) \tag{7}$$

where

$$\gamma = \left(\frac{c}{r^2} - a\right)\Big/\left(\frac{c}{r^2} + a\right)$$

The corresponding expression for the left end of the beam has the sign in front of the radicals reversed. R depends only on the two nondimensional parameters γ and (ϕ_{11}/ϕ_{12}) which are readily evaluated from system properties a, b, c and r. Defining

$$R' = \min\ (R,\ 1/R)$$

references the multimode factor to the dominant mode. For this 2DOF system, the SRSS multimode response factor, $\sqrt{(R')^2 + 1}$, has an upper bound of $\sqrt{2}$.

2.2 Case 2: Flexible Support and Flexible Commodity

Fig. 2 depicts a flexible commodity, modeled as a Bernoulli-Euler beam, resting on equally spaced elastic supports of equal stiffness. The solution of the homogeneous equation of motion

$$EI\frac{\partial^2 y}{\partial x^4} + \frac{\partial^2 y}{\partial t^2} = 0$$

leads to the eigenfunctions

$$\phi_n(x) = A_n \sin\lambda_n x + B_n \cos\lambda_n x + C_n \sinh\lambda_n x + D_n \cosh\lambda_n x \tag{8}$$

where

$$\lambda_n = \sqrt[4]{m\omega_n^2/EI}$$

Eq. (8) together with boundary conditions

$$V = -EI\phi'''(0) = 0$$

$$\phi'(0) = \phi'(L/2) = 0$$

$$V = -EI\phi'''(L/2) = -k\phi(L/2)/2$$

lead to the eigenvalue problem

$$\left.\begin{aligned} & -B_n\lambda_n\sin(\lambda_n L/2) + D_n\lambda_n\sinh(\lambda_n L/2) = 0 \\ & B_n\left[EI\lambda_n^3\sin(\lambda_n L/2) - (k/2)\cos(\lambda_n L/2)\right] + D_n\left[EI\lambda_n^3\sinh(\lambda_n L/2) - (k/2)\cosh(\lambda_n L/2)\right] = 0 \end{aligned}\right\} \quad (9)$$

with corresponding frequency equation

$$\coth\beta_n + \cot\beta_n = A\beta_n^3$$

where $\beta_n = \lambda_n L/2$ and $A = 32EI/kL^3$. The stiffness ratio A is a measure of the beam to support stiffness. Participation factors are defined by

$$\Gamma_n = \frac{\int_o^{L/2}\phi_n(x)\,dx}{\int_o^{L/2}\phi_n^2(x)\,dx}$$

$$= \frac{B_n\sin\beta + D_n\sinh\beta}{B_n^2\left[\frac{\beta}{2} + \frac{\sin 2\beta}{4}\right] + D_n^2\left[\frac{\beta}{2} + \frac{\sinh\beta\cosh\beta}{2}\right] + B_nD_n(\sinh\beta\cos\beta + \cosh\beta\sin\beta)}$$

B_n and D_n are any values which satisfy the first of (9).

The ratio R* of the total SRSS response to the first mode response at position x is

$$R^* = \frac{\sum_j\left[\Gamma_j\phi_j^i(x)\right]^2}{\left|\Gamma_1\phi_1^i(x)\right|} \quad (10)$$

where i connotes the derivative order corresponding to the response parameter of interest. Eq.(10) is evaluated for support shear, midspan deflection and support and midspan moments over the practical range of A values. The results are shown in Fig. 4.

2.3 Case 3: Variable Spans

For this case a 3-span symmetrical structure (Fig. 3) with variable ratios of interior to exterior span lengths is investigated. Two sets of eigenfunctions apply: ϕ_{AB} corresponding to the interior span and ϕ_{BC} to the exterior. To satisfy moment and rotation compatibility at the support,

$$\left.\begin{aligned} \phi'_{AB}(L_1) &= -\phi'_{AB}(L_2) \\ \phi''_{AB}(L_1) &= \phi''_{BC}(L_2) \end{aligned}\right\} \quad (11)$$

The additional boundary conditions are

$$\left.\begin{aligned} \phi'_{AB}(0) &= \phi'''_{AB}(0) = 0 \\ \phi_{BC}(0) &= \phi''_{BC}(0) = 0 \end{aligned}\right\} \qquad (12)$$

Only the response of half the interior span need be considered. Eq's (11), (12), and (8) lead to the frequency equation

$$\cos\lambda_n(L_1+L_2) - \cos\lambda_n L_1 \sin {}_n L_2 (\coth\lambda_n L_2 + \tanh\lambda_n L_1) = 0$$

and corresponding eigenfunctions

$$\phi_{AB,n}(x_1) = B_{1,n}\cos\lambda_n x_1 + D_{1,n}\cosh\lambda_n x_1$$

where

$$D_{1,n} = \left(\frac{-\cos\lambda_n L_1}{\cosh\lambda_n L_1}\right) B_{1,n}$$

The remaining treatment is as for Case 2. For ratios $\alpha = L_2/2L_1$. $R^*(\alpha)$ is plotted in Fig. 5.

3. CONCLUSIONS

Multimode response curves and equations applicable to various types of commodity-support systems have been presented. For systems which are nearly regular in the sense of Case 2, the 1.05 value of SRSS multimode response factor for shear justifies the exclusion of higher mode responses altogether.

The rigid commodity on flexible supports represents a 2DOF system having an upper bound SRSS factor of $\sqrt{2}$ if the factor is referenced to the dominant mode. A less conservative factor can be determined from Eq. (7).

For variable span commodities on rigid supports, the multimode factor exceeds 3 for shear due to the close spacing and high participation of higher modes for multispan structures. This high value, however, does not apply to structures which are analyzed using the correct approximate modeling techniques. For large span ratios $L_2/2L_1$, appropriate models for hand calculations are either a two-span continuous or a fixed-simple single span beam. These structures have higher relative participation in the first mode and corresponding lower multimode factors. Only structures with moderate span ratios exhibit the multimode effects shown in Fig. 5. These observations caution against computer modeling or resonant frequency testing for determination of fundamental mode frequencies for subsequent use with multimode factors.

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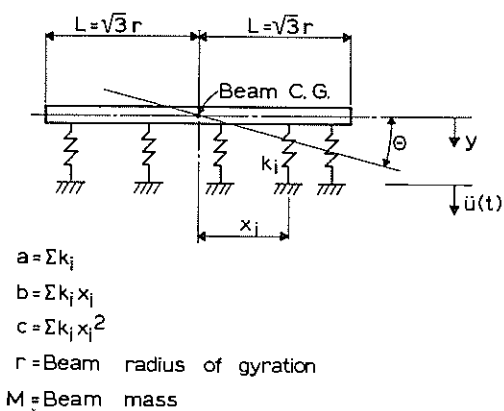

Fig. 1 RIGID COMMODITY WITH FLEXIBLE SUPPORTS

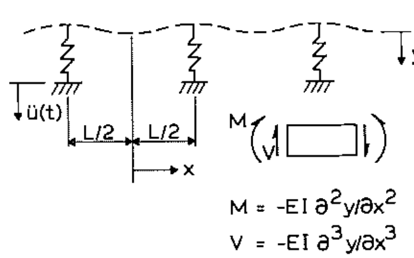

Fig. 2 FLEXIBLE COMMODITY WITH FLEXIBLE SUPPORTS

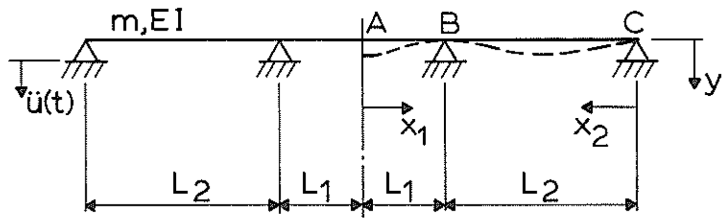

Fig. 3 VARIABLE SPAN COMMODITY

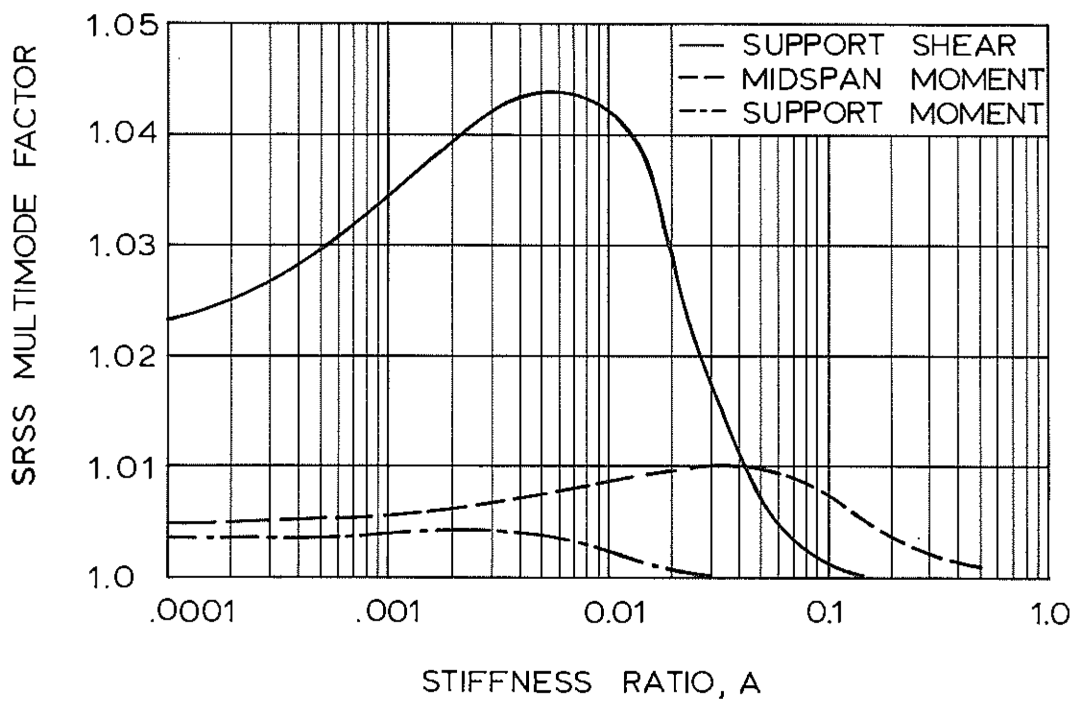

Fig. 4 CASE 2: MULTIMODE RESPONSE OF FLEXIBLE COMMODITY WITH FLEXIBLE SUPPORTS

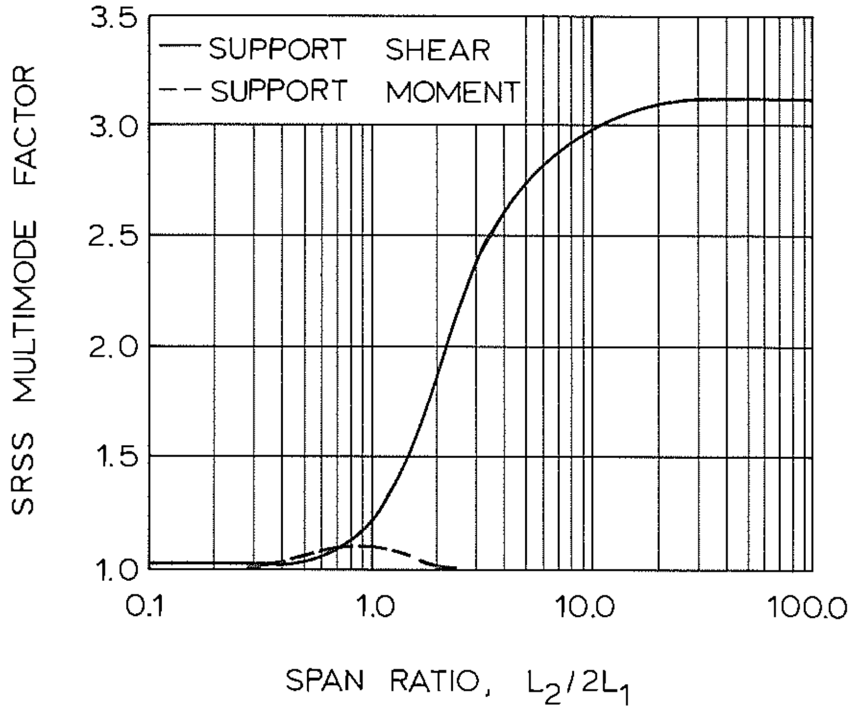

Fig. 5 CASE 3: MULTIMODE RESPONSE OF VARIABLE SPAN COMMODITY