AN INVESTIGATION OF FLUIDELASTIC INSTABILITIES IN TUBE BANKS SUBJECTED TO FLUID CROSS-FLOW

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SUMMARY

The experimental and theoretical work summarised here is part of a continuing general study of heat exchanger tube vibration, and is concerned with the particular problem of the self-excited, steady-state vibrations of arrays of tubes in cross-flow.

The basic experimental work is the measurement, using strain gauges, of the motions of tubes located in the front, centre and back rows of a 10 row array. The Perspex tubes are of 1″ diameter and are mounted in a cantilevered configuration in a wind tunnel of 18″square working section.

In the experiments the damping of the tubes in the array is first determined by exciting the instrumented tubes into free vibration and the array is then mounted in the wind-tunnel and the dynamic deflections of the tubes measured as a function of airspeed. The data from the measurements made so far are in general agreement with other workers, a typical result being

$$\frac{v_c}{fd} = 2.5\sqrt{\frac{m\delta}{\rho d^2}}$$

for an array in which the tubes were laid out in an equilateral triangle configuration with pitch-to-diameter ratio of 1.25.

The theoretical work is a study of the implications which follow when Connors' approach to the instability problem is applied to configurations more typical of heat exchanger practice. It is shown that in the case of single-span tubes which receive the cross-flow over the whole span, the mode-shape is unimportant, a result which seems to disagree with earlier work. When the flow meets only a part of the span, however, the critical velocity is influenced not only by the concentration of the flow, but by the mode-shape of the tube and the part of the span to which the flow is applied. Calculations have also been made for the case of a multi-span tube with partial admission in the first and last bays, and here it is found that the critical velocity can be strongly influenced by conditions in the end bays. These results are believed to be new.

Further experimental work is concerned with verification of the theoretical results on the effects of mode shape and partial admission and of initial turbulence.

In general, the experiments on partial admission agree qualitatively with the theoretical work, but quantitative results are only preliminary.

The effect of turbulence is to reduce the critical velocity. First indications of this lie in the fact that the critical airspeed is lower for the back row of an array than for the front row by about 12%. A turbulence grid reduces the critical velocity of the front row by about 35% and that of the back row by about 8%. There is, however, no significant change in the sharpness of the increase of tube amplitude with airspeed — a result which disagrees with other published work.

1. Introduction

The high cost of modern reactor heat exchangers and the stringent safety requirements associated with such plant put a high premium on mechanical reliability. An essential requirement for mechanical reliability in the medium and long term is the absence of vibration, but vibration is not uncommon in heat exchangers and, furthermore, its incidence seems to be rising.

It is now well-recognised that one of the major causes of severe tube vibration in heat exchangers is the mechanism of fluid-elastic instability, but although the principal features of this mechanism are becoming clear, there are still many aspects of the phenomenon which are vague, particularly with respect to the effects of conditions met with in practical heat exchanger designs.

The work presented in this paper represents the early results of a programme of study which will take up these problems in an attempt to bridge the gap between laboratory studies of fluid-elastic instability and the application of the data so obtained to practical design.

The particular problems dealt with here are the effects of mode-shape and partial admission on the critical speed of a single-span tube, the effect of partial admission in the inlet bay on the critical speed of a multi-span tube, and the effect of turbulence generated upstream on the critical speed of a tube bundle.

2. Test Facility

2.1 The Wind Tunnel

The experimental work reported here has been carried out in a wind tunnel built specifically for heat exchanger tube vibration studies at Harwell. The general arrangement of the wind tunnel is shown in Figure 1. Its working section measures 0.457m (18in) square and the centrifugal fan can provide approach velocities up to 15 m/s.

2.2 Test Sections

The tests completed to date have been carried out on tube bundles having arrays based on an equilateral triangular lattice, with pitch/diameter (p/d) ratios between 1.25 and 1.78. The bundles are assemblies of tubes of outside diameter 25.4mm, and have 10 rows of tubes counted in the flow direction. A cantilevered tube mounting arrangement was adopted in order to keep the tube fundamental frequency sufficiently low to ensure that self-excited vibrations would occur within the operational envelope determined by the tunnel size and speed range. To avoid unnecessarily thin tubes, 'Perspex' was selected as the tube material and the tubes were bonded into a 25.4mm thick 'Perspex' tubeplate. This system resulted in consistency of the end restraint conditions for all of the tubes in the array. The tubes are cantilevered from the top wall of the working section and to ensure that they vibrate freely a clearance of 1mm is allowed between the free end of each tube and the bottom wall. The bottom wall of the test section is also made of 'Perspex' so that the motion of the free ends of the cantilevered tubes can be easily observed.

2.3 Instrumentation

The motion of selected tubes within the array is detected using wire resistance strain gauges, attached to the tube bore at the position of maximum bending strain. For each array, ten tubes are instrumented in accordance with the locations shown in Figure 2, and for each of these tubes the components of displacement parallel and normal to the flow direction are measured. The amplified strain gauge signals are fed to an ultra violet recorder and to an RMS meter.

2.4 Partial Admission

Tests have been carried out in which the airflow was restricted to part of the tube span. This was achieved by inserting a flat plate immediately upstream of the tube bundle and normal to the flow, as shown in Figure 3. Two plates have been used, each 19mm thick, providing flow blockages of 33⅓% and 50% of the cross sectional area of the bundle.

2.5 Turbulence Promotion

A turbulence grid was constructed to study the effect of initial turbulence on the threshold velocity for self-excited vibration. The grid is built up from aluminium bars, 25mm wide by 3mm thick, arranged on a 50mm pitch as shown in Figure 4. In the experiments the grid was located 240mm upstream of the first row of tubes.

3. Theoretical Results

3.1 Connors' Equation

Connors' original experiments on the fluid-elastic instability of tubes in cross-flow were carried out on a single row of tubes. The tubes, which were rigid along their spans but flexibly mounted at their ends, had only two degrees of freedom, in each of which the modal displacement was uniform across the span and the natural frequencies were the same.

It was observed that when the instability occurred, all tubes were set into motion at the natural frequency f, the two translational modes of a particular tube combining to give it an orbital motion, and, on average, the relative motions of the tubes formed a certain pattern which may be called the fundamental array mode.

In analysing the instability, it was assumed that the energy dissipated in the damping of a tube suspension must be derived from work done on the tube by fluid forces, and in a second series of experiments the motion of a tube relative to its neighbours was simulated and the resulting forces measured. It was found that if x and y are displacements measured along and normal to the direction of flow, the component force coefficients could be expressed as:

$$c_{F_x} = \frac{F_x}{\frac{1}{2}\rho u^2 dL} = 0.202\ \frac{y}{d} \qquad \ldots (1)$$

$$c_{F_y} = \frac{F_y}{\frac{1}{2}\rho u^2 dL} = 0.330\ \frac{x}{d} \qquad \ldots (2)$$

in which ρ, u are the fluid density and velocity, and d, L the tube diameter and length.

Now when the exciting force and the displacement response are in quadrature, the work done per cycle in the x-mode is given by $W_x = \pi x_o (F_x)_o$, where the suffix zero denotes amplitude. Thus, using eqs. (1), (2), one can write:

$$W_x = 0.202\ (y_o/d)\ \tfrac{1}{2}\rho u^2 dL \pi x_o \qquad \ldots (3)$$

$$W_y = 0.330\ (x_o/d)\ \tfrac{1}{2}\rho u^2 dL \pi y_o \qquad \ldots (4)$$

The energy stored at any instant in the x-mode is given by $\frac{1}{2}M(dx/dt)_0^2$, where M is the tube mass, and the energy dissipated by damping per cycle is this multiplied by $2\delta_x$, where δ_x is the logarithmic decrement of the free vibrations in this mode. One can therefore write for simple harmonic motion:

$$E_x = 2\delta_x\ \tfrac{1}{2}\ M(dx/dt)_0^2 = 4\pi^2 f^2 x_o^2\ M\delta_x \qquad \ldots (5)$$

$$E_y = 4\pi^2 f^2 y_o^2 M\delta_y \qquad \ldots (6)$$

By equating eqs. (3) and (5), (4) and (6) one can show that if $\delta_x = \delta_y = \delta$,

$$\left(\frac{x_o}{y_o}\right)^2 = \frac{0.202}{0.330} = 0.61 \qquad \ldots (7)$$

and on substituting this in either of the equalities, there follows the result:

$$\frac{u}{fd} = 9.85\sqrt{\frac{(M/L)\delta}{\rho d^2}} = 9.9\sqrt{\frac{m\delta}{\rho d^2}} \qquad \ldots (8)$$

where m = M/L, the tube mass per unit length.

Eq. (8) is Connors' equation. As derived, it is restricted to a single row of tubes, each of which has only two modes of oscillation (in which the modal displacements are uniform across the span and the frequencies are the same) and receives the cross-flow across its full span.

3.2 The effect of mode-shape on the critical speed for a single span

In assessing the effect of mode-shape on the critical speed, the force acting on a tube must be treated as a distributed load. If l is distance measured in the spanwise direction, then for deflections in the flow direction the load distribution may be written as

$$p(l,t) = \tfrac{1}{2}\rho u^2 d\, c_x(l).\sin 2\pi ft$$

in which $c_x(l)$ is the local value of the force coefficient and it is assumed that the tube is vibrating at its natural frequency, f.

The tube displacement may be written

$$x(l,t) = -x_o\, Q_x(l).\cos 2\pi ft$$

where $Q_x(l)$ defines the mode-shape, x_o is a reference amplitude and it is recognized that, (as earlier), the force and the displacement are in quadrature.

With these expressions, the work done per unit time on unit length of tube becomes:

$$d_2W_x = p(l,t)\,\frac{d}{dt}.x(l,t) = 2\pi f.p(l,t).x(l,t)$$

$$= 2\pi f.\tfrac{1}{2}\rho u^2.d.c_x(l).x_o.Q_x(l).\sin^2 2\pi ft$$

and on integrating twice, the work done per cycle on the whole tube is, for this mode:

$$W_x = \pi x_o.\tfrac{1}{2}\rho u^2 d.\int_o^L c_x(l).Q_x(l).dl \qquad \ldots (9)$$

The energy stored in the tube per unit length is given by:

$$dE_x = \tfrac{1}{2}\, m(dx/dt)^2 max = 2\pi^2 f^2 x^2$$

and from this it follows that the total energy dissipated in this mode is:

$$E_x = 2\delta_x \int_o^L 2\pi^2 m\, f^2 x^2 dl = 4\pi^2 m\, \delta_x\, f^2 x_o^{\,2} \int_o^L Q_x^{\,2}(l).dl \qquad \ldots (10)$$

Now from eqs. (1) and (7), it follows that

$$c_x(l) = 0.259\,\frac{x(l)}{d} = 0.259\,\frac{x_o}{d}.Q_x(l)$$

and if this is substituted into eq. (9) and the result put equal to eq. (10), one obtains the result:

$$\pi x_o\, \tfrac{1}{2}\rho u^2 d.0.259\,\frac{x_o}{d} \int_o^L Q_x^{\,2}(l).dl = 4\pi^2 m\delta_x f^2 x_o^{\,2} \int_o^L Q_x^{\,2}(l).dl \qquad \ldots (11)$$

from which it follows that

$$\frac{u}{fd} = 9.9 \sqrt{\frac{m\delta_x}{\rho d^2}}$$

It is seen that the effect of mode-shape cancels and the formula for critical speed is unchanged.

3.3 The effect of partial admission on the critical speed for a single span

The calculation of the previous section is easily extended to the case of partial admission. If the volume flux is such that with full admission the cross-flow velocity is u, then with admission restricted to that part of the span between l_1 and l_2, the cross-flow velocity will be increased to

$$u^1 = u \frac{L}{l_2 - l_1}$$

Then eq. (11) becomes

$$\pi x_o \tfrac{1}{2}\rho u^2 \frac{L^2}{(l_2-l_1)^2}\, d.0.259 \frac{x_o}{d} \int_{l_1}^{l_2} Q_x^2(1).dl = 4\pi^2 m\delta_x f^2 x_o^2 \int_o^L Q_x^2(1)\, dl \qquad \ldots (12)$$

and the formula for critical speed becomes:

$$\frac{u}{fd} = 9.9 \sqrt{\frac{m\delta_x}{\rho d^2}} \quad \frac{l_2-l_1}{L} \quad \left\{ \frac{\int_o^L Q_x^2(1)\, dl}{\int_{l_1}^{l_2} Q_x^2(1)\, dl} \right\}^{-\frac{1}{2}} \qquad \ldots (13)$$

In this case, the critical speed depends upon the mode-shape and on the length of tube to which the flow is restricted.

If the mode-shape is sinusoidal, with

$$Q_x(1) = \sin(\pi l/L) = \sin \pi\lambda$$

then the formula becomes

$$\frac{u}{fd} = 9.9 \sqrt{\frac{m\delta_x}{\rho d^2}} \quad \left\{ \frac{\lambda_2-\lambda_1}{(\lambda_2-\lambda_1) - \frac{\sin 2\pi\lambda_2 - \sin 2\pi\lambda_1}{2\pi}} \right\}^{\frac{1}{2}} \qquad \ldots (14)$$

This may be written as

$$\frac{u}{fd} = C\ 9.9 \sqrt{\frac{m\delta_x}{\rho d^2}} \qquad \ldots (15)$$

which, in conjunction with eq. (14) defines C, the correction factor for the effect of partial admission on the critical speed of a single span tube with a sinusoidal mode-shape. The factor has been calculated and is tabulated in Table 1.

3.4 The effect of partial admission on the critical speed of a multi-span tube

The problem of greatest interest with regard to heat exchanger design is that of a multi-span tube clamped at its ends and simply supported at the intermediate baffles, with partial admission in the first bay and full-span flow in all other bays. An estimate of the correction factor for this configuration can be made using the method of §3.3 if the modal shape is known, and a reasonable approximation for the fundamental mode is obtained by taking for the shape in the first and last bays the static deflection curve of a clamped-pinned beam with a uniformly distributed load, and for the intermediate bays a simple half

sine-wave. Thus, if all spans have length L, one can write the modal shape as:

$Q(\lambda) = 3.8476\ (2\lambda^4-3\lambda^3+\lambda)$ in the first and last bays, and

$Q(\lambda) = A \sin \pi\lambda$ for the intermediate bays, where A is chosen to make the slope continuous at the first and last baffles.

If the flow in the first bay is restricted to that part of the span between $\lambda_1 L$ and $\lambda_2 L$, the velocity in this bay is given by:

$$u^1 = \frac{u}{\lambda_2-\lambda_1}$$

With these results the correction factor for this case may now be shown to be:

$$C = \left[\frac{0.8926 + 0.61\ (N-2)}{\frac{F(\lambda_1,\lambda_2)}{(\lambda_2-\lambda_1)^2} + 0.4463 + 0.61\ (N-2)} \right]^{\frac{1}{2}}$$

where N is the number of spans (and equal to 2 or greater), and the function $F(\lambda_1,\lambda_2)$ is given by:

$$F(\lambda_1,\lambda_2) = 14.8 \left(\frac{4}{9}\lambda^9 - \frac{3}{2}\lambda^8 + \frac{9}{7}\lambda^7 + \frac{2}{3}\lambda^6 - \frac{6}{5}\lambda^5 + \frac{1}{3}\lambda^3\right)_{\lambda_1}^{\lambda_2}$$

This factor has been calculated for several cases; it is tabulated in Tables 2(a) and 2(b) for the cases of a tube with 3 spans and one with 15 spans.

3.5 The effect of turbulence on the critical speed

Analysis of the effects of turbulence on the critical speed has only just been started, nevertheless one can predict with some certainty that upstream turbulence will decrease the critical speed from the simplest of arguments. Thus, if the fluid velocity is written as $(\bar{u} + u^1)$, where u^1 is the fluctuating turbulent component, and it is assumed that the equation for critical speed still applies, one finds, before taking the square root in eq. (8), the result:

$$\frac{(\bar{u} + u^1)^2}{f^2d^2} = 97\ \frac{m\delta}{\rho d^2}$$

which gives, on taking the time average and re-arranging:

$$\frac{\bar{u}^2}{f^2d^2} = 97\ \frac{m\delta}{\rho d^2} - \frac{\overline{u^{12}}}{f^2d^2}$$

Although this is a rather simple-minded argument, it agrees with intuition.

4. Experimental Results

4.1 Fluidelastic Instability

The damping of each of the instrumented tubes in the array was first determined by disturbing the tube and recording the decay of the resulting free vibration. A typical value of logarithmic decrement was found to be 0.25.

With the array mounted in the wind tunnel, the dynamic deflections of selected tubes were measured as a function of airspeed. Typical response curves for tubes in the first, centre and last rows of an equilateral triangular lattice array (flow normal to base of triangle), of p/d ratio equal to 1.25, are shown in Figures 5-7. From the response curve for each tube, a threshold velocity, defined as the intersection of a tangent to the post-threshold response with the velocity axis, was determined. Each tube was found to vibrate at

its fundamental frequency which was in the range 35.5Hz to 36.5Hz.

From the data obtained in these tests, the value of the constant in eq. (8) was found to be 2.50 with a standard deviation of 0.06

4.2 Effects of partial admission and mode shape

The effect of restricting the airflow to part of the tube span has been measured for different configurations and compared with predictions from the theory given in Section 3.2. Because the mode shape of a cantilever is asymmetric, the effect of partial admission is clearly dependent upon the position of the flow blockage. The results from a test in which one third of the span measured from the tip was blocked off are shown in Figure 8. It is seen that the theory overestimates the new critical speed by about 21%. Other tests have been carried out with 50% blockage and in these the theory underestimated the new critical speed when the blockage was applied from the root, and the new critical speed when the blockage was applied from the tip lay beyond the capacity of the wind-tunnel.

4.3 Effect of Initial Turbulence

Further tests were undertaken with the turbulence grid, described in Section 2.5, positioned upstream of the tube array. Figures 5-7 show the effect of increased turbulence on tube response and it can be seen that in every case a decrease in threshold velocity resulted. The effect of the increased turbulence, however, was greatest on tubes in the first row for which a decrease in threshold velocity of 35% was obtained. For tubes in the last row the decrease was only 8%.

5. Discussion of Results

5.1 Fluidelastic Instability

The response curves shown in Figures 5-7 show the rapid increase in tube amplitude which occurs when the threshold velocity is exceeded. The severity of this phenomenon suggests that it is prudent to design a heat exchanger such that this condition is completely avoided. The reported value of 2.5 for the constant in eq. (8) is in general agreement with other workers [1] and adds to the evidence that the values of this constant pertinent to complete tube banks depend on the geometry of the array and are significantly lower than the value of 9.9 reported by Connors [2] for a single row of tubes.

5.2 Effect of partial admission

The theory presented for the effect of partial admission is, as far as the fluid mechanics of blockage is concerned, very elementary. Nevertheless, the experiments show that the direction of the change in critical speed is correct, and it seems likely that when the blockage is modest, estimates are not greatly in error, particularly if the blockage occurs where the modal deflection is greatest. However, it is clear that if the effect of partial admission is to be assessed with confidence, the characteristics of the flows around blockage plates (which in practice are impingement plates and, sometimes, baffle plates) must be investigated.

5.3 Effect of initial turbulence

As might be expected, additional turbulence in the approach airstream had a much greater effect on the tubes in the first row than those deeper into the bundle. Turbulence created by impingement plates and valves immediately upstream of heat exchangers could, therefore, render the tubes at the bundle entrance more prone to vibration.

Although initial turbulence consistently resulted in a decrease in threshold velocity, the shape of the tube response curves was not significantly altered. This result is contrary

to the findings of Southworth and Zdravkovich [3] who reported complete changes in the tube response curves. Their experiments were confined to two tube rows, whereas the present study utilises complete arrays of tubes.

6. Conclusion

The task of designing a vibration-free heat exchanger is a difficult one, particularly if the further condition is imposed that the design should not be excessively expensive. This paper has reported preliminary results from an experimental and theoretical programme in which the effects of certain features of practical installations on the critical speed for tube instability are being examined.

Acknowledgement

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[3] SOUTHWORTH, P.J. and ZDRAVKOVICH, M.M. "Effect of Grid Turbulence on the Fluidelastic Vibrations of In-Line Tube Banks in Cross Flow". Journal of Sound and Vibration (1975) 39(4), 461-469.

TABLE I

Correction factor for partial admission to a single-span tube with a sinusoidal mode-shape

λ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	λ_1
		1.25	.91	.78	.72	.71	.72	.76	.82	.90	1.00	0
			.49	.53	.55	.57	.61	.65	.72	.81	.90	0.1
				.32	.39	.45	.50	.56	.63	.72	.82	0.2
					.25	.34	.41	.48	.56	.65	.76	0.3
						.23	.32	.41	.50	.61	.72	0.4
							.23	.34	.45	.57	.71	0.5
								.25	.39	.55	.72	0.6
									.32	.53	.78	0.7
										.49	.91	0.8
											1.25	0.9
												1.0

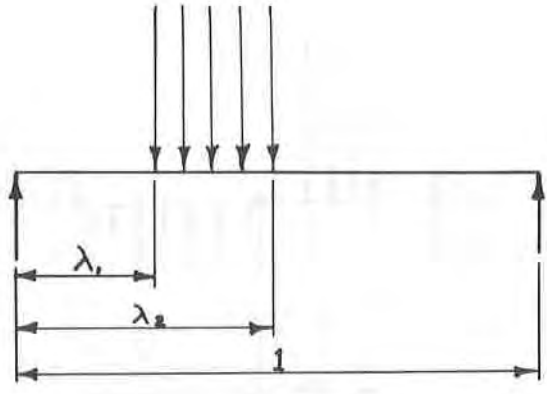

TABLE II(a)

Correction factor for partial admission to a multi-span tube clamped at its ends and with simple intermediate supports

Three equal spans

λ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	λ_1
		.99	.88	.83	.82	.82	.85	.89	.93	.97	1.00	0
			.61	.66	.69	.73	.77	.82	.88	.93	.97	0.1
				.44	.55	.62	.69	.76	.83	.89	.94	0.2
					.38	.51	.61	.70	.79	.86	.92	0.3
						.37	.52	.65	.76	.85	.92	0.4
							.41	.59	.74	.86	.95	0.5
								.50	.73	.89	.99	0.6
									.69	.94	1.06	0.7
										1.00	1.13	0.8
											1.18	0.9
												1.0

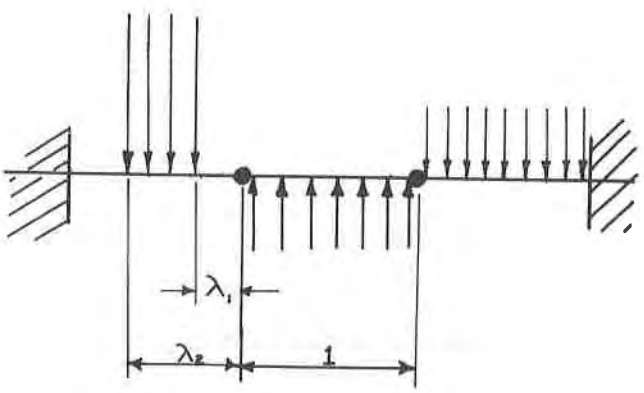

TABLE II(b)

Correction factor for partial admission to a multi-span tube clamped at its ends and with simple intermediate supports
Fifteen equal spans

λ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	λ_1
		1.00	.98	.96	.96	.96	.97	.98	.99	.99	1.00	0
			.88	.91	.92	.93	.95	.96	.97	.99	.99	0.1
				.77	.85	.89	.92	.94	.96	.98	.99	0.2
					.71	.82	.88	.92	.95	.97	.99	0.3
						.70	.83	.90	.94	.97	.99	0.4
							.73	.87	.94	.97	.99	0.5
								.81	.93	.98	1.00	0.6
									.92	.99	1.01	0.7
										1.00	1.02	0.8
											1.03	0.9
												1.0

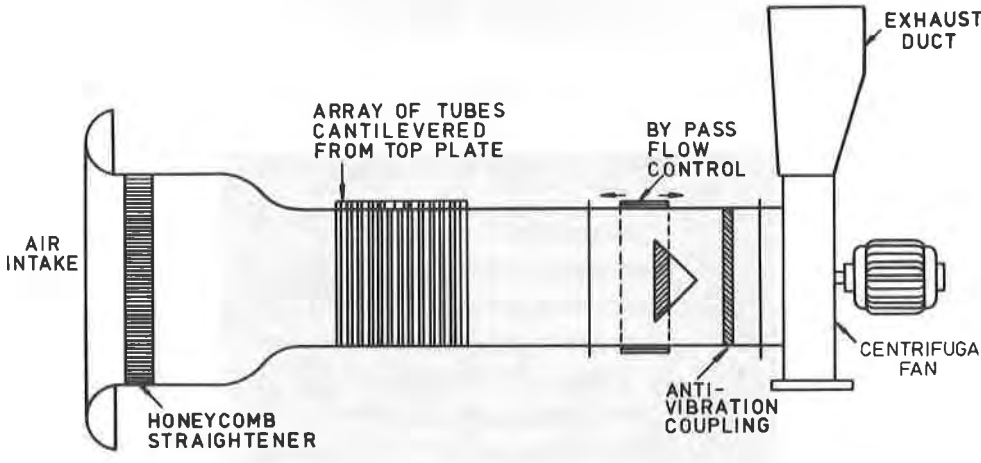

Figure 1 - The Test Facility .

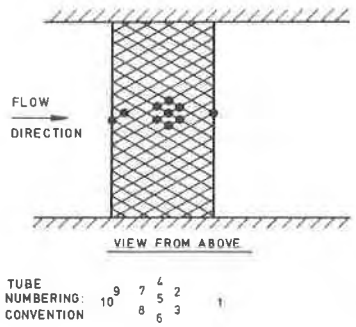

Figure 2 - Positions of instrumented tubes.

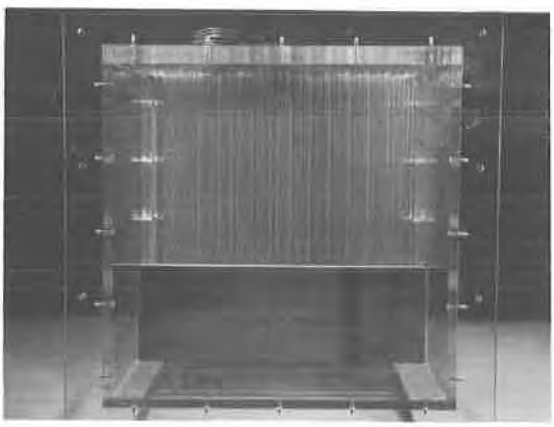

Figure 3 - Flow blockage plate in position.

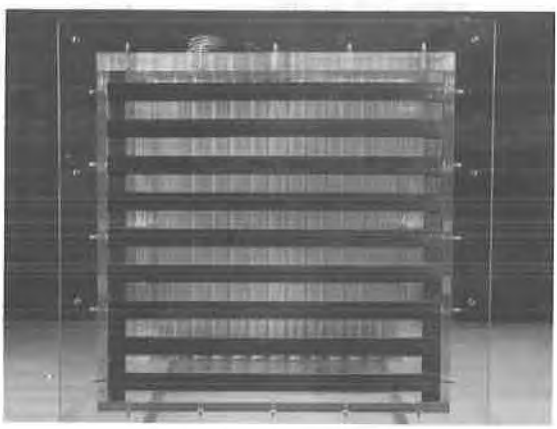

Figure 4 - Turbulence grid in position.

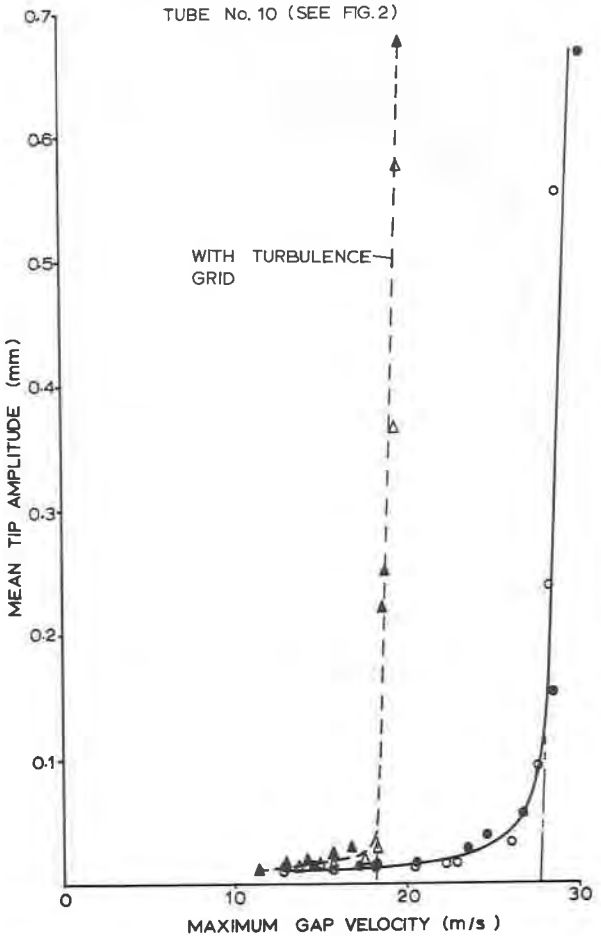

Figure 5 - Response of a tube in the first row.

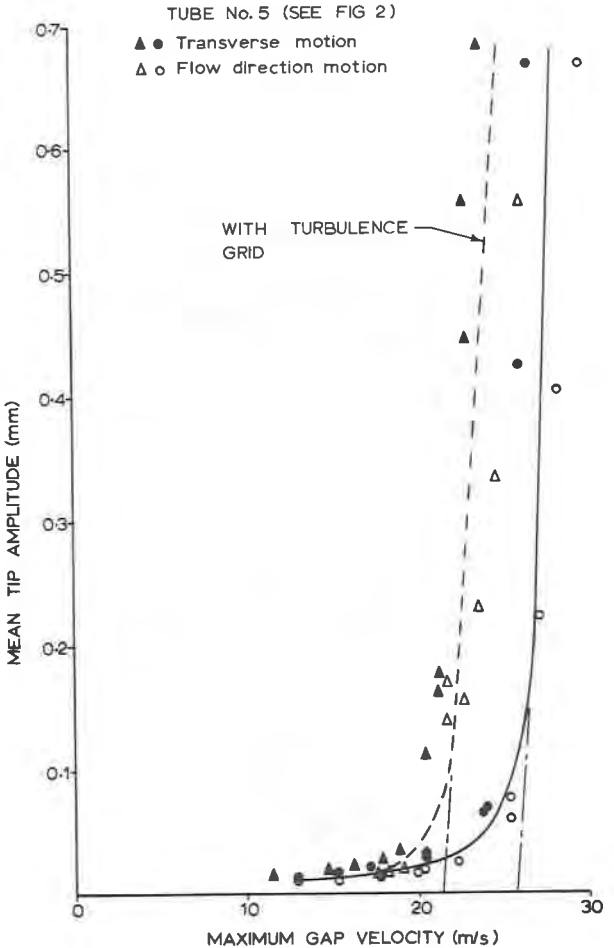

Figure 6 - Response of a tube in the centre row.

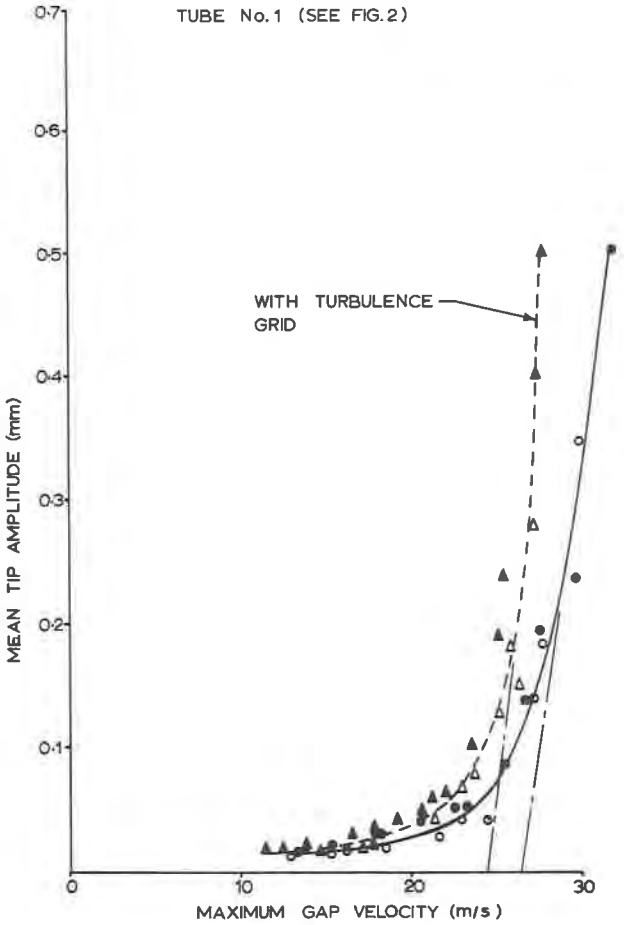

Figure 7 - Response of a tube in the last row.

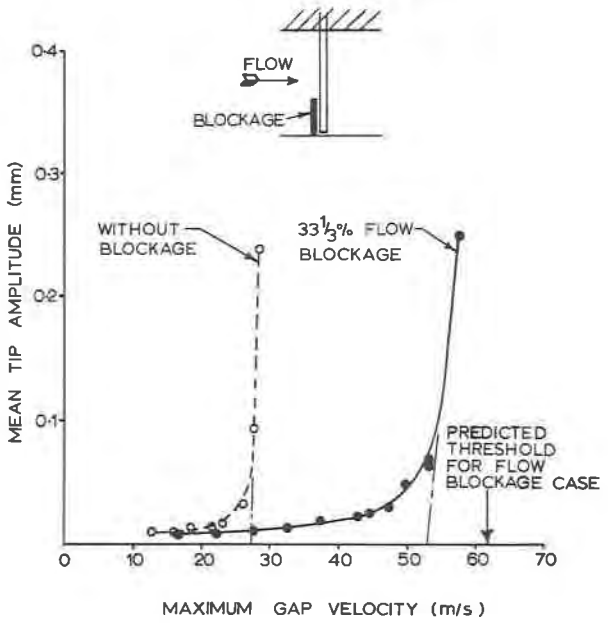

Figure 8 - Effect of partial admission on tube response.