Reference Stress Approach for Estimating Creep Strain of Flat Plate Pressure Vessel Heads

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Abstract

The reference stress method is a useful technique for obtaining creep strains because it requires less expense and labour than numerical methods using large size computers. Furthermore, it has an advantage that the effects of uncertainties contained in the material data can be minimized. In spite of these advantages, application of the reference stress method was limited only to simple structures such as straight pipes, beam structures and cylinders.

Thus, in the previous paper, the author had extended the reference stress method to be applicable for tapered cylindrical vessels by using the energy theorem and proved its applicability through experiments and numerical calculations.

In this paper, the reference stress method has been applied to flat plate pressure vessel heads.

The method is based on the fundamental relation that component deformation curves and reference test curves are geometrically similar to each other and the values of reference stress (σ_{Ri}) of several points(i) in the component can be obtained from this fundamental relation. These reference stresses can be used for estimating creep strains of the same component under various loadings, that is, reference stresses(σ^2_{Ri}) of the component under load P^2 are limited by $\sigma^1_{Ri} \times (P^2/P^1)$, where σ^1_{Ri} are reference stresses of the same component under load P^1.

In the previous paper, only the axial and circumferential strain components were considered. In this study, the Mises equivalent strain which is important in inelastic analysis has been considered as well as strain components and applicability of this method for the Mises equivalent strain has also been proved by the numerical method using the MARC program. Further, the applicability of this method for flat plate pressure vessel heads subjected to variable loadings has been proved through experimental techniques, using model vessels of aluminium alloy under various internal pressures at 150°C.

The stress distribution occurring in flat plate pressure vessel heads is very different from that of tapered cylindrical vessels, but creep deformation can be estimated precisely by this method in both cases. Therefore, the author's method is proved to be applicable to any type of pressure vessel in general use.

1. Introduction

In designing for elevated temperatures, precise estimation of the creep strain of components is always important in order to assure structural integrity.

Numerical methods such as the finite element method of non-linear time-dependent stress analysis can ascertain the creep strain of the component, but they require large computer atorage capacity and appropriate constitutive equations. Consequently, a lot of cost and labour is required.

On the other hand, simplified methods such as the energy theorem /1/, the reference stress method /2/, the inelastic flexibility factor method /3/, etc. have been developed and proved to be effective ways for ascertaining creep strains, but they also imply disadvantages or limitations.

(i) Geometry is limited to simple beam or cylinder.

(ii) Creep law is limited to simple equations like Norton's law.

(iii) Loading conditions or temperature conditions are restricted within narrow limits.

When we intend to design the component for nuclear use, we cannot avoid performing at least one finite element analysis for the main part of the component.

In this case, the problem of deciding for what loading condition the analysis should be performed, arises. For designers or operators of components, it is more important to know the actual creep strain in normal operating conditions than the hypothetical creep strain in accident conditions. Nevertheless, the analysis for accident conditions is performed in many cases to satisfy the code limits.

For upset conditions, we face another problem of how small load or temperature changes should be considered in the calculation of creep strain. It will be a troublesome calculation if we try to pursue the actual load history in detail.

For all of these cases, it will be very useful if we could estimate the creep strain caused in upset or accident conditions from the analysis of normal operating conditions.

Considering these circumstances of design, this paper intends to show a simplified method for estimating creep strain under any loading conditions, from the creep strain under one reference loading condition. The method is derived from the reference stress method and is applicable to any type of component.

At present, the creep law which can be considered in this method is limited to simple Norton's law. But, estimation up to about three times of reference creep strain is possible so that this method has little trouble in practical usage.

2. Theory

The author has already demonstrated that the reference stress method is derived from the energy theorem /4/.

The total rate of energy dissipation at a steady state is bounded from above by the creep energy dissipation rate associated with an equilibrium distribution of stress /1/.

Using this energy theorem and Norton's law as the creep law, the fundamental equation of the reference stress method is derived.

$$\frac{\dot{u}_c}{u_e} = \frac{\dot{U}_c}{U_e} \tag{1}$$

where $\dot{u}_c$: creep strain rate of uniaxial test specimen

u_e : elastic strain of uniaxial test specimen

$\dot{U}_c$: creep strain rate of component

U_e : elastic strain of component

This equation shows that the reference test curves and component deformation curves are geometrically similar under a stress level referred to as the reference stress.

From equation (1),

$$\frac{\dot{U}_c}{U_e} = EB\ \sigma_R^{\,n-1} \qquad (2)$$

where, E : elastic constant

B : creep constant

n : creep index

σ_R : reference stress

Therefore,

$$\sigma_R = \left(\frac{1}{EB} \cdot \frac{\dot{U}_c}{U_e}\right)^{\frac{1}{n-1}} \qquad (3)$$

By using this equation, the reference stress at any location of the component can be obtained from the initial elastic strain and creep strain rate at the same location of the component. This reference stress becomes a function of location, and is distributed in the component.

On the other hand, if the collapse load of the component is known, the reference stress can be obtained using a equation as below /2/.

$$\sigma_R = (P/P_L)\ \sigma_y \qquad (4)$$

where P_L : collapse load

σ_y : yield stress

This equation was derived from the fact that the stress distribution at the collapse of the component is similar to that which appears in the component whose material is assumed to have the creep index of $n = \infty$. Therefore, in strict terms, this equation is only valid when $n = \infty$.

In the case where the stress state does not reach the stationary state and the redistribution of stress takes place, equation (4) becomes

$$\sigma_R = k\ (P/P_L)\ \sigma_y \qquad (5)$$

where k is a constant expressing the degree of stress redistribution and depends on the magnitude of load. Therefore, the ratio of the reference stresses under load P^1 to P^2 is not proportional to the ratio of loads, but

$$\sigma_R^2/\sigma_R^1 = k^2P^2/k^1P^1 \qquad (6)$$

or

$$\sigma_R^2 = K\ (P^2/\ P^1)\ \sigma_R^1 \qquad (7)$$

where $K = k^2/k^1$

As equation (7) is valid for all kinds of components, the same relation is valid for the reference stresses which are distributed in the component, that is,

$$\sigma_{R_i}^2 = K\,(P^2/P^1)\,\sigma_{R_i}^1 \qquad (8)$$

where, suffix i (i = 0,1,2,-,-) denotes the location in the component.

Therefore, if we obtain the reference stresses ($\sigma_{R_i}^1$) of each location (i) under load P^1, and the reference stress (σ_{Ro}^2) of a reference point (o) in the component under load P^2 by using equation (3), we can obtain K and consequently the reference stresses of all locations (i) under load P^2 by using equation (8).

In this paper, equation (3) is used to obtain the reference stress of complicated components such as pressure vessels for which the collapse load is difficult to obtain. For simple structures, the reference stress is also obtained from equation (3). In the appendix, the reference stress of a beam in a simple bending obtained by equation (3) is described.

3. Verification by numerical analysis

The creep analysis of pressure vessels with flat plate head was performed by numerical analysis. The geometry of this pressure vessel is shown in Fig.1.

The material characteristics employed in the analysis are as follows.

Young's modulus	98100 MPa
Poisson's ratio	0.3
Creep law (t : hour)	$\dot{v} = 4.496 \times 10^{-17}\,\sigma^6$

These material characteristics do not represent a specific material, but were set as a typical creeping material. This creep law was determined so that a creep strain of about 2000×10^{-6} may be caused in 100 hours when $\sigma = 88.3$ MPa.

For the analysis, the general purpose finite element program MARC /5/ was used. The element used was an axisymmetric shell (Element Type No.15). The mesh division diagram is also shown in Fig.1.

The internal pressures considered in the analysis were 0.98 MPa, 1.08 MPa and 1.18 MPa. The creeping time was 99 hours in all cases. The elastic stress distribution is shown in Fig.2, which corresponds to an internal pressure of 0.98 MPa. Referring to this elastic stress distribution, locations to investigate the reference stress were selected, and shown as A to H in Fig.1.

The creep strain rates and corresponding reference stresses in each location for three loading cases are shown in Table I.

From table I, it is known that the ratio of $\sigma_{R_i}^1$ with respect to P^1 = 0.98 MPa to $\sigma_{R_i}^2$ with respect to P^2 = 1.08 MPa is almost constant and K defined in equation (7) falls in between 0.979 and 0.968.

The same relation holds true with P^1 and P^3. K is between 0.958 and 0.934. In this case, the value of $\sigma_{R_i}^3/\sigma_{R_i}^1$ is scattering from 1.121 at H(ℓ) to 1.150 at E(ℓ). If the value at E(ℓ) were 1.121 instead of 1.150, the creep strain rate would be 14.002 which may be said to be a better approximation compared with 15.902. Similarly, if the value at H(ℓ) were 1.150, the creep strain rate would be 4.6878 which is also a satisfactory value compared with 4.127.

As K does not exceed 1.0 in this case, the upper limit of the creep strain rate could be obtained if we assume K to be 1.0.

In Table II, the Mises equivalent creep strain rates and corresponding reference stresses in each location for three loading cases are shown. Here, the Mises equivalent elastic strain was obtained by $\bar{U}_e = \bar{\sigma}/E$, where $\bar{\sigma}$ is the Mises equivalent stress.

From Table II, the value of $\sigma_{R_i}^2/\sigma_{R_i}^1$ is almost constant and K is between 0.977 and 0.953. The value of $\sigma_{R_i}^3/\sigma_{R_i}^1$ is also constant and K is between 0.961 and 0.914.

In the case of Mises equivalent strains, the K range is greater than that of the strain component shown in Table I. But this is due to the small values of G and H, and we can omit these two locations because the stress of these locations is low and these locations are not important for design purpose. The K range in Table II becomes equal to that of Table I if we omit the K values at G and H. Therefore, we can estimate the Mises equivalent creep strain rate as well as the strain components.

The author has already demonstrated that this method is applicable for tapered cylindrical vessels. The stress distribution occurring in a flat plate pressure vessel head is considerably different from that of a tapered cylindrical vessel, but creep strain rate can be estimated by this method as described above. Therefore, this method is applicable to any type of pressure vessel being generally used.

4. Experimental approach

In the previous section, this method was proved to be applicable for pressure vessels under constant loads by numerical approach.

In this section, this method is proved for pressure vessels under variable loadings by experimental approach.

Using completely annealed aluminium alloy (A5052-0), three pressure vessel test specimens as shown in Fig.1 were machined out, where inside corner radius and outside corner radius at the junction of head plate and cylinder are 10 mm and 5 mm, respectively.

The constitutive equation of this aluminium alloy was determined as follows, as a result of uniaxial tensile creep test /4/.

$$\varepsilon_{total} = e + p + v$$

where

$$e = \sigma/47100$$

$$p = \sigma/2890, \qquad \sigma_y = 82.1 \text{ MPa}$$

$$\dot{v} = 4.226 \times 10^{-47}\,\sigma^{21.63}$$

The test temperature was set at 150°C.

The creep experiments using each of three test specimens were performed by applying a variable internal pressure with the aid of a hydraulic device.

The typical experimental result is shown in Table III. In this case, the internal pressure was varied in three steps; 1.13 MPa, 1.32 MPa and 1.42 MPa. The elastic strain includes an increase of strain due to the hardening effect.

It had been originally planned to measure the creep strain in all points from A to H in Fig.1. But, due to bad conditions such as high temperature, long time duration and large strain, many strain gauges were detached and the only significant data recorded is shown in Table III.

Fig.3 shows the result of creep curves which corresponds to Table III.

The estimation of creep strain rate at each load step is possible as discussed earlier.

The results of the estimation of the creep strain at C' and F, where A is used as a reference point, and the results of the estimation of creep strain at A and C', where F is used as a reference point, are also shown in Fig.3.

The difference between experimental and estimated creep strain is about 20%, which can be said to be a good approximation.

Note that the experiments were performed in the plastic region but equation (1) is still valid in this case as explained in ref. 4.

5. Conclusion

The author has extended the reference stress method to be applicable for estimating the creep strain of general components such as tapered cylindrical vessels or flat plate pressure vessel heads.

This method has been confirmed by numerical calculations and experiments.

6. Appendix

This appendix shows that the reference stress of a rectangular beam subjected to simple bending can be obtained by using equation (3).

The elastic strain and the creep strain rate at the outer fiber of the beam is obtained as follows /2/.

$$U_e = 3M/2Ebd^2$$

$$\dot{U}_c = B\left[\frac{(2n+1)}{2n}\left(\frac{M}{bd^2}\right)\right]^n$$

where b : width of the beam

2d : depth of the beam

Therefore, using equation (3), σ_R can be obtained as follows.

$$\sigma_R = \left[\frac{1}{2}\left(\frac{2n+1}{n}\right)^{\frac{n}{n-1}} \Big/ 3^{\frac{1}{n-1}}\right] \cdot \frac{M}{bd^2}$$

For $2 \leqq n \leqq \infty$, σ_R varies only between $1.042 \cdot \frac{M}{bd^2}$ and $1.000\frac{M}{bd^2}$. This reference stress is a very exact value, because, for this case, Mackenzie obtained $1.014\frac{M}{bd^2}$ as the reference stress and Ponter obtained $\frac{M}{bd^2}$ as the approximate reference stress which corresponds to $n = \infty$.

References

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/2/ P.K. Penny, D.L. Marriot "Design for Creep" McGraw Hill, 1971, pp. 122-156.

/3/ J.T. Boyle, J. Spence "The Analysis of Piping Systems for Creep" 4th-SMIRT, L2/7, 1977.

/4/ F. Matsuda, T. Fujikawa "Reference Stress Approach for Estimating Creep Strain of Tapered Cylindrical Vessels" International Conference on Engineering Aspects of Creep, C241/80, Sheffield, 1980.

/5/ P.V. Marcal : MARC-CDC General Purpose Finite Element Analysis Program, MARC Inc. 1978.

Table I. Results of numerical analysis of elastic strains, creep strain rates and reference stresses

Location (Direction)	Elastic Strain U_e (x 10^{-6})			Creep Strain Rate U_c (x 10^{-6})			Reference Stress σ_R (MPa)			σ_R^2/σ_R^1	σ_R^3/σ_R^1
	Case I	Case 2	Case 3	Case 1	Case 2	Case 3	Case 1	Case 2	Case 3		
A(ℓ)	875.6	963.1	1051	4.455	6.865	10.234	64.95	69.48	73.95	1.070	1.139
A(t)	878.1	965.9	1054	4.493	6.968	10.289	65.02	69.65	73.99	1.071	1.138
B(ℓ)	-730.9	-803.9	-876.9	3.503	5.451	8.199	64.18	68.79	73.35	1.072	1.143
B(t)	-733.4	-806.7	-879.9	3.541	5.485	8.309	64.27	68.83	73.50	1.071	1.144
C(t)	767.9	844.6	921.4	2.932	4.588	6.823	61.32	65.81	70.01	1.073	1.142
D(t)	-623.1	-685.4	-747.7	2.262	3.553	5.337	60.71	65.19	69.50	1.074	1.145
E(ℓ)	-1072	-1179	-1286	6.587	10.487	15.902	67.45	72.62	77.57	1.077	1.150
F(ℓ)	1204	1324	1445	8.225	12.936	19.313	68.89	74.00	78.79	1.074	1.144
G(ℓ)	-672.3	-739.5	-806.8	1.100	1.711	2.476	51.77	55.48	58.70	1.072	1.134
H(ℓ)	897.7	987.5	1077	1.942	2.932	4.127	54.74	58.32	61.37	1.065	1.121

Direction (ℓ) : Meridional direction, Direction (t) : Circumferential direction

Case 1 : Internal pressure P^1 = 0.98 Mpa
Case 2 : " P^2 = 1.08 MPa , P^2/P^1 = 1.1
Case 3 : " P^3 = 1.18 MPa , P^3/P^1 = 1.2

Table II. Results of numerical analysis of the Mises equivalent strains and creep strain rates, and reference stresses.

Location	Mises Elastic Strain U_e (x 10^{-6})			Mises Creep Strain Rate $\dot{\bar{U}}_c$ (x 10^{-6})			Reference Stress σ_R (MPa)			σ_R^2/σ_R^1	σ_R^3/σ_R^1
	Case 1	Case 2	Case 3	Case 1	Case 2	Case 3	Case 1	Case 2	Case 3		
A	1253	1378	1503	8.939	13.470	20.244	69.49	74.01	78.91	1.065	1.136
B	1046	1150	1255	7.159	10.780	16.433	68.92	73.39	78.46	1.065	1.139
C	962.4	1059	1155	4.306	6.389	9.519	63.30	67.20	71.53	1.062	1.130
D	758.3	834.1	909.9	3.352	4.962	7.350	63.15	67.01	71.24	1.061	1.128
E	1055	1160	1266	7.096	11.181	17.357	68.68	73.80	79.19	1.075	1.153
F	1173	1290	1408	9.133	14.193	21.643	70.72	75.78	81.02	1.072	1.146
G	684.9	753.4	821.9	1.816	2.537	3.456	57.01	59.80	62.52	1.049	1.097
H	879.6	967.6	1056	2.433	3.382	4.690	57.49	60.25	63.21	1.048	1.099

Case 1 : Internal pressure P^1 = 0.98 MPa
Case 2 : " P^2 = 1.08 MPa , P^2/P^1 = 1.1
Case 3 : " P^3 = 1.18 MPa , P^3/P^1 = 1.2

Table III. Results of experimental analysis of elastic strains, creep strain rates and reference stresses

Location (Direction)	Elastic Strain U_e (x 10^{-6})			Creep Strain Rate $\dot{U}_c$ (x 10^{-6})			Reference Stress σ_R (MPa)			σ_R^2/σ_R^1	σ_R^3/σ_R^1
	Load 1	Load 2	Load 3	Load 1	Load 2	Load 3	Load 1	Load 2	Load 3		
A(ℓ)	1460	1714	1841	4.736	12.734	14.970	79.60	82.86	83.22	1.041	1.046
C'(ℓ)	800	939	1009	1.895	4.245	5.988	78.39	80.89	81.96	1.032	1.046
D(ℓ)	2230	2618	2812	9.473	39.264	52.894	80.64	85.73	86.68	1.063	1.075

Direction (ℓ) : Meridional Direction

Load 1 : Internal pressure P^1 = 1.13 MPa
Load 2 : " P^2 = 1.32 MPa P^2/P^1 = 1.174
Load 3 : " P^3 = 1.42 MPa P^3/P^1 = 1.261

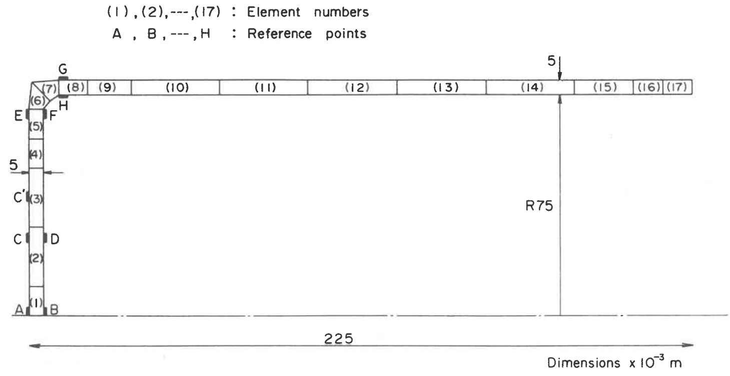

Fig. 1 Flat Plate Pressure Vessel Head

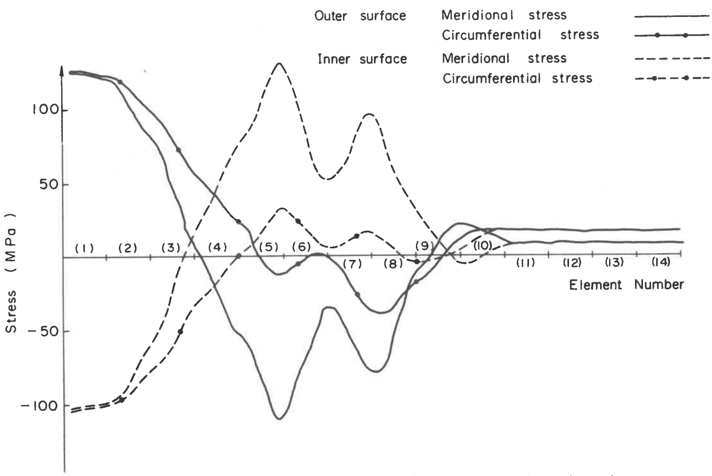

Fig. 2 Elastic Stress Distribution

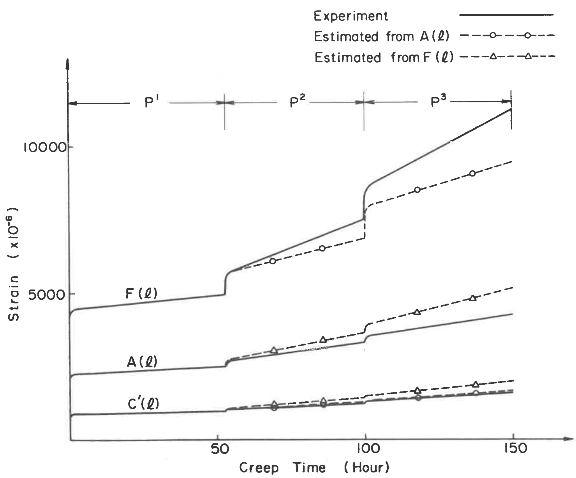

Fig. 3 Experimental Result of Creep Strains