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Evaluation of Crack Propagation under Thermal Striping Load Using Green's Function Approach

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ABSTRACT

In this study, an efficient numerical approach for the evaluation of crack propagation under a thermal striping load using the Green's function approach has been presented. The present approach for crack propagation analysis is based on multi Green's functions pre-determined for each stage of the incremental crack growth. The Green's function method(GFM) can be applied to evaluate not only thermal stresses for fatigue analysis, but also crack initiation to crack instability. The analysis results for crack propagation by GFM were in agreement with those of the actual observation for the piping structure subjected to a thermal striping load.

1. INTRODUCTION

Thermal fluctuations are the dominant sources of both high frequency fatigue and fracture on many components of a liquid metal fast breeder reactor (LMFBR). Calculating the service life of structures often involves an analysis of fatigue crack growth and requires accurate stress intensity factor (SIF) solutions to predict both the crack propagation rate and the fracture strength of the cracked body. When a crack is subjected to complicated mechanical loading, the corresponding SIFs are efficiently obtained using the weight function method[1-4]. Computation of SIFs for a cracked body subjected to a transient thermal loading by finite element method requires step-by-step computation for the entire time range. When a structure is subjected to thermal loads, the corresponding SIFs can be efficiently obtained using Green's function method(GFM). The Green's function separates influences of geometry and loading. Once Green's function for the geometry is known, the SIFs for the other loading cases can be easily determined. There have been studies on the analysis of the SIFs for stationary cracks under transient thermal loads using the GFM[5,6] and thermal weight function method[7]. In the present study the Green's function approach for the analysis of crack propagation has been presented under transient thermal loading.

The thermal striping phenomenon, which occurs due to an imperfect mixing of sodium streams with different temperatures is one of the most significant problems in a LMFBR. Thermal stresses arising from thermal striping can initiate surface cracks by high cycle fatigue. Through-the-wall failures of mixing tees have been reported several times in the test loop and LMFBR plant, which demonstrates this mechanism for component failure. As an example analysis with multi SIF Green's function, the thermal striping problem of the pipe tee junction was solved and the efficiency of the present approach has been demonstrated.

2. THEORETICAL BACKGROUND ON GREEN'S FUNCTION METHOD

2.1 Green's Function Concept for Evaluation of the SIF

The Green function is defined as the response of a system to a standard step or impulse input. The Green's function contains all essential information of the system when it is properly defined. Under the quasi-static thermoelasticity and fracture theory, the SIF in an elastic body is a unique function of temperature. Based on the Green's function concept and the Duhamel theorem, the SIF under mode I quasi-static thermal load can be expressed as following

$$K_I(t)=\int_0^t G_{K_I}(t-\tau)\frac{d\Theta(\tau)}{d\tau}d\tau \tag{1}$$

where τ, t are times, Θ(τ) is the time varying boundary condition and $G_{K_I}(t)$ is the SIF Green's function, which physically means the SIF of the cracked body due to a unit step change of the boundary temperature. As shown in Fig. 1, the Green's function, $G_{K_I}(t)$, is determined for a unit step load and always converges to a constant value after a "decay period". Therefore, the integration range can be shortened substantially only to the decay period,

$$K_I(t)=\int_{t-t_d}^t G_{K_I}(t-\tau)\frac{d\Theta(\tau)}{d\tau}d\tau. \tag{2}$$

2.2 Green's Function Approach for Crack Propagation Analysis

In the present study, the Green's function approach has been applied for the crack propagation analysis. The Green's functions should be pre-determined for each individual crack length. When the initial crack length is a_o and the amount of incremental crack growth is Δa, the SIF Green's function, G_i at i-th stage of crack growth is

$$\begin{array}{ll} G_o & \text{for} \quad a=a_o \\ G_1 & \text{for} \quad a=a_o+\Delta a \\ \cdots & \\ G_i & \text{for} \quad a=a_o+i\Delta a. \end{array} \tag{3}$$

Then the corresponding SIFs for the Green's function are

$$\begin{aligned} K_0(t) &= \int_{t-t_d}^t G_0(t-\tau)\frac{d\Theta}{d\tau}d\tau \\ K_1(t) &= \int_{t-t_d}^t G_1(t-\tau)\frac{d\Theta}{d\tau}d\tau \\ &\cdots \\ K_i(t) &= \int_{t-t_d}^t G_i(t-\tau)\frac{d\Theta}{d\tau}d\tau \end{aligned} \tag{4}$$

For fatigue crack propagation analysis under transient thermal load, the SIF ranges for each crack length should be determined. The general functional relationship for crack growth is

$$\frac{da}{dN}=f(\Delta K,R) \tag{5}$$

where $\Delta K \equiv K_{max} - K_{min}$, $R \equiv \frac{K_{max}}{K_{min}}$ and $\frac{da}{dN}$ is crack growth per each cycle. Fatigue crack propagation can be evaluated using the following Elber's equation or Forman's equation

$$\frac{da}{dN} = C(\Delta K_{eff})^m \tag{6}$$

$$\frac{da}{dN} = \frac{C\Delta K^{m-1}}{\frac{K_{crit}}{K_{max}} - 1} \tag{7}$$

where C and m are material constants, ΔK_{eff} is the effective SIF range and K_{crit} is the fracture toughness.

Using Eqs. (6) and (7), it is easy to estimate the fatigue crack propagation lifetime. The above procedure of crack propagation analysis by the present Green's function approach shortens the huge amount of computation time in numerical analysis such as FEM under a general thermal transient load in a cracked body.

2.3 Effects of Material Properties on Green's Function

The SIF Green's function should be determined at a specific temperature in order to apply the linear superposition principle. In general, the effect of varying temperature in the Green's function should be considered in performing the analysis as realistically as possible. However, it was shown that the effect of material properties is small due to the compensation of thermal and mechanical properties over the temperature range. As the temperature of 304SS increases from 300°C to 500°C which is the operating temperature range of LMFBR, Young's modulus(E) and density(ρ) decrease 10.17% and 1.09%, respectively while the thermal expansion coefficient(α), specific heat(C_p) and conductivity(K) increase 7.57%, 4.97% and 15.78%, respectively. That is, the properties of ρ, C_p and K interact with one another in heat transfer analysis while the properties of E and α interact with each other over the temperature ranges in stress analysis so that the temperature effect may be minimized. Considering that the actual temperature variations of the structure in LMFBR is usually within 200°C, calculating the SIF under transient thermal load with only one SIF Green's functions determined at the mean temperature will be reasonable. Fig. 2 shows an example of the Green's function determined at 200°C to 500°C for the present problem, which shows that the Green's functions are close to one another near the peak value where the contribution in Duhamel integration is high.

The validity of GFM for the SIF calculation of the cracked piping structure under triangular thermal loads for a temperature variation of ΔT=±45°C from the average temperature of 384.6°C with 0.033 Hz is shown in Fig. 3, which shows that the SIFs by GFM are in good agreement with those by standard FEM.

3. EXAMPLE ANALYSIS

The present example problem is for the evaluation of fracture integrity due to the thermal striping load in the tee junction of the LMFBR secondary piping system[8]. The mixing of two flows in the tee junction will induce high frequency thermal fatigue in the the piping. The sodium in a branch line flows into the main pipe of the secondary circuit as illustrated in Fig. 4. A small pipe, connected with a tee junction to the main pipe discharges sodium at 430°C into the main pipe. Two convergent flows with temperature differences of 90°C are mixed in the tee junction area. There is a circumferential weld at 160 mm downstream from the horizontal axis of the small pipe. The thermal striping damage is to be evaluated after 90,000 hours of operation. The material of the base metal for both pipes is AISI 304 stainless steel, while the weld material of the main pipe is 16Cr-8Ni-2Mo.

3.1 Finite Element Modeling

For the crack propagation analysis, an axisymmetric model with 1540 isoparametric quadratic elements for the heat affected zone of the welded joint as shown in Fig. 5 were used. In this analysis, the ABAQUS[9] was used for heat transfer, thermal stress and fracture mechanics analyses. The temperature in the pipe is assumed to be changed sinusoidally. The most damaging thermal striping frequency is usually 0.0l~l0Hz, depending on the size of the problem[10]. In the present analysis, the crack propagation analysis was carried out for the striping frequency of 0.1 Hz and circumferential crack at the location of A in Fig. 5.

3.2 Evaluation of Crack Propagation

The material constants of Eq. (6) for AISI 304 stainless steel are $C = 7.5\times10^{-13}$ and n = 4. The crack propagation analysis using GFM requires determination of the SIF range, ΔK as a function of the incremental crack lengths, and the fatigue lifetime can be easily determined by Eq. (6) or Eq. (7). The SIF Green's functions for each stage of incremental crack growth(Δa=0.5mm) from the initial crack length of 0.5 mm are shown in Fig 6. It is shown that the Green's functions increase as the crack propagates.

The variation of the SIFs at each stage of the crack growth under the sinusoidal loading with the striping frequency of 0.1 Hz are shown in Fig. 7. It is shown that the SIF increases as the crack propagates under the same pattern of the load.

In the meantime the effect of the temperature on the Green's functions is shown in Fig. 8, which shows that the maximum difference between the results based on the Green's function for 300°C and that for 500°C is estimated to be 1.89%, and the two results agree well with those of the standard FEM. It should be noted from Fig. 2 for the SIF Green's function that the results for 4 temperature cases agree well one another in the initial stage having high weight for the Duhamel integration in the calculation of the SIF.

The variations of ΔK during crack growth are shown in Fig. 9. The polynomial expression of ΔK as a function of crack length is expressed for the same sinusoidal loading as

$$\Delta K = 7.71 - 1853.12a + 1.58\times10^{6}a^{2} - 5.71\times10^{8}a^{3} + 1.08\times10^{11}a^{4} - 7.88\times10^{12}a^{5}.\ (MPa\sqrt{m}) \qquad (8)$$

The estimated lifetime from a=0.5 mm up to a=5 mm under the sinusoidal thermal load with the frequency of 0.1 Hz was 940.7 hours. As for the crack propagation for a>5.0 mm which is over 70% of the thickness, the validity of Paris law is uncertain because plastic deformation occurs throughout the remaining ligament. The instability analysis based on J-integral showed that the calculated tearing modulus at welded joint was 11.06 while the tearing modulus for this material at 427 °C based on the multiple-specimen J_R-curve procedure was 612[11]. Therefore, the crack will be arrested between 5mm and 7mm along the thickness direction. The estimation result is in agreement with that of the actual observation of the plant.

4. CONCLUSIONS

The Green's function approach using multi Green's functions for the crack propagation analysis was carried out. The effect of temperature on the Green's functions was investigated.

The Green's function determined at the mean temperature was shown to predict properly the actual behavior of the cracked body. The differences in SIF Green's functions between 300°C and 500°C was only 1.89% for the present example problem. It was shown that the present Green's function approach can perform efficiently the analysis for crack propagation as well as fatigue lifetime by simple numerical integration.

An example analysis of the mixing tee for LMFBR secondary piping under thermal striping load was carried out. The crack propagation analyses showed that crack would be propagated up to 5 mm through the thickness direction for 940.7 hours. The instability analysis with tearing modulus showed that the crack would be arrested at the location between 5 mm and 7 mm along the thickness direction.

Since the calculation with a simple integration in the present Green's function approach takes less than a second, it can be used for a real time fracture monitoring of the components in nuclear power plants or industries.

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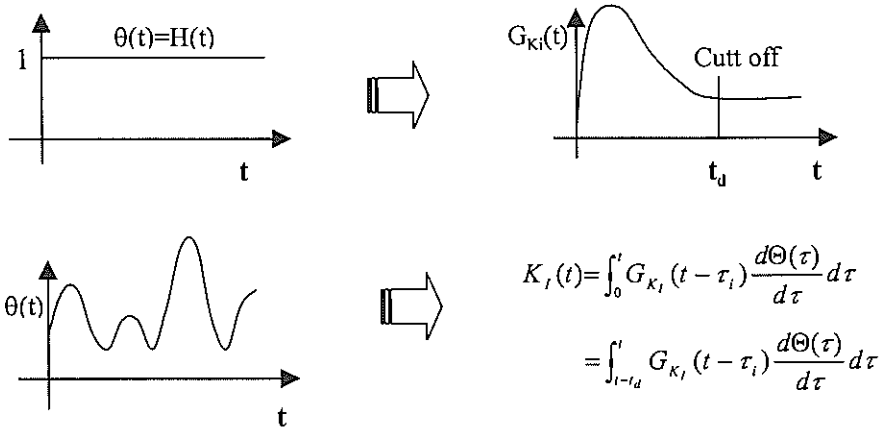


Fig.1 Concept of Green's function approach to fracture analysis

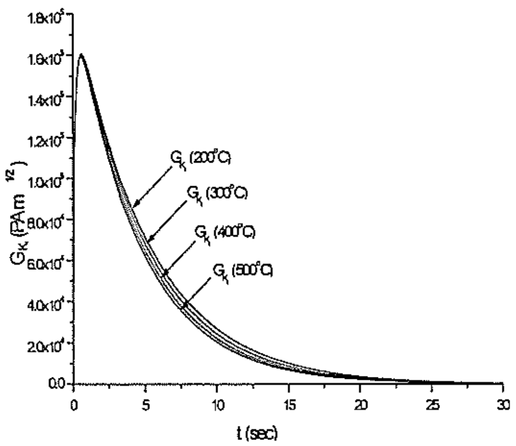
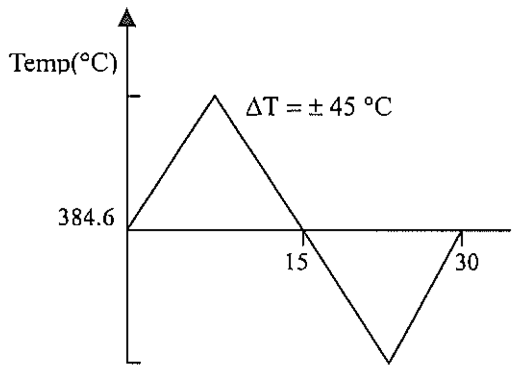
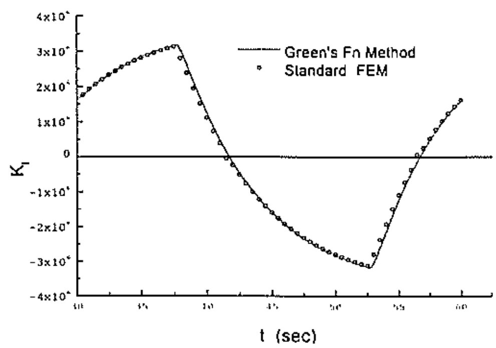


Fig. 2 The SIF Green's functions determined at various temperatures



(a) input thermal load



(b) Comparison of the SIFs by GFM and FEM

Fig. 3 Validation of the Green's function method in fracture evaluation

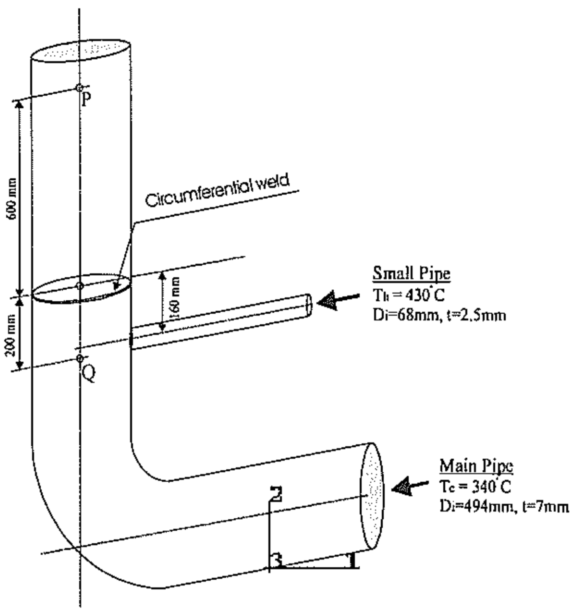


Fig. 4 Geometrical configuration of secondary piping

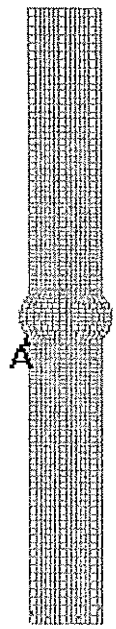


Fig. 5 Axisymmetric FE model of main pipe near welded joint

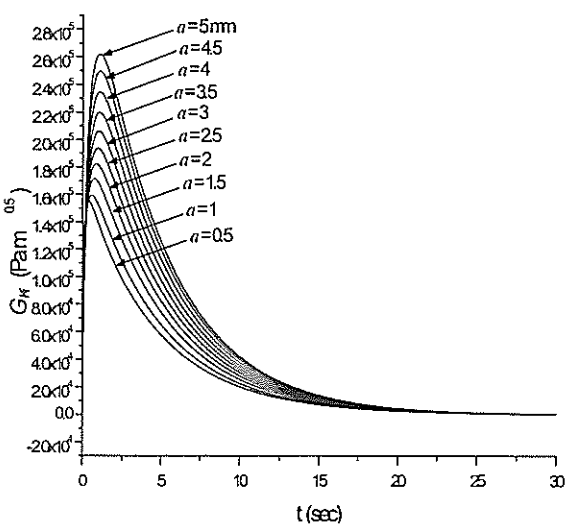


Fig. 6 Green's functions for the variable crack lengths

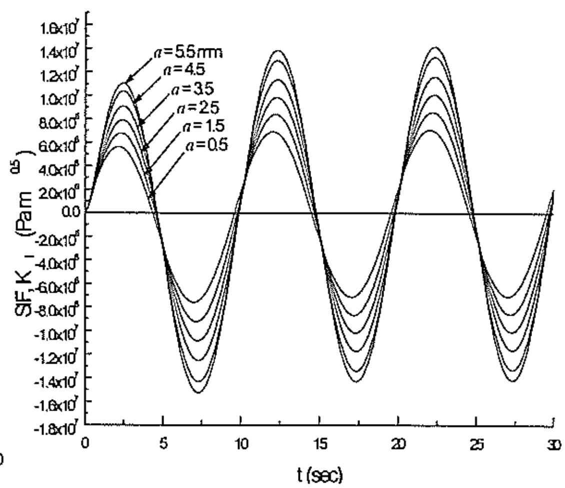


Fig. 7 Variations of the SIFs for incremental crack lengths under sinusoidal loading

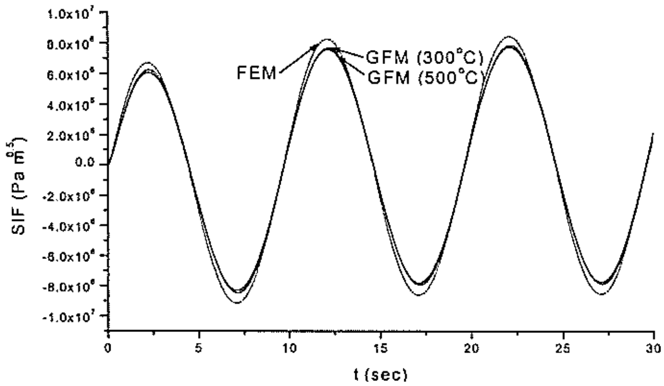


Fig. 8 Comparison of the SIFs by Green's function method at 300°C and 500°C.

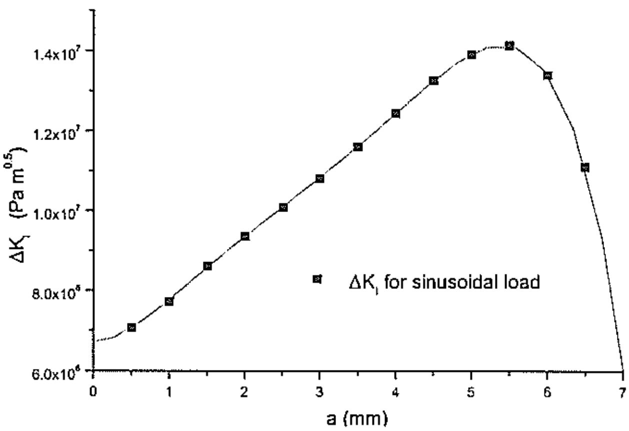


Fig. 9 Variation of stress intensity factor ranges during crack growth