A Fatigue Analysis of a Tube under Thermal Loading

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ABSTRACT

At JRC, experiments have been done in which a tube is repeatedly loaded by severe thermal shocks on the outer wall. This paper describes the analysis made in order to support these experiments. The temperature distribution is approximated by a one dimensional solution with a changing heat transfer coefficient. By varying the governing parameters of the problem, the best fit with the experimental result is obtained.

As the tests were designed to investigate amongst others the skin effect, it was necessary to calculate the reversed plastic strain behaviour arbitrary close to the wall where the shocks took place. Furthermore, the tests were done under generalised plane strain conditions. Therefore, no use could be made of elasto-plastic FEM codes and an ad hoc program was made, based on an analytical approach. The plasticity is evaluated by means of the sublayer model. After a number of shocks, the stress distribution in the tube stabilizes and the maximum reversed plastic strain is used to predict the number of shocks before failure using a Coffin law.

In order to be able to predict the crack propagation once it is initiated, K-factor curves in time are calculated for longitudinal cracks taking the change in stress distribution due to plasticity into account. Used is the code and method as described by Reynen.
As this code is elastic, the effect of the plasticity is taken into account by means of a modified temperature field. The temperatures and the strains coming from the previously mentioned ad hoc program are used. The FEM code used for the K-factor calculations gives the K-factor curves through the whole wall, at a certain time, in only one run. However, if crack closure occurs, due to residual stresses, the curve is constructed point by point.

INTRODUCTION

At JRC Ispra, experiments have been done in which a tube is repeatedly loaded by severe thermal shocks on the outer wall. The experiments and the results have been presented in /1/. In this paper, the analysis is described into more detail. First the crack initiation and afterwards the crack growth is discussed.

1. Crack initiation

As in the test pieces the generalized plane strain condition prevailed, that could not be taken into account by available FEM codes, an ad hoc semi-analytical computer program -SKIN2D- was made. This approach has also the advantage that the strains could be evaluated arbitrarily close to the wall.

1.1 Temperature analysis

As the crack initiation was initially considered to be governed by the skin effect, in which the geometry of the test piece does not play a role, the temperature distribution through the tube wall was approximated by a one-dimensional expression. Afterwards it turned out that this approximation was precise enough for the whole time range of interest, as compared to the measured temperature distribution.
Mathematically the problem is defined by:

- the diffusion equation
$$\lambda \frac{\partial^2 T}{\partial x^2} = \rho c \frac{\partial T}{\partial t} \tag{1}$$

- the boundary conditions
$$\frac{\partial T}{\partial x} = 0 \quad \text{for } t > 0 \text{ and } x = 0 \tag{2}$$
$$\frac{\partial T}{\partial x} = -h\,(T - T_m) \quad \text{for } t > 0 \text{ and } x = L \tag{3}$$

- the initial condition
$$T(t=0) = T_o - T_m \qquad 0 \leqslant x \leqslant L \tag{4}$$

where

λ	= conductivity	ρ	= density
c	= specific heat	T	= temperature
t	= time	x	= coordinate through the wall
L	= wall thickness	T_o	= wall temperature
T_m	= fluid temperature		

The solution of this problem in terms of a Fourier series is well known and can be found in handbooks on temperature

$$T(x, t) = T_m + \sum_{i=1}^{\infty} A_i \, e^{-k\beta_i^2 t} \cos \beta_i x \tag{5}$$

$$A_i = \frac{2(T_o - T_m)\sin\beta_i L}{\beta_i L + \cos\beta_i L \sin\beta_i L} \tag{6}$$

β_i are the roots of $\beta L \operatorname{tg}\beta L = \frac{hL}{\lambda}$ and $k = \frac{\lambda}{\rho c}$.

As in particular the heat transfer coefficient changes during the shock (due to an immerging and an arrest period), this solution was extended to take data changing a finite number of times into account. It turned out that the only difference with the standard solution is that the coefficients A_i (6) in the Fourier series had to be calculated according to

$$A_i^* = \frac{2(T_m - T_m^*)\sin\beta_i^* L}{\beta_i^* L + \sin\beta_i^* L \cos\beta_i^* L} +$$

$$\frac{2\beta_i^* L\{\frac{hL}{\lambda} - \frac{hL^*}{\lambda^*}\}}{\beta_i^* L + \sin\beta_i^* L \cos\beta_i^* L} \sum_{j=1}^{\infty} A_j \frac{\cos\beta_i^* L \cos\beta_j L}{(\beta_i^* L)^2 - (\beta_j L)^2} e^{-k\beta_j^2 t'} \tag{7}$$

where t' is the time at which the data change and $*$ denotes the new data, β_i and β_i^* are the roots of the same equation but with different data.

It might happen that β_i and β_i^* are equal, so that this contribution in the summation becomes undefined. In this case, the term in the series is calculated as

$$\frac{A_i}{2} \frac{\sin\beta_i L \cos\beta_i L}{\beta_i L + \sin\beta_i L \cos\beta_i L} e^{-k\beta_i^2 L}$$

In the test, the reheating of the tube (after it was pulled out of the sodium) was done very slowly and both at the inside and outside. As this process could not be simulated by the simple model, three different approximations were tried and compared with each other. No significant differences in the stress-strain distribution and reversed plastic strains could be noticed. The three approaches are:

a) after the cooling phase, instantaneously back to the initial temperature;

b) modifying the $\lambda/\rho c$, h^* and T_m^* in such a way that the heating took place over a time range similar to that used in the experiments;

c) the heating was done like the cooling but in the reversed way.

As there is no preference between the three methods, method c) has been used in all the calculations. The temperature distribution has been compared with a FEM solution and a good agreement was obtained.

A series of parametric calculations were run, in which the heat transfer coefficients were changed in several ways. The best fit as compared to the experimental results was found by changing the heat transfer coefficient after 0.14 sec from $h = 30\ kW/m^2 {}^oC$ to $h = 15\ kW/m^2 {}^oC$. We refer to /1/ for other data.

Fig. 1 shows the temperature distribution through the wall and in Fig. 2 the temperature history at three distances from the outer wall are given.

1.2 Stress-strain analysis

For the generality, the stress-strain analysis has been done in two dimensions which also allows the analysis of other types of problems such as ratcheting a.o. The two equilibrium conditions in cylindrical coordinates are

$$\frac{\partial \sigma_\rho}{\partial \rho} + \frac{\sigma_\rho - \sigma_\varphi}{\rho} = 0 \tag{8}$$

$$\int_{R_1}^{R_2} \sigma_z \rho \, d\rho = F \tag{9}$$

where

σ_ρ, σ_φ and σ_z are the stress components, R_1 and R_2 are the inner and outer radius of the tube, F is the axial force, ρ is radial coordinate.

These equilibrium equations are solved in the way indicated in /2/, besides that in the Hooke's law the plastic strain components and the temperature terms are present. The generalized plane strain condition

$$\frac{\partial w}{\partial x} = \text{constant}$$

is eliminated from the expressions for the stresses by applying (9). Due to space limitations, only the result for the σ_z component is given here

$$\sigma_z = \frac{E}{1-\nu^2}\left[\nu \int_{R_1}^{\rho} \frac{\dot\epsilon_\rho - \dot\epsilon_\varphi}{\rho} d\rho - (1+\nu)\alpha T - (1-\nu)\dot\epsilon_z + \nu \dot\epsilon_\rho + \right.$$

$$\left. + \frac{z}{R_2^2 + R_1^2}\left\{(1+\nu)\int_{R_1}^{R_2} \alpha T \rho \, d\rho + \frac{2-\nu}{2}\int_{R_1}^{R_2} \dot\epsilon_z \rho \, d\rho - \frac{\nu}{2} R_2^2 \int_{R_1}^{R_2} \frac{\dot\epsilon_\rho - \dot\epsilon_\varphi}{\rho} d\rho \right\}\right] \tag{10}$$

where $\dot\epsilon_\varphi$, $\dot\epsilon_\rho$, $\dot\epsilon_z$ are plastic strain components.

The other two stress components have a similar expression. The integrals are calculated by means of a Simpson rule in which the increments in the coordinates have a different size to allow a refinement at places where the gradient in the stresses is steeper. As the material of the test pieces showed a kinematic hardening behaviour, the plastic strains are evaluated using a parallel sublayer model, in which the total stresses are calculated as the weighted sum of all layers.

The normality rule is used for the incremental plastic strains and the total values which have to be introduced in (10) are found as the sum of them.

By means of an iteration through (10) - which expresses the equilibrium condition - the final values for the plastic strains are obtained. Usually no more than 6 iterations are required. After about 3 to 4 loading cycles, the stability in the stress-strain distribution was reached. This situation (for sodium as a cooling medium) is given in Fig. 3. For the data used, see /1/. The stress-strain law is used as a monotonic bi-linear curve with a hardening slope of half the value of the slope in the cyclic stress-strain relation. From experiments it was found that, for this material, the hardening was only a function of the strain range (independent from the total strain) if the number of cycles was large enough. Due to the fact that throughout the whole tube wall the minimum strain during a cycle is almost zero, the strain range is about equal to the total strain. Except the way of using the stress-strain relation, the analysis was performed in the sense of the ORNL recommendations /6/. Besides the tests in sodium, another test was run with water as a cooling medium. In this case, the order of magnitude of the number of cycles before failure could be predicted, using the maximum reversed plastic strain at the outer wall. Failure occurred between 5000 and 10000 cycles, the predicted number was about 10000. For the tests in sodium, the prediction was that failure could not be expected before 50,000 to 100,000 cycles, using data from /3,4/. The tests were stopped after 77,000 cycles and no crack initiation had taken place.

2.1 Crack propagation

In order to investigate the crack propagation behaviour of the tube, K-factors were needed to be introduced in a Paris law. As a complete elastic-plastic cyclic FEM calculation was not feasible, due to the complexity of the problem and doubtful final results, an "approximate" method was applied. Reynen /5/ has reported a very attractive way to calculate the linear elastic intensity factors. He has modified a Bersafe code in such a way that it can handle substructures and, moreover, it can calculate the crack closure work along a defined path in only one run. A post processing program calculates from this crack closure work the stress intensity factors.

The same code is used here, in a way that the plastic strains due to thermal shock are taken into account. The axial crack in the tube was modelled by 27 constant strain triangles over the wall thickness (Fig. 4). In circumferential direction, half of the cross section was modelled using substructures. The effect of the plasticity caused by the thermal loading, was introduced in this elastic code by means of a modified temperature field,

$$T^{*} = T + \frac{\overset{\bullet}{\epsilon}_z}{\alpha} \tag{11}$$

where T^{*} is the modified temperature, T is the temperature calculated in SKIN2D and $\overset{\bullet}{\epsilon}_z$ is the plastic strain component in the uncracked tube.

In order to check this approach, the stress distribution in the uncracked tube calculated by FEM and by the SKIN2D program have been compared for several time steps. The crack closure work is calculated by uncoupling the nodes along the crack path; this is automatically done by the program for the whole crack in one run.

For times close to zero, large residual stresses are present in the tube wall, so that the crack does not open. The calculated work, however, is always positive. In these cases the crack has been modelled by uncoupling the nodes one after the other in separate runs to see if the crack really opens or not. The obtained K-curves are given in Fig. 5.

As the code is linear elastic, the plasticity caused by the presence of the crack is not taken into account by this procedure. This plasticity due to the realignements of the stresses in the remaining lignement, can become very important for larger crack lengths. To have an idea of this effect, an approximate calculation has been done of the change in K-factors. It turned out that till the point where the reversed plasticity occurred, at the opposite side of the crack, the difference in K-factors is less than 15%. The experimental results for the crack growth could not be explained by a Paris law relation, which indicates that the crack growth must be considered strain controlled rather than stress controlled /7/.

3. Conclusions

The presented analytical approach has proven to give a good picture of the stress-strain behaviour in a tube wall under cyclic thermal loading. Parametric studies could be performed in a minimum of time (less than 20 sec CPU for three loading cycles). Although not mentioned in this paper, the influence of an axial force could be analysed too. The stress intensity factors are calculated in an elastic way using the stress field in the plastified tube wall (which is completely different from the elastic one).
The method is believed to give good results up to 60 - 70% of the wall thickness.

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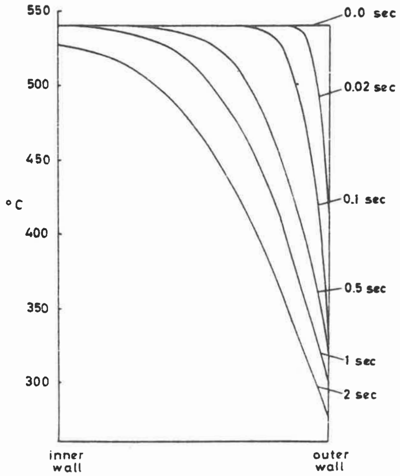

Fig.1 Temperature distribution in the tube wall for several times after the shock

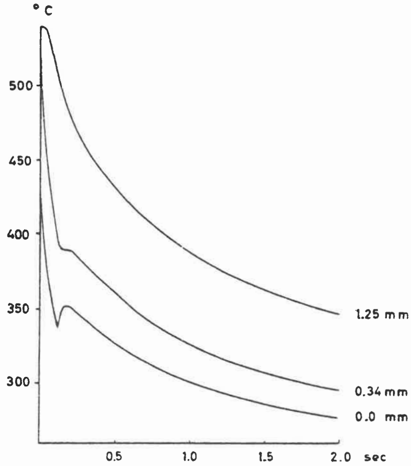

Fig. 2 The temperature history at 0.0, 0.34 and 1.25 mm from the outer wall during ½ a cycle

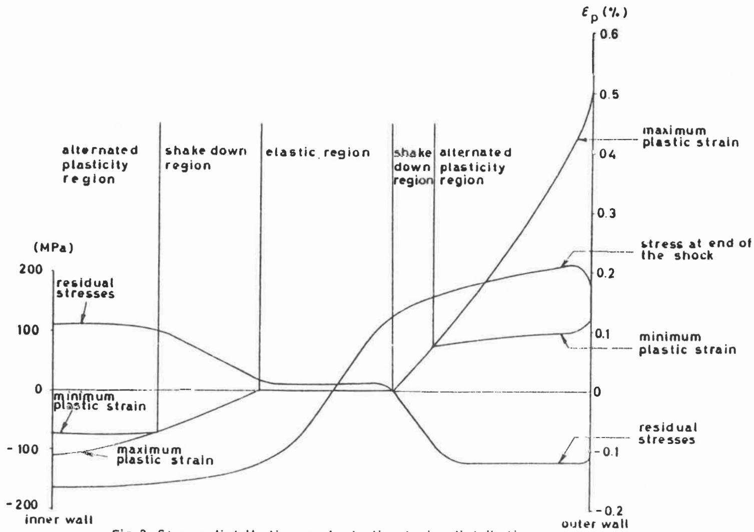

Fig. 3 Stress distribution and plastic strain distribution in the tube wall during a shock (uncracked tube)

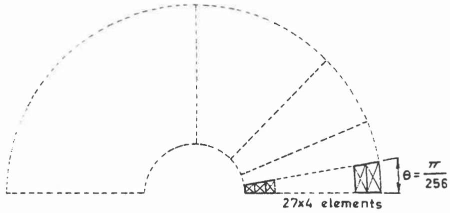

Fig.4 Substructure model for K-factor calculation

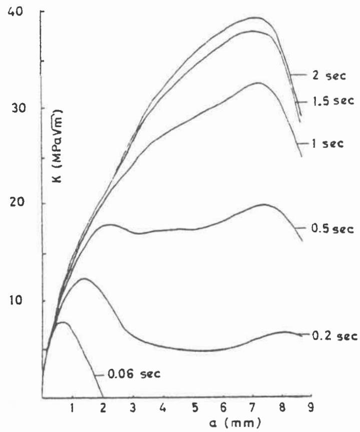

Fig. 5 Stress intensity factor as a function of crack depth for a number of times after the shock