Vibration of Thick Plates with Damping

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ABSTRACT

In this paper, a vibration equation of elastic thick plates with external and inner damping is deduced from Hamilton's principle. The general expressions for responses of thick plates with damping under arbitrary loads are given. The generalized coordinates are solved by using the third B-spline function. The numerical results for rectangular plates are given herein. The results are compared with those obtaind from the improved theory and the classical theory of plates.

INTRODUCTION

The structures with thick plates have been used extensively in national defence, mechanical engineering, chemical engineering, nuclear engineering, civil engineering, etc. Various theories have been established to deal with the problems of elastic plates, which include the classical theory of thin plates, the improved theory of thick plates, three-dimensional elastic theory. The classical theory cannot be expected to hold for plates, whose thicknesses are large with respect to the width. It also cannot be expected to hold when the wavenumber is large and the distribution of loads is nonuniform. However, it is difficult to obtain the analytical solutions for plates with various boundary conditions on the basis of three-dimensional elastic theory. Recently, more attentions have been paid to the improved theory, in which some of the suppositions in the classical theory are neglected and the effects of rotatory inertia and shear deformation are retained. It is applied more extensively than the classical one.

In this paper, the damping effects are considered. The vibration equation of elastic thick plates with external and inner damping are deduced from Hamilton's principle. General expressions of the dynamic responses of thick plates with damping under arbitrary loads are given. The generalized coordinates are solved by using the MSC. The numerical results for rectangular plates subjected dynamic loads are given herein. The results are compared with those obtained from the improved theory and the classical theory of plates.

VIBRATION EQUATION OF THICK PLATES WITH DAMPING

The essential viewpoint of Donnell's modification of the classical deflection method for plates is that the deflection w_s caused by the transverse shear strains would be added to the bending deflection w_f obtained from the classical theory for plates, making the total deflection of the middle plane of the plate, i.e.

$$w_t = w_f + w_s \tag{1.1}$$

Then the relationship between w_f and w_s is made

$$w_t = [1+L']w_f \tag{1.2}$$

From Hamilton's principle we can obtain the vibration equation of elastic thick plates with damping

$$[\rho hL \frac{\partial}{\partial t^2} + (c^* \rho hL + c\nabla^2 D\nabla^2)\frac{\partial}{\partial t} + \nabla^2 D\nabla^2]w_f = [1+L']q(x,y,t) \tag{1.3}$$

where, the operators $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $L' = -\frac{(8-3\nu)h^2}{40(1-\nu)}\nabla^2$, $L = [1+L']^2 - \frac{h^2}{12}\nabla^2$, c^* and c are the external and inner damping factors, ν is Poisson's ratio, h is the thickness of the plate, ρ is the mass per unit volume, D is the flexural rigidity of the plate, q is the transverse load. For convenience we make

$$q_t(x,y,t) = [1+L']q(x,y,t) = q(x,y,t) + q_s(x,y,t) \tag{1.4}$$

GENERAL EXPRESSIONS OF THE DYNAMIC RESPONSES OF THICK PLATES WITH DAMPING

Assuming that

$$w_t(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} [1+L']W_{mn}(x,y)\ T_{mn}(t) \tag{2.1}$$

$$q_t(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} [1+L']W_{mn}(x,y)\ q_{mn}(t) \tag{2.2}$$

From (1.3), (2.1) and (2.2) and after some calculation we can obtain

$$T_{mn}(t) = e^{-\xi_{mn}\omega_{mn}t}[T_{mn}(0)\cos(\omega'_{mn}t) + \frac{T_{mn}(0) + \varepsilon_{mn}T_{mn}(0)}{\omega'_{mn}}\sin(\omega'_{mn}t)]$$
$$+ \frac{\omega^2_{mn}}{\rho h\omega^2_{mn(c)}}\int_0^t e^{-\xi_{mn}\omega(t-t_1)} q_{t_{mn}}(t_1)\sin[\omega'_{mn}(t-t_1)]\,dt_1 \tag{2.3}$$

where

$$\zeta_{mn} = (c^*+c\omega^2_{mn})/2\omega_{mn} \tag{2.4}$$

$$\varepsilon_{mn} = \xi_{mn}\omega_{mn} \tag{2.5}$$

$$\omega_{mn} = \omega_{mn}\sqrt{1-\zeta^2_{mn}} \tag{2.6}$$

$$q_{t_{mn}}(t) = \int_0^a\int_0^b [q(x,y,t)+q_s(x,y,t)]W_{mn}(x,y)dxdy \Big/ \int_0^a\int_0^b [W_{mn}(x,y)]^2 dxdy \tag{2.7}$$

For thick rectangular plates with simply supported edges we have

$$W_{mn}(x,y) = \sqrt{2/a}\,\sin(m\pi x/a)\ \sqrt{2/b}\,\sin(n\pi y/b) \tag{2.8}$$

$$\omega_{mn(c)} = \frac{\pi^2}{a^2}\sqrt{\frac{D}{\rho h}}\,(m^2+\frac{a^2}{b^2}n^2) \tag{2.9}$$

$$\omega_{mn} = \omega_{mn(c)}\{1+\frac{\pi^2h^2}{a^2}(m^2+\frac{a^2}{b^2}n^2)[\frac{1}{12}+\frac{8-3\nu}{20(1-\nu)}$$
$$+(\frac{8-3\nu}{40(1-\nu)})^2\ \frac{\pi^2h^2}{a^2}(m^2+\frac{a^2}{b^2}n^2)]\}^{-1/2} \tag{2.10}$$

For thick rectangular plates with other edges we can introduce the general variational principle.

Substituting (2.4) into (2.1) we can obtain the general expressions of the dynamic responses of thick plates with both external and inner damping.
In the case that a thick plates is subjected to a concentrated load at (x_p,y_p), i.e.

$$q(x,y,t) = P(t)\delta(x_p,y_p) \tag{2.11}$$

In order to calculate $L'q(x,y,t)$ in (1.3), i.e. $\nabla^2 q(x,y,t)$, it is necessary to apply the generalized function. Thus we have

$$q(x,y,t) = P(t)[1+L']W_{mn}(x,y) \tag{2.12}$$

Substituting (2.2) into (2.3) and using (2.1) we can obtain the solution in this case.

SOLUTION ON THE GENERALIZED COORDINATES BY USING THE MSC

In order to solve the dynamic responses of structures it is necessary to solve the differential equation

$$\ddot{T}_{mn}(t) + 2\xi_{mn}\omega_{mn}\dot{T}_{mn}(t) + \omega^2_{mn}T_{mn}(t) = P_{mn}(t) \tag{3.1}$$

We can solve (3.1) by the MSC (Method of Spline Collocation). The spline function as follows is used

$$\Omega_3(x) = \begin{cases} (x+2)^3 & x\in[-2,-1] \\ (x+2)^3 - 4(x+1)^3 & x\in[-1,0] \\ (2-x)^3 - 4(1-x)^3 & x\in[0,1] \\ (2-x)^3 & x\in[1,2] \\ 0 & |x|>2 \end{cases} \tag{3.2}$$

By using the MSC we can obtain

$$\tilde{T}_{mn}(t_{i+1}) = \tilde{A}_{mn}\tilde{T}_{mn}(t_i) + \tilde{B}_{mn}P_{mn}(t_{i+1}) \tag{3.3}$$

where
$$\tilde{T}_{mn}(t_{i+1}) = \begin{Bmatrix} T_{mn}(t_{i+1}) \\ \dot{T}_{mn}(t_{i+1}) \\ \ddot{T}_{mn}(t_{i+1}) \end{Bmatrix} \qquad T_{mn}(t_i) = \begin{Bmatrix} T_{mn}(t_i) \\ \dot{T}_{mn}(t_i) \\ \ddot{T}_{mn}(t_i) \end{Bmatrix}$$

$$\tilde{A}_{mn} = \begin{bmatrix} \frac{5}{6} - \frac{\beta_{mn}}{6}(L_{1,mn}+L_{2,mn}) & \left(\frac{2}{3} - \frac{\beta_{mn}L_{2,mn}}{6}\right)\Delta t & \frac{7+\beta_{mn}}{36}(L_{1,mn}-2L_{2,mn})]\Delta t^2 \\ \frac{-1-\beta_{mn}(L_{1,mn}+L_{2,mn})}{2\Delta t} & -\frac{\beta_{mn}L_{2,mn}}{2} & \frac{1+\beta_{mn}}{12}(L_{1,mn}-2L_{2,mn})]\Delta t \\ \frac{-1-\beta_{mn}(L_{1,mn}+L_{2,mn})}{\Delta t^2} & \frac{-2-\beta_{mn}L_{2,mn}}{\Delta t} & \frac{-5+\beta_{mn}}{6}(L_{1,mn}-2L_{2,mn})] \end{bmatrix}, \qquad \beta_{mn} = \frac{1}{L_{3,mn}}$$

$$\tilde{B}_{mn} = \begin{Bmatrix} \beta_{mn}\Delta t^2/6 \\ \beta_{mn}\Delta t/2 \\ \beta_{mn} \end{Bmatrix}, \qquad \begin{Bmatrix} L_{1,mn} \\ L_{2,mn} \\ L_{3,mn} \end{Bmatrix} = \begin{Bmatrix} 1 - \xi_{mn}\omega_{mn}\Delta t + \omega^2_{mn}\Delta t^2/6 \\ -2 - 2\omega^2_{mn}\Delta t^2/3 \\ 1 + \xi_{mn}\omega_{mn}\Delta t + \omega^2_{mn}\Delta t^2/6 \end{Bmatrix}$$

NUMERICAL RESULTS AND CONCLUTION

The calculations are performance for the rectangular plates with simply supported edges subjected three kinds of dynamic loads as shown in Fig.1-3, where the period of the fundamental vibration mode $t^* = 2\pi/\omega_n$, The modules of elasticity $E=2.06\times10^5 N/mm^2$, Possion's ratio $\nu=0.3$, height-span rations h/b = 0.05, 0.1, 0.2, 0.3, the length-width ratios a/b = 1.0, 1.25, 1.50, 1.75, the external damping $c'=0.98\times10^{-3}$, 1.96×10^{-3}, 2.94×10^{-3}, $4.9\times10^{-3}N\ sec/mm^3$ (where $c' = c^*\rho h$), the inner damping factor $c = 10^{-4}$, 2×10^{-4}, 3×10^{-4}, 5×10^{-4}sec. The results on the responses are given in Fig.4-12, where the full and dotted lines represent the nondimensional deflection w_t and w_f in the centre of the plate using the improved theory and the classical theory respectively. They are equal to w_t/w^* and w_f/w^* respectively, where w^* represents the static deflection in centre of the plate calculated by the classical theory. Fig.4-6, Fig.7-9, Fig.10-12 correspond to the cases without, with external, with inner damping respectively.

From the numerical results it is evident that the peak value of responses is decrease with the increase of damping, that the response with high frequency is more influenced by inner damping; conversely, the one with low frequency is more influenced by external damping, and that the differences between the theories are increased with the increase of the thickness-width ratio and with the decrease of the length-width ratio.

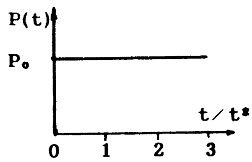

Fig. 1

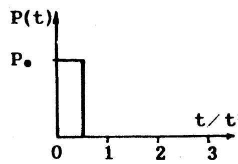

Fig. 2

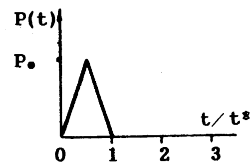

Fig. 3

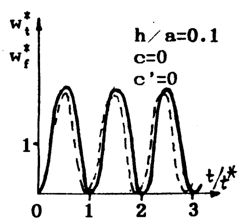

Fig. 4

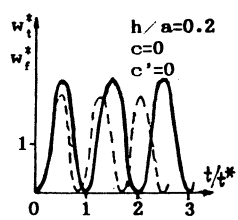

Fig. 5

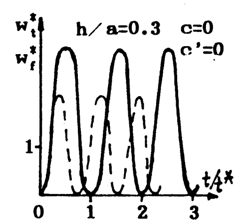

Fig. 6

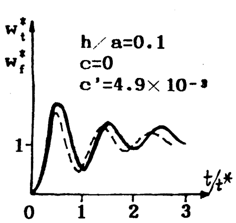

Fig. 7

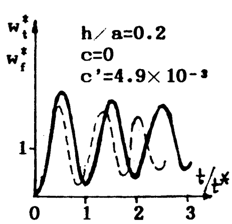

Fig. 8

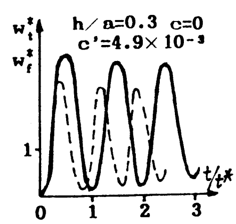

Fig. 9

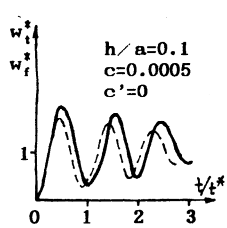

Fig. 10

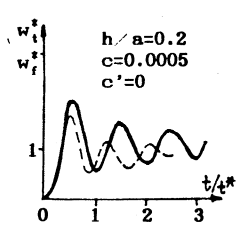

Fig. 11

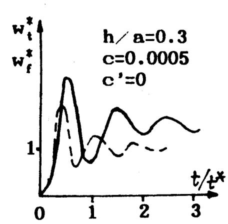

Fig. 12

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