Periodically Layered Composites for Mountings

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Abstract

Present work is a study of the dynamic response of periodically layered composites with an aim to establish their use as mountings. Some typical attenuation characteristics of such composites subjected to harmonic excitation are given. Attenuation of stress in an elastic bar with such a mounting is also demonstrated in this paper.

1. Introduction

Integrity of reactor components and structures against seismic, impact or other accident induced loads often warrants the use of suitable mountings for reduction of stresses or deflections, which otherwise would call for strengthening of such components. In case of seismic loading the problem is more severe for components at higher elevations where the floor acceleration may exceed the ground acceleration. Conventional mounts or shock absorbers, particularly for the large components are generally quite massive and space limitation may conceivably rule out their use. Again, the choice of materials to be used in the reactor environment is also restricted.

This motivates a search for simple and compact mountings relying not only on the commonly used principles of frequency dependent attenuation or dissipation in the structural mountings. Quite often it turns out that the requirements on the frequency of the mounted system for force attenuation and strength are conflicting. This suggestes a shift required in the basic mechanism of attenuating dynamic forces and stresses induced in a component or a structure. Analysis of periodically layered elastic and viscoelastic composites (Fig.1) reveals that such materials are potential attenuators of dynamic stresses and these can be used as mountings. The attenuation mechanisms in such media are the scattering and dispersion due to layering, which essentially splits a primary wave in time, into a refracted and a reflected wave. Thus, in a composite built of elastic constituents, the reduction of the stress is effected by partial transmission from an interface at a given instant of time.

The present work addresses the one dimensional problem only. This only highlights the efficacy of the composites in attenuating plane stress waves which is not possible in a homogeneous elastic medium.

There are some studies [1,2,3] on the propagation of plane stress waves normal to the layering of a periodically layered medium. However, the assumption of quasi-periodicity of the solution in [1] appears to be untenable for realistic boundary conditions, thus, such an analysis can be applied to infinite media only. The attenuation results given in [2] and [3] are applicable to very low and very high frequency excitation respectively.

Ref [4] presents an analysis of plane stress waves in periodically layered media which does not depend on the assumptions made in 1 or other similar works, is applicable to all types of loading and material combination and offers an exact solution. Some typical results of this analysis on the attenuation of harmonic stress waves are given in this paper.

Next, as an illustration of use of such composites as mountings, the stress response of a bar to harmonic excitation is presented.

2. Analysis of Periodically Layered Elastic Composites

2.1 Governing Equations

It is assumed that the dimensions are such that a one dimensional theory is applicable. The force equilibrium equation for the ith layer is

$$\frac{\partial \sigma_i}{\partial x} = \rho_i \frac{\partial^2 u_i}{\partial t^2} \tag{1}$$

x and t denote the space coordinate and time respectively. E_i, ρ_i, u_i and σ_i denote the Young's Modulus, density, displacement and stress respectively. For a linear elastic material,

$$\sigma_i = E_i \frac{\partial u_i}{\partial x} \tag{2}$$

Thus, the equation of motion for the ith layer is

$$\frac{\partial^2 u_i}{\partial t^2} = \frac{E_i}{\rho_i}\frac{\partial^2 u_i}{\partial x^2} = C_i^2 \frac{\partial^2 u_i}{\partial x^2} \tag{3}$$

where C_i is the longitudinal wave velocity in that layer.

2.2 Solution

Solution of equation (3) is obtained by first considering the Laplace transform with zero initial conditions yielding,

$$U_i = A_i e^{\frac{sx}{c_i}} + B_i e^{-\frac{sx}{c_i}} \tag{4}$$

where $U_i(s)$ is the transform of the displacement $U_i(t)$,

$$U_i = \int_0^\infty e^{-st} u_i(t)dt \tag{5}$$

and A_i and B_i are arbitrary constants. The transform $F_i(S)$ of the stress is then given by the following equation

$$F_i = E_i \frac{s}{C_i}\left[A_i e^{\frac{sx}{c_i}} - B_i e^{-\frac{sx}{c_i}}\right] \tag{6}$$

These solutions i.e. equations (5) and (6) are now applied to the constituent layers 2j-1 and 2j of the jth period, to obtain the transformed displacements and stresses at the two ends of this period in terms of the four constants A_{2j-1}, B_{2j-1}, A_{2j} and B_{2j}. Making use of continuity of displacement and stress at the interface of these two layers the constants of integration are eliminated and the end displacements and stresses are related by the transfer matrix $[T_j]$.

$$\begin{bmatrix} U_{j,R} \\ F_{j,R} \end{bmatrix} = \begin{bmatrix} T_j \end{bmatrix}\begin{bmatrix} U_{j,L} \\ F_{j,L} \end{bmatrix} \tag{7}$$

Subscripts R and L refer to the conditions at the right and left ends of the jth period. By continuity, again,

$$\begin{bmatrix} U_{j,L} \\ F_{j,L} \end{bmatrix} = \begin{bmatrix} U_{j-1,R} \\ F_{j-1,R} \end{bmatrix} \tag{8}$$

Thus, for a n-period composite where the constituent layers in each period are identical

$$\begin{bmatrix} U_{n,R} \\ F_{n,R} \end{bmatrix} = \begin{bmatrix} T \end{bmatrix}^n \begin{bmatrix} U_{1,L} \\ F_{1,L} \end{bmatrix} = \begin{bmatrix} T^{(n)} \end{bmatrix}\begin{bmatrix} U_{1,L} \\ F_{1,L} \end{bmatrix} \tag{9}$$

where

$$[T] = [T_j], \quad j = 1, 2, \ldots n \tag{10}$$

$[T]^n = [T^{(n)}]$ is obtained by Sylvester's theorem[5].

As a specific case, a composite built-in at the left end, x = 0 and excited by a force $f_R(t)$ per unit area at the right end x = l is considered. The transformed boundary conditions are,

$$U_{1,L} = 0 \tag{11}$$

$$F_{n,R} = \int_0^\infty e^{-st} f_R(t)\, dt \tag{12}$$

Thus, the stresses at the end of the jth period are obtained as,

$$F_{j,R} = \frac{t_{22}^{(j)}}{t_{22}^{(n)}} F_{n,R} \tag{13}$$

$$F_{j,L} = F_{j-1,R} = \frac{t_{22}^{(j-1)}}{t_{22}^{(n)}} F_{n,R} \qquad (14)$$

The time domain response is evaluated by inversion of the Laplace Transforms. For elastic composites, the poles of equations (13) and (14) are found to be simple, purely imaginary and occuring in pairs and the inversion is carried out by the method of residues.

2.3 Numerical Results

The response of composites to harmonic loading has been studied to obtain the attenuation of stress waves. The results are presented in terms of the nondimensional frequency ωT_s and the acoustic impedance ratio Z.

$$T_S = \frac{h_1}{C_1} + \frac{h_2}{C_2} \qquad (15)$$

and

$$Z = (\rho_2 E_2 / \rho_1 E_1)^{1/2} \qquad (16)$$

h_1 and h_2 are the thicknesses of the constituent layers. It is observed from Fig.2 that for harmonic loading the attenuation increases with frequency and the most significant variation is around ωT_s = 1 i.e. when the wavelength is of the order of the length of a period.

It is quite natural to expect increased attenuation with Z since higher the mismatch in the acoustic impedance, lower will be the transmission of the stress wave, a greater portion being reflected back. As pointed out earlier, the attenuation in an elastic composite is not by dissipation of mechanical energy but rather, it is achieved by splitting the incident wave in time - in a refracted and a reflected wave. Thus, it is also expected to have higher attenuation with increased number of layers, providing more interfaces. Fig.3 shows the effect of number of layers on attenuation. It may be seen that the increase in attenuation is marginal after the first few layers. This is in contrast to the predictions of other dispersion theories and is construed to be the effect of multiple reflections in a finite medium - a factor not accounted for in the earlier theories.

These results indicate that such composites may be used for attenuation of stresses in components and structures subjected to dynamic loading.

3. Analysis of an Elastic Bar with a Composite Mounting

The case of a homogeneous bar built-in at one end and excited by a force at the other end is considered. It is further assumed that the one-dimensional theory is applicable. Then it is known that the plane stress wave will propagate unattenuated through the medium, build up to twice its magnitude at the fixed end where it gets reflected and further history will be determined by the interference between the incident and reflected waves including those from the open end. The stress response when such a bar is interfaced with a periodically layered composite (Fig.4) is presented in this section.

3.1 Governing Equations

In the notation of the previous section if the overall transfer matrix of the homogeneous medium is called $[T_h]$ and that of the composite mounting is called $[T_m]$ then the overall transfer matrix $[T_c]$ of the combined system can be expressed in the following way.

$$\begin{bmatrix} U_R \\ F_R \end{bmatrix} = [T_m] \begin{bmatrix} U_I \\ F_I \end{bmatrix} = [T_m][T_h] \begin{bmatrix} U_L \\ F_L \end{bmatrix} = [T_c] \begin{bmatrix} U_L \\ F_L \end{bmatrix} \qquad (17)$$

U_I and F_I are the transformed displacement and stress at the interface of the mounting and the homogeneous bar. In the homogeneous section, the maximum stress will be developed at the built-in end. Hence, it is adequate to determine the stress at that point only. The boundary conditions are the same as those described by equations (11) and (12). The final solution for the stress $f_L(t)$ at the built-in end may be written as

$$f_L(t) = \sum_m \text{Res} \; \frac{e^{s_m t} F_R}{t_{22(c)}} \tag{18}$$

The stress $f_i(t)$ at the interface is obtained as

$$f_i(t) = \sum_m \text{Res} \; \frac{\cosh(sT_h) F_R e^{st}}{t_{22(c)}} \tag{19}$$

3.3 Numerical Results

Some typical results of response to harmonic excitation are presented in Figs.5 and 6. The attenuation due to the mounting is clearly seen. The corresponding material properties are shown in Table-I.

4. Conclusion

The problem considered in this paper concerns the propagation of plane longitudinal stress waves. It has been shown that such stress waves, propagating normal to the layering get attenuated in a periodically layered elastic composite - which is not possible in a homogeneous elastic medium. As an illustration the attenuation of stresses in an elastic bar attached to such a mounting has been presented. It may be mentioned, though such results are not presented here due to space limitation, that such media can be effectively used for reduction of bending stresses and also for attenuating shear waves.

References

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[3] Barker, L.M. - 'A Model for Stress Wave Propagation in Composite Materials' - Jl. Composite Materials - 5, 1971, 140.

[4] Ghosh, A.K. - 'Dynamics of Periodically Layered Media' - Ph.D thesis, Indian Institute of Technology, Bombay-1981.

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TABLE-I

MATERIAL PROPERTIES (Ref.Fig.4)

Sl.No.	Homogeneous Bar			Composite						No. of periods
				Layer-1			Layer-2			
	h (cms)	E (Kg/cm^2)	ρ (gms/cc)	h (cms)	E (Kg/cm^2)	ρ (gms/cc)	h (cms)	E (Kg/cm^2)	ρ (gms/cc)	
1 (Ref. Fig.5)	20	0.91x10^6	6.1	2.5	2x10^6	7.8	2.5	3x10^4	1.1	4
2 (Ref. Fig.6)	200									

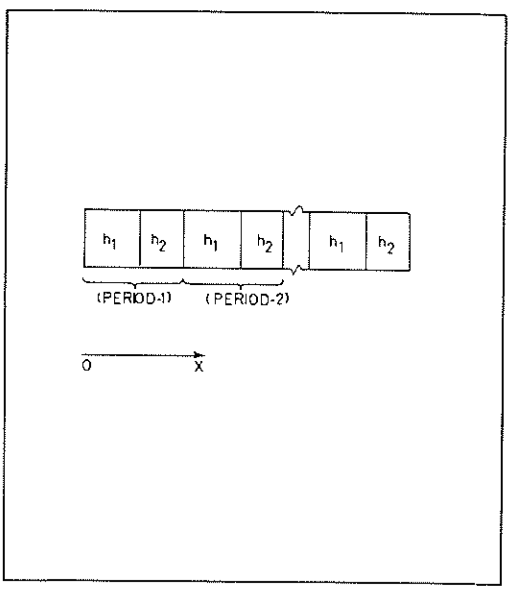

FIG.1 DESCRIPTION OF A PERIODICALLY LAYERED COMPOSITE

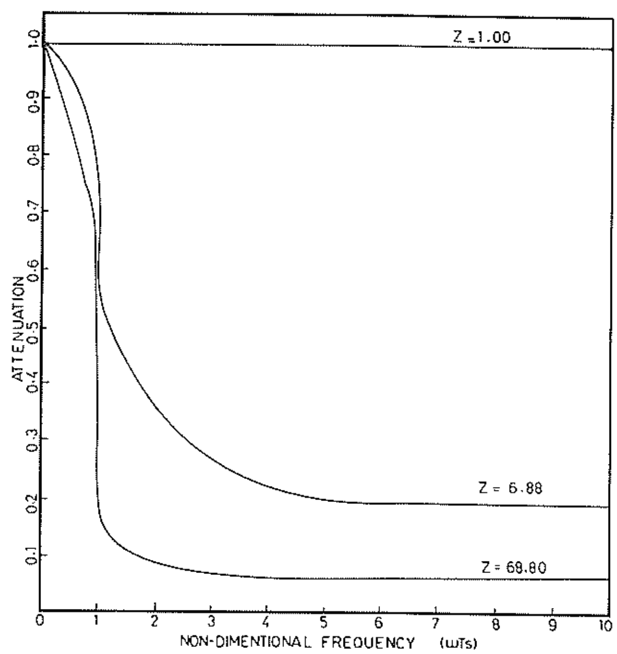

FIG. 2 EFFECT OF THE ACOUSTIC IMPEDANCE RATIO ON ATTENUATION IN AN ELASTIC COMPOSITE

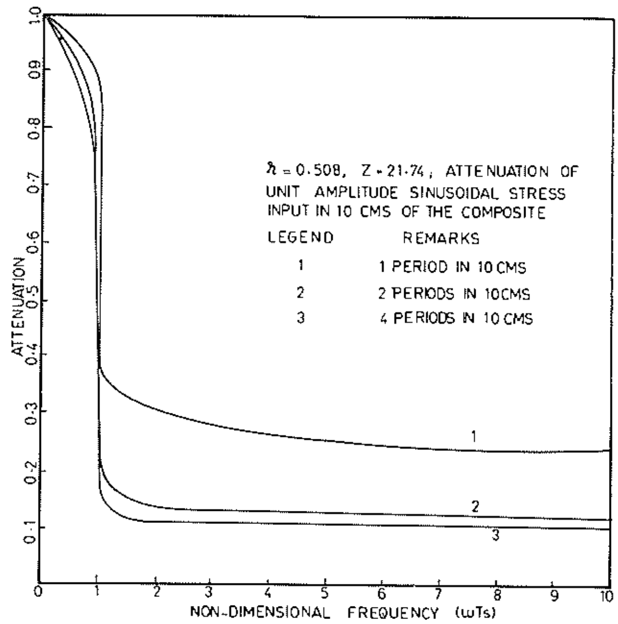

FIG.3 EFFECT OF NUMBER OF LAYERS ON ATTENUATION IN AN ELASTIC COMPOSITE

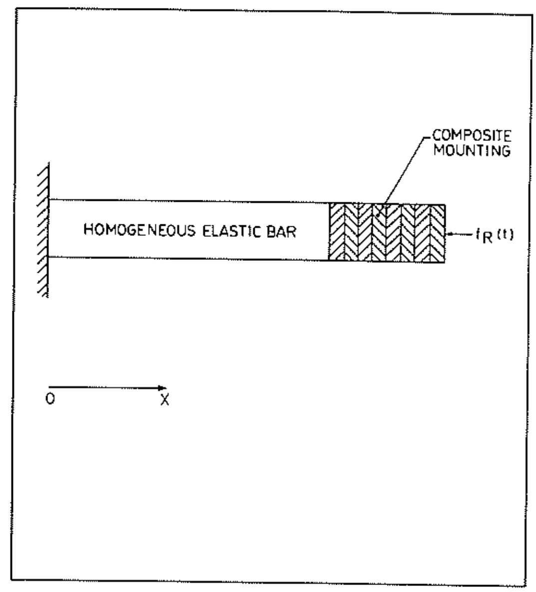

FIG.4 ELASTIC BAR WITH A COMPOSITE MOUNTING

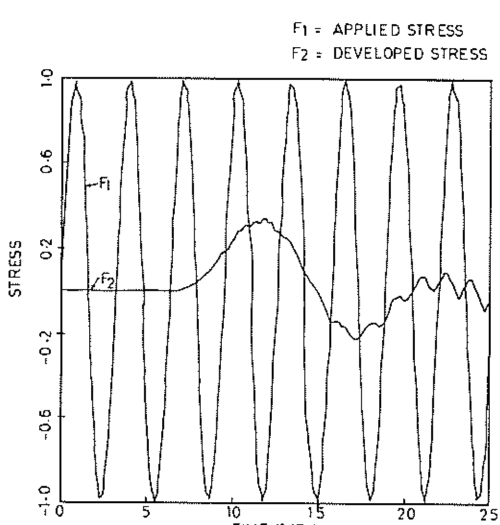

FIG.5 STRESS AT THE BUILT-IN END OF A 20cm. BAR ; ωTs = 20

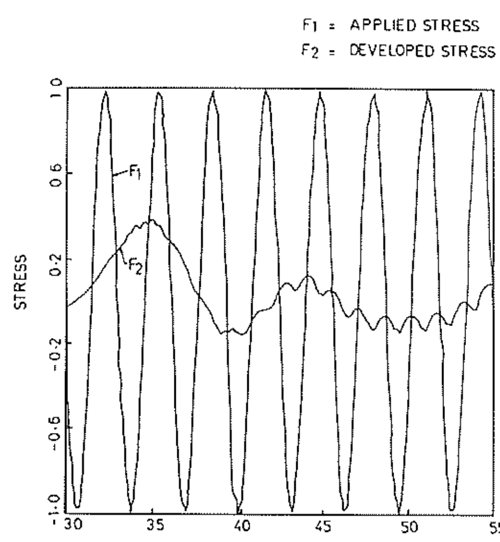

FIG.6 STRESS AT THE BUILT-IN END OF A 200cm. BAR ; ωTs = 2.0