NEWMARK'S FORMULA FOR ESTIMATING SLIDING DISPLACEMENT OF AN UNANCHORED BODY SUBJECTED TO EARTHQUAKE EXCITATION

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ABSTRACT

A free standing rigid body under the action of base excitation can move in many different ways. In this study, the sliding response of a body placed on a horizontal base which undergoes a one-dimensional horizontal motion is considered. In 1965, Newmark gave a simple formula to determine the sliding distance of a free-standing body subjected to a single rectangular acceleration pulse of short duration at the base. The objective of this study is to see if this formula can be applied to estimate the sliding displacement of a body under the action of real earthquakes. Newmark's formula calls for the maximum velocity of the base which information is usually not directly available. To make use of the response spectrum commonly available to the engineers, Newmark's formula is first re-derived and expressed in terms of the maximum displacement of the base which can be determined from the absolute displacement response spectrum in low frequency range. An ensemble of 75 real earthquakes are then employed, the equation of sliding motion is solved numerically and the average of the maximum sliding displacement of the body relative to the base is computed. The computed displacement is then compared with that obtained by Newmark's formula. This is done for a body placed on the ground as well as on the floors of a building. It is shown that Newmark's formula can be used if an adjustment factor, as suggested in this study, is applied.

INTRODUCTION

Engineers are sometimes interested in knowing whether or how an unanchored body responds to base excitation. This problem has been studied by many researchers. It has been shown that a free-standing rigid body can move in many ways (see, for example, [1]). This study is concerned with the slide mode of response. That is, it is assumed that the configuration of the body, the base excitation and the coefficient of friction between the body and the base are such that the body, initially at rest, will remain at rest or undergo the slide mode of motion.

Practitioners would like to be able to estimate the sliding distance of a body by some simple formula if possible. In 1965, Newmark [2] provided such a simple formula by assuming that the base acceleration is a single, horizontal rectangular pulse with a short duration. Variations of this formula have been proposed by engineers in the nuclear industry. It is of interest to see if this formula, modified or interpreted in various ways, may indeed be applied to estimate the sliding distance of a body. Newmark's formula is expressed in terms of the maximum velocity of the base which is usually not immediately available to the engineers. To make use of the information contained in the response spectrum which is usually available, Newmark's formula is first re-derived and expressed in terms of the maximum displacement of the base which is in fact the ordinate of the response spectrum of the absolute displacement in its low frequency range.

An ensemble of real earthquakes is employed as input, the equation of sliding motion is solved numerically and the mean of the maximum relative displacement of the body with respect to the base is computed and compared with that obtained using Newmarks's formula.

This exercise is extended to the case in which a body is placed on the floors of a building. A five-story shear frame is selected as the building's model. The shear frame is uniform in mass and stiffness and has 5% modal damping. The mass and stiffness are adjusted so that the natural periods of vibration of the shear frame can be varied. The 75 ground acceleration time histories are filtered through the shear frame and the acceleration time histories of the floors are computed and used as input to the equation of sliding motion of the body. As in the case in which the body is placed on the ground, the maximum relative displacement of the body is computed and compared with that obtained by Newmark's formula.

It is shown that if Newmark's formula is to be used, an adjustment factor, as suggested in this paper, should be applied.

EARTHQUAKE RECORDS

The earthquake records are taken from a series of strong motion accelerograms referred to as CIT-SMARTS, given by the Strong Motion Center and Earthquake Engineering Research Laboratory at California Institute of Technology. Seventy five earthquakes are selected. They are listed in [3]. These 75 earthquakes are of different characteristics in terms of energy content in various frequency bands, duration and variation of intensity with time. The statistical properties of the sliding response of a body obtained using these earthquakes as input must therefore be interpreted accordingly.

The time histories of the ground acceleration of these earthquakes are first normalized. That is, each of the absolute maximum accelerations, which defines the strength of the earthquake, denoted by A_{gx}*g where g is gravitational acceleration, is scaled to a value of unity. When one is interested in the statistics of the sliding response of a body to A_{gx} other than unity (say, 0.4), all the acceleration time histories of the ensemble are scaled by multiplying them by the value of 0.4.

NEWMARK'S FORMULA

Newmark's work on the sliding of a body subjected to base motion was carried out in connection with the study of the movement of embankment and of bodies of soil or rock on the surface of a dam during an earthquake. In his study [2], he assumed that the base's acceleration is a single rectangular pulse (of magnitude $A_{gx}g$ of short duration t_0 (see Fig.1). In this way, a closed form solution for the displacement of the body was easily obtained. Recognizing that the time history of ground acceleration in an earthquake is better represented by a series of pulses, Newmark suggested that an adjustment factor be applied to the simple formula he derived. Newmark, however, did not provide the adjustment factor. Newmark's formula and approach to estimating the sliding distance of a body has been used, with modifications, by engineers in the nuclear industry. There has not been, however, any serious effort to verify the effectiveness of the formula.

Newmark's formula[2] takes the form:

$$R_d = \frac{V^2}{2\mu g}\left(1 - \frac{\mu}{A_{gx}}\right) \tag{1}$$

where R_d is the maximum displacement of the body relative to the base, V is the maximum velocity of the base and μ is the coefficient of friction. The value of V, however, is usually not readily available to the engineer. To facilitate it's use and in consideration that for earthquake design, response spectra are usually available, Newmark's formula is recast in terms of μ, A_{gx} and S_d, the maximum displacement of the base, which is identified as the value of the absolute displacement response spectrum in low frequency range. Fig. 1 gives the time history of base acceleration. If A_{gx}*g exceeds μg, the body undergoes sliding motion with velocity μgt, t being time. Fig.2 gives the velocities of the base and the body. It is easy to see that $t_e = A_{gx}t_0/\mu$. The area OAB=area OABC-areaOBC gives the relative maximum displacement R_d of the body with respect to the base which may be verified to be given by Eq. 1. Substituting $V = A_{gx}gt_0$ and noting that t_0 is related to

$$S_d = areaOABC = \left(\frac{A_{gx}gt_0^2}{2}\right)\left(\frac{2A_{gx}}{\mu} - 1\right) \tag{2}$$

it may be seen that

$$R_d = \frac{S_d}{2}\left(\frac{(A_{gx}/\mu) - 1}{(A_{gx}/\mu) - 0.5}\right) \tag{3}$$

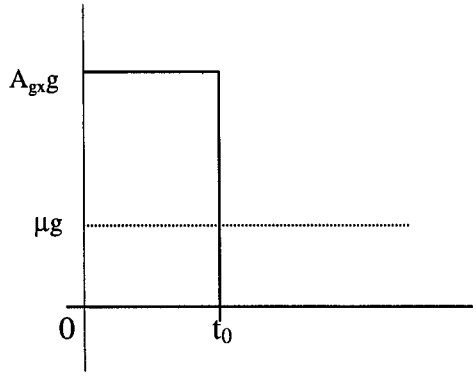

Fig. 1. Rectangular acceleration pulse

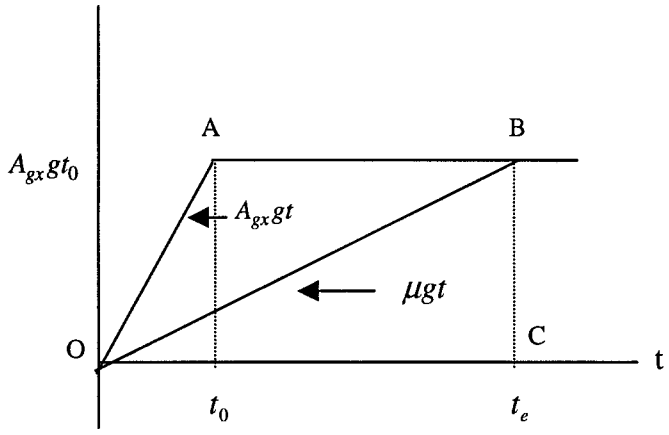

Fig. 2 Velocities of the base and the body

RESULTS

Newmark's formula is verified for a body placed on the ground as well as on the floors of a building. For that purpose, a five-story shear frame of uniform floor mass and story stiffness with 5% modal damping ratio, shown in Fig.3, is used. The values of the mass and stiffness are given various values so that the structure's natural periods of vibration are varied. The time histories of the 75 earthquake accelerations are first fed into the shear frame and the time histories of the accelerations of the floors are obtained. For each floor, for each of the 75 floor acceleration time history, the maximum acceleration is computed, normalized and scaled as desired as explained in **EARTHQUAKE RECORDS** section. The average absolute displacement response spectra of the floors (as well as the ground) are produced. Comparison is made of the mean maximum relative displacement of the body as computed by the equation of motion (hereafter, for convenience, denoted R_c) and the maximum relative displacement R_d obtained from Eq.3. The S_d value in Eq.3 is the mean of the values of the 75 absolute displacement response spectra in low frequency range. For the ground, the average absolute displacement response spectrum is shown in Fig.4.

Fig.5 gives the mean of the maximum relative displacement of the body placed on the ground versus A_{gx} for three values of μ (=0.1,0.2 and 0.3). The solid curves represent R_c values and the dotted curves, R_d values from Eq.3. It is seen that for the case of μ=0.1 in Fig.5, R_c may or may not exceed R_d but for the cases of μ=0.2 and 0.3, R_d always exceeds R_c.

Figures of R_c and R_d versus A_{gx} are prepared for all the floors of three shear frames with Tn=0.2, 0.3 and 0.5 sec for μ=0.1,0.2 and 0.3. These results indicate that in order to use Eq.3 to estimate sliding distance of a body, an adjustment factor need be introduced.

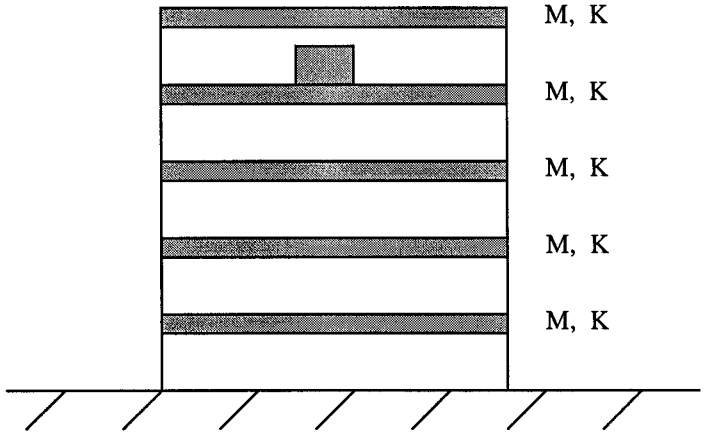

Fig. 3. Uniform shear frame(Tn=0.2, 0.3, and 0.5sec, damping ratio = 5%)

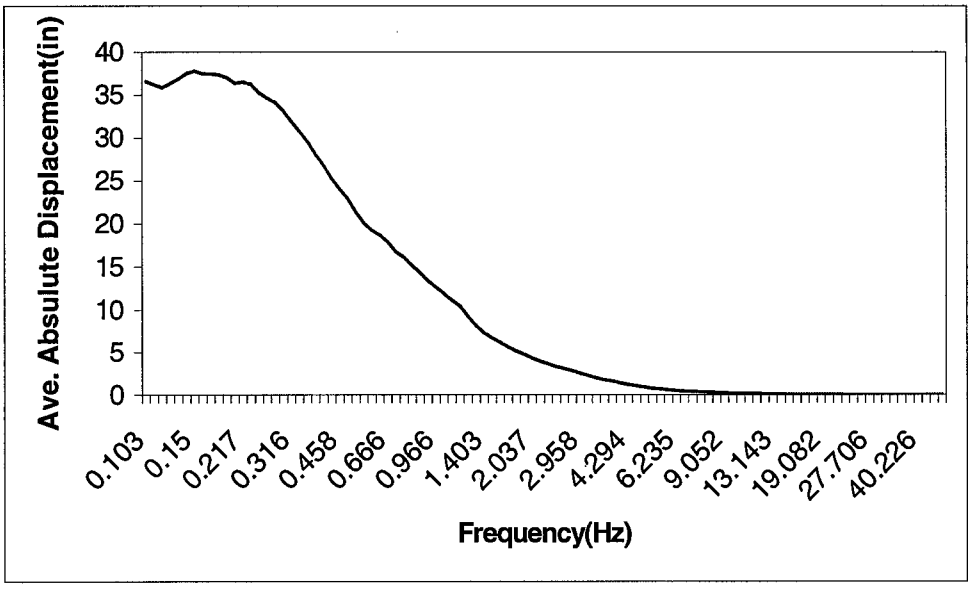

Fig. 4. Average displacement response spectrum(1 in = 2.54cm)

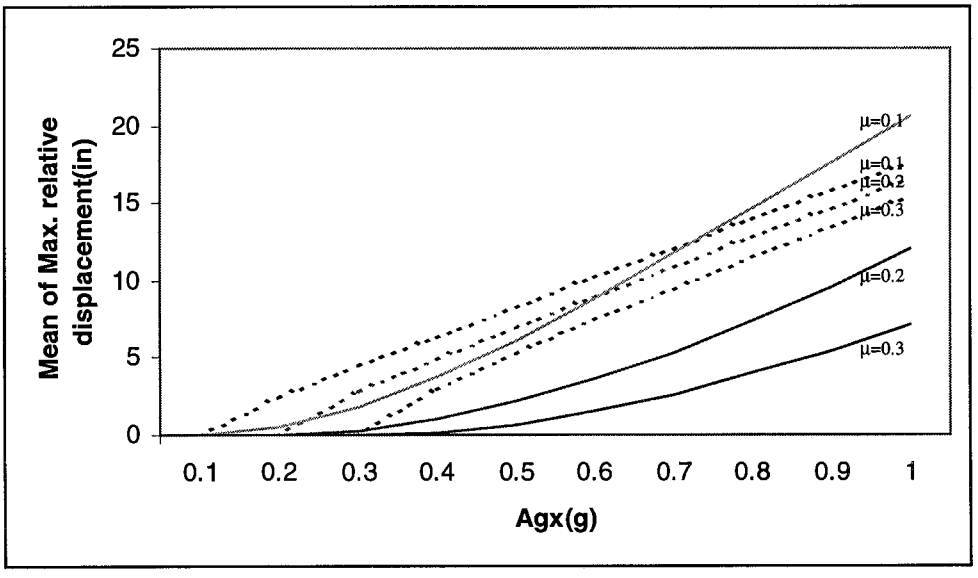

Fig. 5. Mean of the maximum relative displacement of a body placed on the ground (1 in = 2.54cm)

ADJUSTMENT FACTOR AND RESULTS

R_d is affected by the coefficient of friction μ, the maximum base acceleration A_{gx} and characteristics of the building reflected primarily by the fundamental natural period of vibration T_n, The structures considered in this study are rather stiff with $T_n < 0.5$ sec ($f_n = 1/T_n > 2$Hz). It was found that the influence of the flexibility of a structure is not very important. For this reason, it is expected that the adjustment factor, to which R_d in Eq. 3 is multiplied, may be a function of only μ and A_{gx}. By trial and error, the following adjustment factor is obtained:

$$Adj = \left(\frac{0.19}{\mu + 0.06} \right) \times A_{gx}^{\frac{1}{2}} \tag{4}$$

This equation reflects the fact that the larger the coefficient of friction, the smaller the adjustment factor but the adjustment factor is proportional to the square root of the size of the base motion.

Figs. 6,7,8,9 and 10 give the adjusted R_d and R_c plotted against A_{gx} using μ as parameter for the ground and for the first and fifth floors of the shear frame for T_n=0.3 and 0.5 sec. The same exercise is performed for other floors of the two shear frames. These results show that after applying the adjustment factor in Eq.4 to Eq.3, R_d and R_c are in reasonably good agreement.

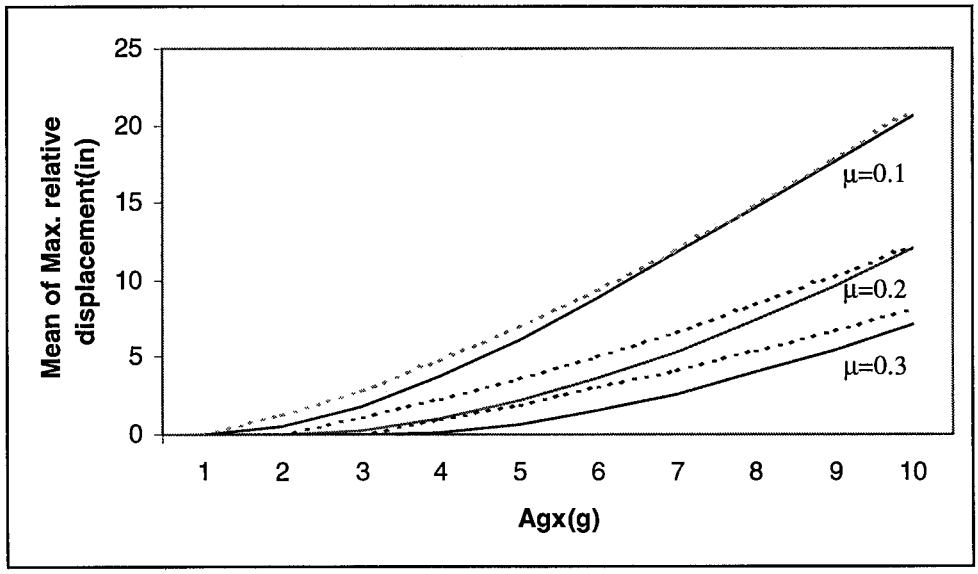

Fig. 6. Mean of the maximum relative displacement of a body placed on the ground

(1 in = 2.54cm)

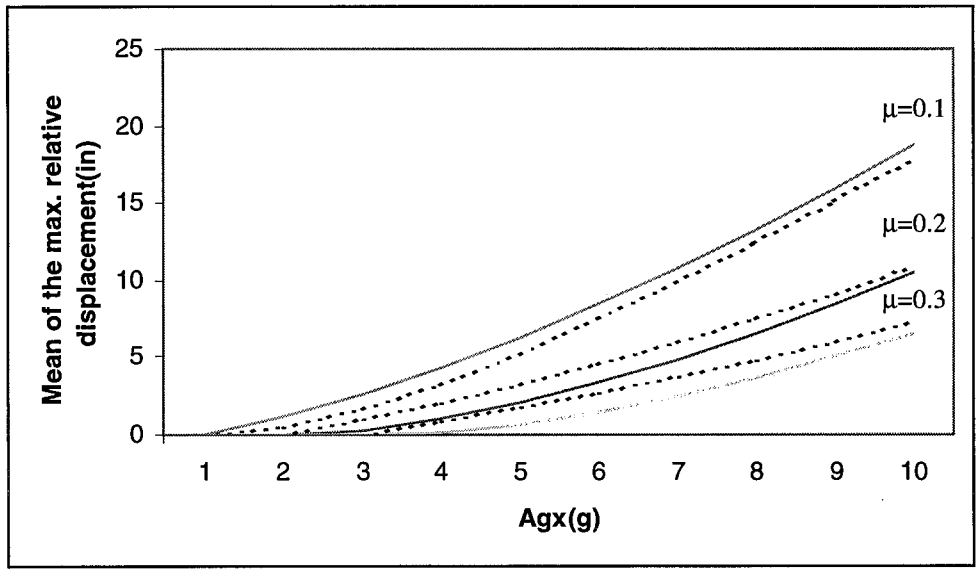

Fig. 7. Mean of the maximum relative displacement of a body placed on the 1st floor(Tn=0.3sec)

(1 in = 2.54cm)

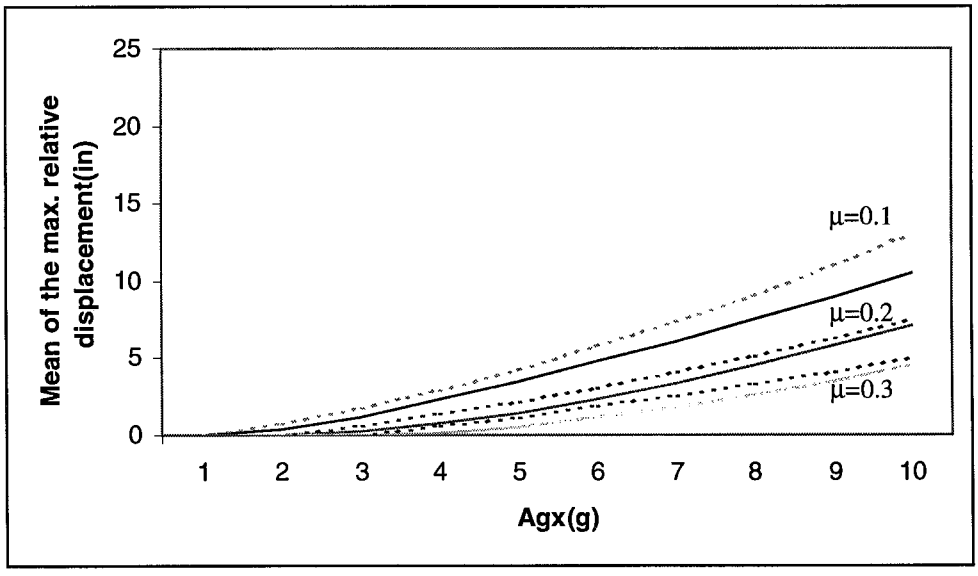

Fig. 8. Mean of the maximum relative displacement of a body placed on the 5th floor(Tn=0.3sec)

(1 in = 2.54cm)

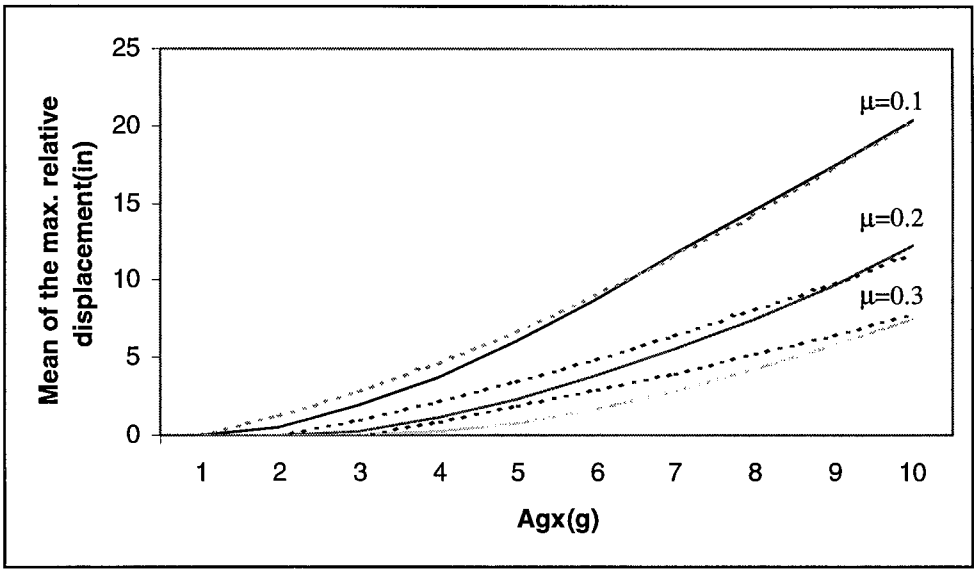

Fig. 9. Mean of the maximum relative displacement of a body placed on the 1st floor(Tn=0.5sec) (1 in = 2.54cm)

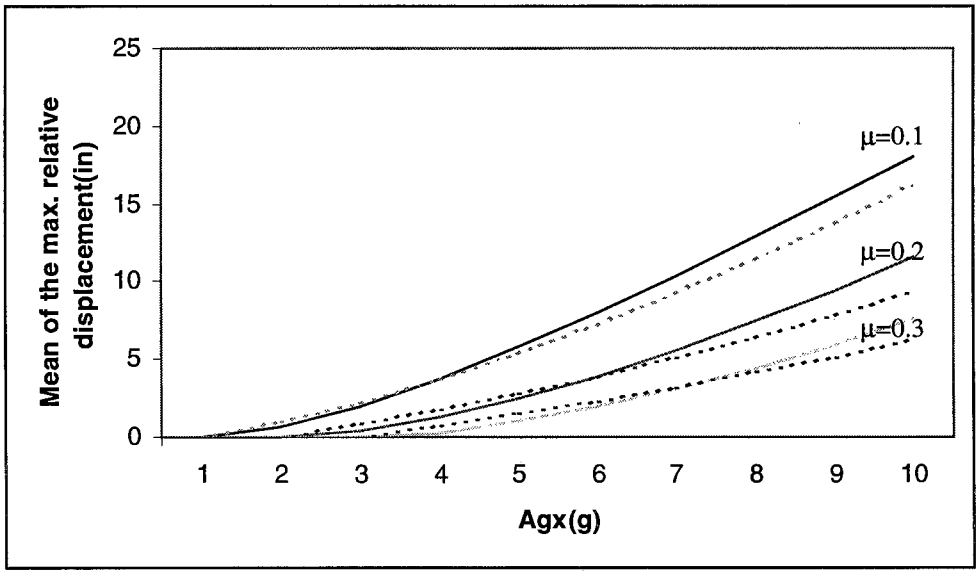

Fig. 10. Mean of the maximum relative displacement of a body placed on the 5th floor(Tn=0.5sec) (1 in = 2.54cm)

CONCLUSIONS

This study shows that Newmark's formula may be used to estimate the sliding distance of an unanchored body placed on a moving base undergoing earthquake induced motion with reasonable accuracy if an adjustment factor, as suggested in this paper is applied. Unlike the original formula requiring knowledge of the maximum base velocity which is usually not readily available, the formula used in this paper is expressed in terms of the maximum displacement of the base which quantity may be easily extracted from the absolute displacement response spectrum using its value in the low frequency end. In using the formula, it should be kept in mind that the formula is verified using 75 one-dimensional horizontal real earthquakes and gives only the mean value of the maximum sliding distance relative to the base with no information regarding the dispersion of the response, and the building is a rather special type of five-story shear frame of uniform floor mass and story stiffness whose fundamental natural period of vibration only covers the range of less 0.5 sec. The last condition, however, is not considered unduly restrictive since most five-story structures would have fundamental natural period of vibration less than 0.5 sec.

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