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General Theory of Electrically Conducting Orthotropic Toroidal Shells for Tokamak Fusion Reactors

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SUMMARY

We discuss in this paper the general theory of electrically conducting orthotropic toroidal shells of revolution of finite thickness, acted upon by rotationally symmetric electromagnetic and thermal loads. The former are produced by the Lorentz forces as the result of electromagnetic interaction of poloidal currents in the shell and the toroidal magnetic induction generated by those currents, as well as the poloidal magnetic induction generated by a set of toroidal currents, above and below the shell surfaces. The toroidal shell is of a sandwich type in the toroidal direction, and represents a structural model of a toroidal magnet system. Combined with its numerical implementation, the theory represents a very efficient semianalytical technique for structural analysis of toroidal magnet systems, for tokamaks.

A basic set of equations of orthotropic toroidal shells of revolution, of finite thickness has been derived on the basis of the approach developed by E. Trefftz and E. Reissner. The equations take into account the effect of transverse shear and normal stresses on the effective midsurface deformations and the body-type nature of the applied mechanical loads. In the process of derivation consistently retained were terms of the order h/R in comparison with unity, and neglected terms of the order h^2/R^2.

There are seven equilibrium equations, and eight constitutive equations which establish the relationships between the stress resultants and couples, and effective midsurface linear and angular displacements. The constitutive relationships are obtained by minimization of the strain energy of the shell, with equilibrium equations as side conditions, and effective midsurface displacements as Lagrangean multipliers.

An efficient finite difference procedure for the numerical solution of these equations has been developed and implemented into a computer code which performs the structural analysis of orthotropic toroidal shells of arbitrary meridional shape, with thickness and elastic moduli varying along the meridian, and subject to poloidal and toroidal electromagnetic loads, and temperature variations.

Discussed also are the results of structural analysis of several closed magnetic confinement machines with a variety of meridional shapes.

Global Inelastic Structural Analysis of the MARS Tandem Mirror Blanket Tubes Including Radiation Effects

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ABSTRACT

The effects of radiation on the structural performance of fusion reactor structures is recognized as a major issue for the development of fusion reactor technology. Neutron irradiation changes the mechanical properties of structural components resulting in a general degradation of these properties. In addition to the mechanical loads (pressure and weight) and the thermal strains, non-uniform inelastic strain fields are induced by radiation swelling and creep in fusion structures. In this paper, we describe a new computer code, STAIRE, for STress Analysis Including Radiation Effects. This code is based on standard beam theory for pipe-bends. The theory is modified in two areas: (1) consideration of the pipes cross-section deformation as the radius of curvature changes; (2) inclusion of inelastic radiation and thermal strains (swelling and creep). An efficient analytical/numerical approach is developed for the solution of indeterminate beam problems. As an application of the method, the stress distribution and deflections of toroidal blanket pipes in the Mirror Advanced Reactor Study (MARS) are evaluated. Swelling strains are identified as a major source of stress and deformation in the proposed blanket design, and possible solutions to the problem are outlined.

1. INTRODUCTION

The design of fusion reactor blankets depends critically on the imposed mechanical, thermal and radiation loads. During the past decade, many designs have evolved for blankets that will give maximum nuclear and thermodynamic performance in a given reactor concept. Fusion reactor designers have primarily concentrated on parameters related to the ability of the blanket to absorb neutrons (tritium breeding ratio and energy multiplication factor), and on thermodynamic parameters (pumping power and cycle efficiency). Structural analysis, however, has usually been performed at a crude level to satisfy major failure criteria described by the ASME boiler and pressure vessel code. Radiation effects were included by assigning an arbitrary limit on a given property (usually swelling or loss-of-ductility). With this situation in hand, a great deal of confusion has resulted for estimates of the useful lifetime of fusion reactor structural members. Recenctly, however, various investigators have attempted to analyze structural aspects of radiation effects in a fusion environment.

Since 1972, a large number of publications dealt with the problem of estimating the End of Life (EOL) of fusion structures. Researchers typically used EOL limits on swelling of 2-10% $\frac{\Delta v}{v}$ and uniform elongation of 0.5-2.0% strain [1]. Numerous reviews have been written by Conn [2], Harkness and Cramer [3], and Power and Reich [4]. However, only a few investigators have performed inelastic stress analysis including radiation effects [5-7]. In the analysis of references [5-7] significant restrictive assumptions had to be invoked for the solution of the inelastic problem. For example, Watson [1] developed a 1-D inelastic stress analysis code, TSTRESS, for the calculation of the long-term redistribution of the stresses in a generic thin-walled plate element that is subjected to membrane loads. The plate is assumed to be free to expand but is constrained from bending.

Our approach to the problem is substantially different from previous attempts [5-7]. In the next section we develop a theoretical model for the simultaneous determination of stress distribution and blanket module deformation due to various inelastic strains. The model is applicable to indeterminate curved beams of arbitrary cross-section. In section 3, we illustrate the application of the model by analyzing the tubular structure of the MARS tandem mirror reactor blanket [8]. Conslusions and recommendations then follow in section 4.

2. THEORETICAL MODEL

2.a. Motivation

Classical beam theory has been developed on the basis of virtual work principles for the simultaneous determination of support reactions, beam deflections and stress distributions. In this approach, a structure of degree r of static indeterminancy is considered. The equations of statics must be supplemented by a number of equations of geometry equal to the degree of redundancy, r, of the structure. The r unknowns or redundant forces are the unknowns in these equations [9]. In curved beams of hollow cross-sections, the beam deflections must be related to the deformation of the cross-section as the deflection increases. This was found to effectively increase the flexibility of the beam resulting in a reduction of the magnitude of the axial stress [10]. The ASME boiler and pressure vessel (B and PV) code has been based on such calculations for the elastic analysis of curved beams [11].

In a fusion reactor, inelastic strains are more common than elastic ones. For a complete analysis of the structure, the following types of strains must be included:

(1) Thermal strain due to thermal expansion.

(2) Swelling strain which is a function of both temperature and neutron fluence.

(3) Irradiation creep strain. This is a function of the stress (linear), and of the neutron fluence.

(4) Thermal creep strain.

Standard beam theory as used in the ASME B and PV Code [11] is therefore not directly applicable.

With the significance of accurate lifetime estimates in fusion reactor structures, it is clear that the development of an appropriate structural analysis method is a worthwhile task. In this regard, two approaches seem to be reasonable. The first is a modification of an existing finite element elasto-plastic structural code to include radiation effects. The second approach is an extension of classical beam theory to account for the new inelastic strains. Throughout this section we will describe our analysis based on the second approach. Advantages over the finite element method can be summarized as:

(1) Simplicity of governing equations.

(2) Easier interaction with the computer code.

(3) Self-consistent allowance for cross-section deformation with beam deflection. In a finite element code, the geometry of the element must also change with deflection.

(4) Faster computations.

2.b. Governing Equations

If a beam is unrestrained externally (statically determinate), then inelastic stresses can only occur due to the requirement that plane cross-sections remain plane after bending, provided that the beam does not exhibit sharp curvature (radius of curvature to pipe diameter ratio $\geq$ 5) [12]. When the beam is statically indeterminate, additional inelastic stresses will result from the redundant restraining forces. Besides the assumption of plane cross-sections remaining plane after bending, it is also assumed that the variation of the temperature and beam cross-section along the length of the beam is continuous and smooth. Under the first assumption, our solutions are expected to be valid at distances from the beam ends greater than the order of the dimensions of the cross-section (St. Venant Principle).

The analysis begins with determination of the redundant end reactions which result from the applied strains. Figure (1) shows a cross-section of the MARS Tandem Mirror Reactor [8], in which each blanket module is composed of two rows of tubes and one row of square beams for added stiffness. Figure (2) illustrates the geometry used in our analysis of the first row of blanket tubes. R_c is the distance to the central axis, $\xi = R - R_c$, where R is current distance, θ is the toroidal angle, s is the arc length, x and y are coordinates, X_F is the redundant axial force, X_p is the redundant radial force, and X_M is the redundant moment.

Because the tube is curved and statically indeterminate, we are unable to separate the axial problem from the radial and rotational problems. Using virtual work principles, it is possible to develop the flexibility matrix for the pipe [12]. This matrix gives the deflections at the free (cut) end of a singly clamped pipe due to unit loads at that end. This is given by the matrix, F [12].

$$\underline{\underline{F}} = \begin{bmatrix} f_{MM} & f_{MF} & f_{MP} \\ f_{FM} & f_{FF} & f_{FP} \\ f_{PM} & f_{PF} & f_{PP} \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} \int dx & \int yds & \int xds \\ \int yds & \int y^2 ds & \int xyds \\ \int xds & \int xyds & \int x^2 ds \end{bmatrix} \qquad (1)$$

where the first row of the matrix equation gives the rotations due to unit moment, unit axial force and unit radial force, respectively. The second row will similarly give the axial deflections and the third row gives the radial deflections due to unit loads at the cut. E is the modulus of elasticity and I is the moment of inertia of the cross-section.

The matrix, F, is used to form an equation for the rotation and axial and radial deflections of the free end of the pipe due to both the end forces and applied strains. The end deflections due to end forces are $\underline{\underline{F}}\underline{X}$ where $\underline{X}$ is a vector of unknown end reactions:

$$\underline{X} = \begin{bmatrix} X_m \\ X_F \\ X_p \end{bmatrix} \qquad (2)$$

If we add the deflections due to the input strains given by a vector E, to the deflections due to end forces, we obtain a vector, D, containing the resultant end rotation and deflection. The final equation becomes

$$\underline{\underline{F}}\underline{X} + \underline{E} = \underline{D}$$

The vector D is generally determined by the boundary conditions. For example, if both ends of the pipe are clamped, there are no end rotations or deflections and $\underline{D}$ is identically zero. Prescribed end translations or rotations can also be included here.

Assuming $\underline{\underline{F}}$, $\underline{E}$ and $\underline{D}$ are known, the unknown reactions are easily found by solving the matrix equation (3) for the vector $\underline{X}$. The result is $\underline{X} = \underline{\underline{F}}^{-1}[\underline{D}-\underline{E}]$. In the following, we describe a method for the determination of the vector $\underline{E}$ from input thermal and radiation strains.

The general form of E is shown by reference [12] to be of the form:

$$E = -\begin{bmatrix} \int w' ds \\ \int w' yds - \int \bar{\epsilon}' \cos\theta ds \\ \int w' xds + \int \bar{\epsilon}' \sin\theta ds \end{bmatrix} \qquad (4)$$

where w' is the change in curvature due to inelastic strains and $\bar{\epsilon}'$ is the average inelastic strain at a given cross section. The average strain can be found easily, $\bar{\epsilon}' = \frac{1}{A}\int_A \epsilon'' dA$, but w' requires additional information. Here ϵ'' is the total inelastic strain due to thermal expansion, αT, swelling, $\frac{\Delta v}{3v}$, irradiation creep, ϵ^c_{irr}, and thermal creep, ϵ^c_{th}.

$$\epsilon'' = \alpha T + \frac{\Delta v}{3v} + \epsilon^c_{irr} + \epsilon^c_{th} \qquad (5)$$

To determine the change of curvature under loading conditions, we begin with a strain-curvature relation. In standard beam theory the strain is given as:

$$e = - \xi w \qquad (6)$$

This equation is valid if the beam's cross section remains unchanged during deformation, but in our case of a thin-walled, curved tube, it requires modification. Hovgaard [10] postulates the following:

$$e = - \xi w - \frac{\Delta\xi}{R_c} \quad , \qquad (7)$$

where $\Delta\xi$ is the radial displacement of a point on the tube relative to the neutral axis. The shape of the final cross section is nearly elliptical. Hovgaard found:

$$\Delta\xi = - K_I \xi^3 w \quad , \qquad (8)$$

where K_I is a constant determined by the pipe geometry:

$$K_I = \frac{6r^2}{6t_c^2 R^2 + 5r^4} \qquad (9)$$

where r is the pipe radius and t is the pipe wall thickness. This relation for $\Delta\xi$ leads to a near-elliptical cross section during bending.

Plugging the $\Delta\xi$ relation into equation (7), gives:

$$e = - (1 - K_I \xi^2) \xi w \quad . \qquad (10)$$

For a given curvature change (w), the strain away from the neutral axis is less than beam theory predicts. This leads to stress relaxation at the outer fibers and an increased pipe flexibility. This is primarily because more rotation is required to provide equilibrium for a given force at any cross section. In our case, the "loads" are input strains, rather than applied forces, so the increased pipe flexibility leads to smaller reactions resulting from the given strains. The stresses in the pipe are therefore less than standard beam theory would predict.

In order to find the curvature as a function of inputs only, we next require a constitutive relation. Assuming that the total strain can be separated into elastic and inelastic portions, the constitutive equation is given by

$$e = \frac{\sigma}{E} + e' \qquad (11)$$

where e is the total strain, $\frac{\sigma}{E}$ is the elastic strain, and e' is the inelastic strain.

Substituting into the strain-curvature relation (equation (10), multiplying by $(\xi - \Delta\xi)$ and integrating over the cross-section we obtain,

$$\int_A \frac{\sigma}{E} (\xi - \Delta\xi) dA + \int e' (\xi - \Delta\xi) dA = - w \int (\xi - \Delta\xi)(1 - K_I \xi^2) \xi dA \qquad (12)$$

The applied moment, M, on any cross section is given by $M = \int_A \sigma(\xi - \Delta\xi) dA$. Using this fact and ignoring second order terms, we can solve for w:

$$w = \frac{-M}{K_{III} E} - \frac{1}{K_{III}} \int_A e' \xi dA \qquad (13)$$

where

$$K_{III} = K_{II}\, I + K_I R_C \int \varepsilon'' \xi^3 dA \tag{14}$$

and

$$K_{II} = \frac{12t^2R_c^2 + r^4}{12t^2r_c^2 + 10r^4} \quad . \tag{15}$$

The constant K_{II} effectively reduces the pipe's moment of inertia, accounting for much of the pipe's increased flexibility. Since w' is the change in curvature due only to the inelastic strains, it is found by setting the applied moment equal to zero: $w' = -\frac{1}{K_{III}} \int_A e'\xi \, dA$. This completes the requirements for the determination of the redundent reaction vector $\underline{X}$.

Now that the end reactions are known, the axial stresses in the pipe can be found. The first step is to find the resultant forces and moment acting on any cross section of the pipe using simple statics.

$$M = X_m + xX_p + yX_f \tag{16}$$

$$F = X_f\cos\theta - X_p\sin\theta \tag{17}$$

$$P = X_f\sin\theta + X_p\cos\theta \tag{18}$$

M, F and P are the moment and forces on a section at an angle θ.

In determining the stresses, we must modify the constitutive equation to allow for a strain, e_o, at the neutral axis. This strain has no effect on the determination of w, but it is significant here. By integrating the new constitutive equation, $e = e' + \frac{\sigma}{E} + e_o$, over the area and using $F = \int \sigma dA$, we obtain an equation for e_o:

$$e = \frac{-F}{AE} - \frac{1}{A}\int e' dA \tag{19}$$

we now have,

$$\frac{\sigma}{E} = e - e' + \bar{e} + \frac{F}{AE} \quad . \tag{20}$$

Substituting for both w and e yields the final result:

$$\sigma_{axial} = \frac{F}{A} + \xi(1 - K_I\xi^2)\left(\frac{M}{K_{III}} - Ew'\right) - E\,(e' - \bar{e}). \tag{21}$$

Our last goal is to find the deflections as a function of toroidal angle. The process followed is similar to determination of the end reactions. The deflections are found by conceptually cutting the pipe at the angle θ and using the equation, $\underline{\underline{F}}\underline{X} + \underline{E} = \underline{D}$. In this case, $\underline{D}$ is unknown and $\underline{\underline{F}}\underline{X}$ and $\underline{E}$ must be determined. $\underline{\underline{F}}$ and $\underline{E}$ are as given before, except any integrals must be performed only from the cut to the pipe end and the origin of x and y is always at the cut with x tangential to the pipe. The vector $\underline{X}$ is now filled with the forces at θ: M, F and P. $\underline{D}$ then gives us the deflection at the cut.

We must still describe the nature of e'. For thermal strains, we have $e_{th} = \alpha T$ where α is the thermal expansion coefficient and T is the difference between the operating temperature and the zero stress temperature. For the swelling strains, ε^s, we use a design equation developed by Ghoniem and Conn for ferritic alloys [13]:

$$\varepsilon^s = \frac{1}{3}\frac{\Delta v}{v} = \frac{1}{3} \quad \exp\left\{\left(\frac{T-T_p}{\gamma}\right)^2\right\}\{.036\ \delta - .074\}\{\phi(c_r)\} \qquad (22)$$

where T_p = the peak swelling temperature (°C)

γ = the Gaussian width

δ = displacement dose (dpa)

$$\phi(C_r) = \begin{cases} .067\ C_r^2 - .457\ C_r + 1.0 & C_r < 5\% \\ .037\ C_r + .237 & C_r > 5\% \end{cases} \qquad (23)$$

This should complete the requirements for our analysis.

2.c. Computational Aspects

Our first task was to compute the effects of the thermal strains only. These calculations would apply during startup, because radiation damage has not had a chance to accumulate. For the assumptions of linear temperature variation with ξ and θ, we were able to formulate closed-form solutions for the reactions and deflections. The corresponding computer code, named STAIRE-I (STress Analysis Including Radiation Effects), is therefore very straightforward.

The second code, STAIRE-II, includes the effects of irradiation swelling. Because the swelling strains are Gaussian in temperature, the necessary integration must be performed numerically. This complicates the code slightly, but the overall method is the same as in STAIRE-I. Although the swelling strains are time-dependent, STAIRE-II is a quasi static code because the calculations require no time-stepping. The stresses and deflections depend only on the instantaneous swelling and thermal strain.

STAIRE-III, which is not included in this paper, will require more complex computation methods because it will include radiation and thermal creep. The fact that creep phenomena are stress dependent requires us to calculate creep strains at a given time from the stresses a short time earlier. STAIRE-IV will incorporate failure criteria, allowing a final determination of the pipe lifetime and necessary design adjustments.

3. RESULTS AND CONCLUSIONS

The results presented here indicate the effects of swelling on a structure as well as design modifications that may enter component lifetimes. The magnitude of the stresses obtained should be considered as upper limits on the actual stresses because creep processes not included here should provide considerable stress relief.

Figure 3 allows us to compare standard beam theory calculations with our modified theory. Beam theory predicts a linear stress distribution over the cross-section, the maximum occuring at the outer fiber. The peak stress on the deformed cross-section is roughly half the beam theory peak and it occurs on an inner fiber.

One important design consideration is the maximum stress occuring in the pipe. Figure 4 shows that a doubly clamped beam can tolerate the initial thermal stresses during full power operation, but the stresses after two years of swelling (138 dpa) will probably exceed the design stress of 28 ksi (HT-9) even after including creep effects. All is not lost, though; figure 5 shows the stress levels after allowing the header to translate .5

cm to accommodate thermal strains and .5 cm/year thereafter to accommodate swelling strains. The maximum stress is about 50 ksi, which will likely be reduced below the design stress by two years of thermal and radiation creep.

After assuring safe stress levels, the blanket designer must allow sufficient room for the expansion of the pipe. Figures 6 and 7 show the radial deflection of the pipe's central fiber. These deflections will be somewhat enhanced by creep, but they should not create any severe problems. The clamped pipe deflects a maximum of about 1.5 cm, whereas the peak is reduced to just over 1 cm when the header is allowed to translate. There displacements will dictate the necessary pipe spacing in the design.

These results indicate that the two year life expected in the MARS design is realizable and longer life can be achieved with minor design modifications. Further analysis should verify this, leading to a more economical design.

ACKNOWLEDGMENT

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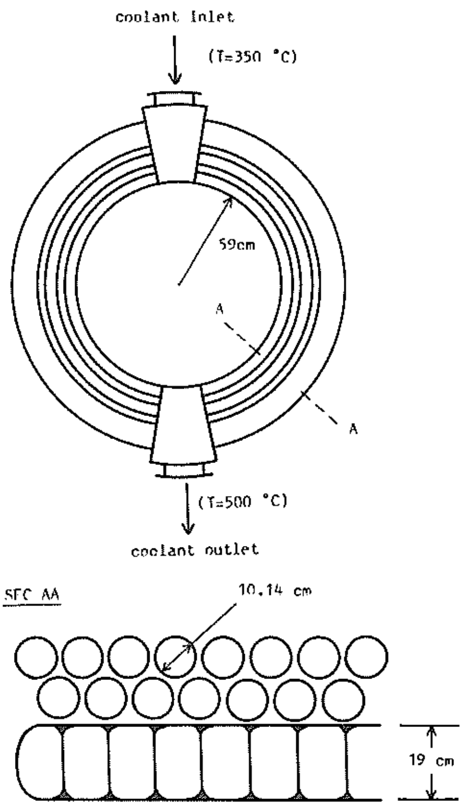

Fig. (1) : Cross-section of the Mirror Advanced Reactor Study (MARS) blanket.

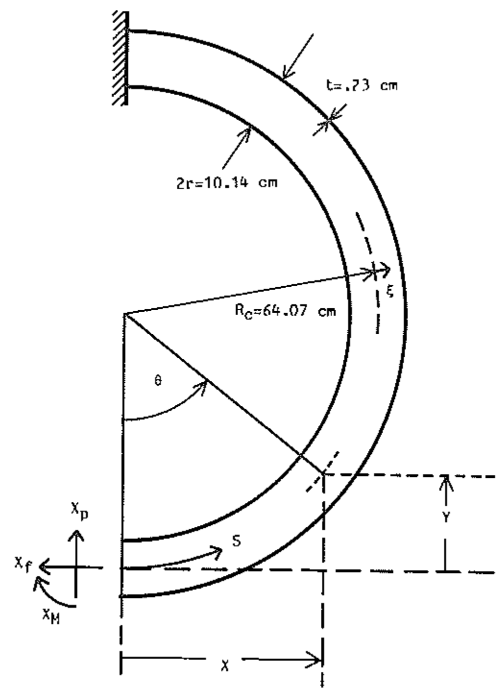

Fig. (2) : Model geometry for the analysis of blanket tubes.

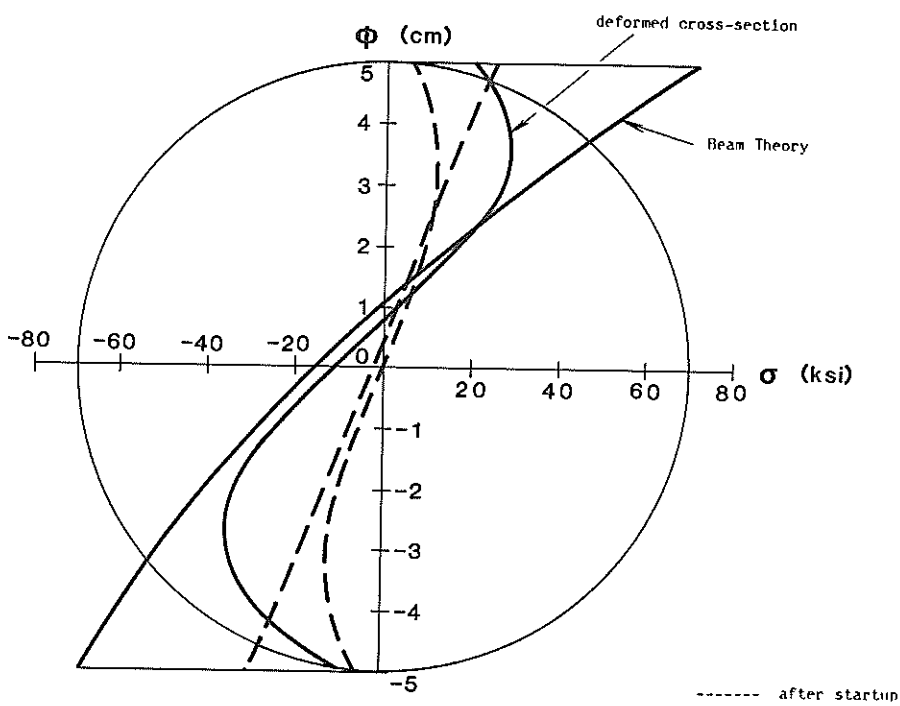

Fig. (3) : Comparison of stress distributions over the cross-section for standard beam theory and that including deformation of the cross-section.

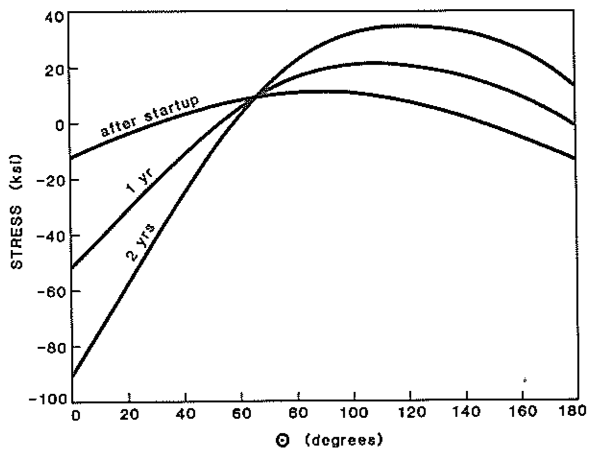

Fig. (4) : Stress distributions over the blanket pipe without header translation.

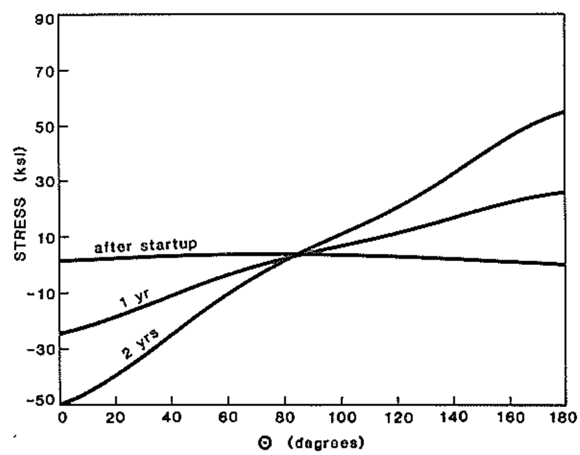

Fig. (5) : Stress distributions with header translation.

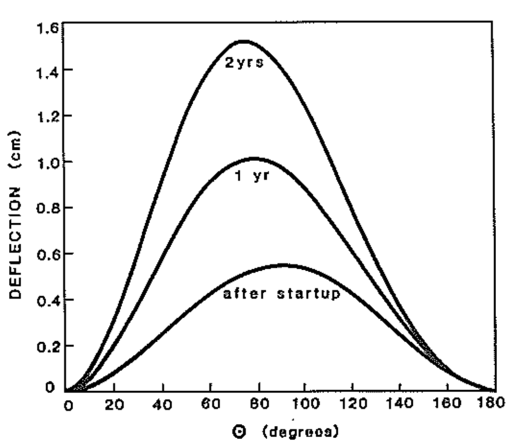

Fig. (6) : Pipe radial deflections with 30° radial temperature drop.

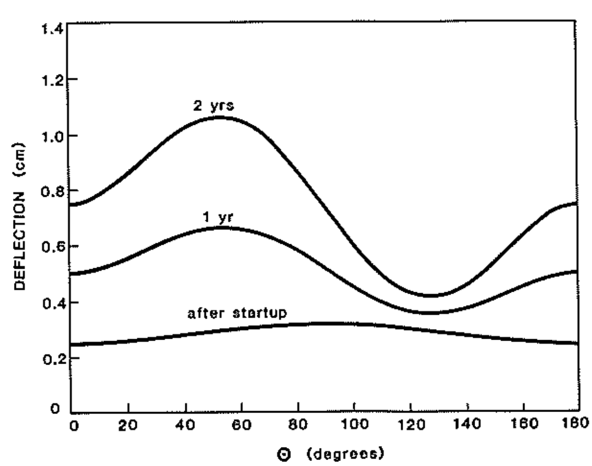

Fig. (7) : Pipe radial deflections with assumed header translation.