BEACON: A THREE-DIMENSIONAL STRUCTURAL ANALYSIS CODE FOR BOWING HISTORY OF FAST BREEDER REACTOR CORES

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Abstract

The core elements of an LMFBR are bowed due to radial gradients of both temperature and neutron flux in the core. Since all hexagonal elements are multiply supported by adjacent elements or the restraint system, restraint forces and bending stresses are induced. In turn, these forces and stresses are relaxed by irradiation enhanced creep of the material.

The analysis of the core bowing behavior requires a three-dimensional consideration of the mechanical interactions among the core elements, because the core consists of different kinds of elements and of fuel assemblies with various burnup histories. A new computational code BEACON has been developed for analyzing the bowing behavior of an LMFBR's core in three dimensions. To evaluate mechanical interactions among core elements, the code uses the analytical method of the earlier SHADOW code (the Subchannel Deformation Analysis Code for Wire-Wrapped Assemblies), which was presented in SMiRT-4 paper D 2/3. BEACON analyzes the mechanical interactions in three directions, which form angles of 60° with one another.

The main features of BEACON can be summarized as follows :

1. It analyzes the whole core bowing behaviors,at various power levels during startup after refueling ; after a certain time of operation at full power ; after cooling down to the refueling temperature and subsequent relaxation of the restraint system.
2. The finite element method is applied to the stress analysis of wrapper tubes, treating the tubes as three-dimensional Rahmen finite elements. This makes it possible to calculate the creep strains by proceeding in small constant-stress time increments.
3. The code can treat the heterogeneity of a core, which contains core elements with different bending stiffnesses and burnup histories. It also handles 30°, 60°, 120° and 180° sectors of the whole core, assuming a boundary condition of rotational or reflective symmetry.

BEACON is applied to the 60° sector of a typical LMFBR's core for analyzing the bowing history during one equilibrium cycle. 120 core elements are treated, assuming the boundary condition of rotational symmetry. The application confirms that the code can be an effective tool for parametric studies as well as for detailed structural analysis of LMFBR's core.

1. Introduction

The core of a Liquid Metal Cooled Fast Breeder Reactor (LMFBR) consists of different kinds of core elements, such as core and blanket fuel assemblies, control rod elements and reflectors. Under operating conditions, these core elements are bowed due to radial gradients of both temperature and neutron flux in the core. During startup and transients, temperature differences across the cross sections of core elements are built up, giving rise to thermal bowing of the core. And in full power operation, differential swelling increases with fuel burnup, which causes time-dependent core bowing. When the core element deflection is large enough to cause mechanical interactions among adjacent elements or the restraint system, restraint forces and bending stresses are induced. In turn, these forces and stresses are relaxed by irradiation enhanced creep of the material. Consequently, the overall bowing behavior of LMFBR's cores will be determined by a complex interaction of thermal, swelling and creep strains during reactor operation.

Many computational codes have been developed for analyzing bowing history of LMFBR's cores [1 - 4]. Since these codes treat a radial spoke through a reactor core in a two-dimensional model, the resulting forces, displacements and bowed shapes must be interpreted as row-averaged values. In an actual core, each hexagonal core element interacts with six neighboring elements or the restraint system. Furthermore, the core consists of various core elements with different bending stiffnesses and of fuel assemblies with different burnup histories. These aspects require a three-dimensional consideration of the mechanical interactions among the core elements [5, 6].

A new computational code BEACON has been developed for analyzing the bowing behavior of an LMFBR's core in three dimensions. In the following sections, the outline of BEACON code and some numerical results obtained with this code are described.

2. Analytical Method

2.1 Mechanical Interactions among Core Elements

For analyzing the mechanical interactions among core elements, BEACON is based on the analytical method used in the SHADOW code [7], which treats the deformations of fuel pins within a hexagonal wrapper tube. It analyzes the mechanical interactions in three directions, which form angles of 60° with one another. The mathematical relationship between the lateral displacements and the contact forces is applied to all core elements on a line in one of the three directions. The three-dimensional bowed shape of each core element is calculated by composing the displacement in each of the three directions.

2.1.1 Decomposition of Displacement into Three Directions

If the core element is loaded by the contact forces from the adjacent element or the restraint system as shown in Fig. 1, in addition to being affected by the bending moment due to thermal, swelling and creep strains, the components of its displacement are expressed as follows,

$$EI \frac{d^2 y_\phi}{dz^2} = -M_\phi(Z) + \sum_{\phi'}^{\phi' \neq \phi} f_{\phi,\phi'} \cdot R_\phi \cdot (\ell_{\phi'} - Z) \cdot U(\ell_{\phi'} - Z)$$
$$+ R_\phi \cdot (\ell_\phi - Z) \cdot U(\ell_\phi - Z), \qquad (1)$$

$(\phi = 0^\circ, 60^\circ, 120^\circ)$

where E, I : Young's modulus and moments of inertia ;

ϕ : direction of a row, denoting the angle from the reference direction ;

y_ϕ : component of displacement towards the ϕ direction ;

M_ϕ : bending moment in the ϕ - Z plane ;

R_ϕ, ℓ_ϕ : contact force in the ϕ direction and its height ;

$f_{\phi,\phi'}$: ϕ component of the unit contact force $R_{\phi'}$;

$U(t)$: unit step function defined by

$$U(t) = \begin{cases} 1 & \text{for} \quad t \geq 0 \\ 0 & \text{for} \quad t < 0 \end{cases} .$$

Since the component y_ϕ in the considered direction is affected by the contact forces $R_{\phi'}$ in the other two directions, three equations given by eq. (1) must be treated simultaneously. However, the iterative procedure makes it possible to handle these equation independently by using the contact forces $R_{\phi'}$ obtained in the previous iteration. This leads to analyzing the mechanical interactions among core elements in only one of the three directions.

2.1.2 Relationship between Displacements and Forces

Let ℓ^i_k and R^i_k be the axial heights of contact points and contact forces, respectively, as shown in Fig. 2. The displacement, $y_i(Z)$, of the i-th core element at axial height Z can be divided into the free bowing, $y^T_i(Z)$, and the forced bending, $y^R_i(Z)$. The former is calculated using conventional beam theory, assuming the core element is subjected to a bending moment, $M_\phi(Z)$, only. The latter can be expressed in the explict form with the unknown variables, R^i_j, as in the following,

$$y^R_i(Z) = \sum_{j=1}^{m(i)} t_i \cdot h(\ell^i_j, Z) \cdot R^i_j - \sum_{j=1}^{m(i+1)} t_i \cdot h(\ell^{i+1}_j, Z) \cdot R^{i+1}_j , \qquad (2)$$

where

$$t_i = - \frac{1}{6EI_i} \qquad (3)$$

$$h(Z_1, Z_2) = \begin{cases} (Z_2)^3 - 3Z_1 (Z_2)^2 & \text{for } Z_1 \geq Z_2 \\ (Z_1)^3 - 3Z_2 (Z_1)^2 & \text{for } Z_1 \leq Z_2, \end{cases} \qquad (4)$$

and I_i is the moment of inertia of the i-th core element. The contact forces, R^i_j, can be obtained by solving the following matrix equation,

$$\begin{bmatrix} \frac{1}{2}A_1 & C_1 & & & & & \\ B_2 & A_2 & C_2 & & & 0 & \\ & \ddots & \ddots & \ddots & & & \\ & & B_i & A_i & C_i & & \\ & & & \ddots & \ddots & \ddots & \\ & 0 & & & B_n & A_n & C_n \\ & & & & & B_{n+1} & \frac{1}{2}A_{n+1} \end{bmatrix} \begin{bmatrix} \{R_1\} \\ \{R_2\} \\ \vdots \\ \{R_i\} \\ \vdots \\ \{R_n\} \\ \{R_{n+1}\} \end{bmatrix} = \begin{bmatrix} \{\phi_1\} \\ \{\phi_2\} \\ \vdots \\ \{\phi_i\} \\ \vdots \\ \{\phi_n\} \\ \{\phi_{n+1}\} \end{bmatrix} , \qquad (5)$$

where the matrices A_i B_i and C_i are given by

$$A_i = (t_{i-1} + t_i) \begin{bmatrix} h(\ell_1^i, \ell_1^i) & \cdots & h(\ell_{m(i)}^i, \ell_1^i) \\ \vdots & h(\ell_j^i, \ell_k^i) & \vdots \\ h(\ell_1^i, \ell_{m(i)}^i) & \cdots & h(\ell_{m(i)}^i, \ell_{m(i)}^i) \end{bmatrix}, \tag{6}$$

$$B_i = -t_{i-1} \begin{bmatrix} h(\ell_1^{i-1}, \ell_1^i) & \cdots & h(\ell_{m(i-1)}^{i-1}, \ell_1^i) \\ \vdots & h(\ell_j^{i-1}, \ell_k^i) & \vdots \\ h(\ell_1^{i-1}, \ell_{m(i)}^i) & \cdots & h(\ell_{m(i-1)}^{i-1}, \ell_{m(i)}^i) \end{bmatrix}, \tag{7}$$

$$C_i = -t_i \begin{bmatrix} h(\ell_1^{i+1}, \ell_1^i) & \cdots & h(\ell_{m(i+1)}^{i+1}, \ell_1^i) \\ \vdots & h(\ell_j^{i+1}, \ell_k^i) & \vdots \\ h(\ell_1^{i+1}, \ell_{m(i)}^i) & \cdots & h(\ell_{m(i+1)}^{i+1}, \ell_{m(i)}^i) \end{bmatrix}, \tag{8}$$

$$(t_0 = 0, \quad t_{n+1} = 0)$$

and the vector $\{R_i\}$ is given by

$$\{R_i\}^T = \{ R_1^i, R_2^i, \text{-------------}, R_{m(i)}^i \} . \tag{9}$$

The vector $\{\phi_i\}$ is calculated using the free bowing, $y_{i-1}^T(Z)$ and $y_i^T(Z)$, and initial gaps between the (i-1)-th and the i-th core elements at contact positions.

In the above equations, the axial heights of contact points, ℓ_k^i, are determined by the iterative procedure represented in Fig. 3. This figure shows the calculational scheme for locating the contact points, in which the inner loop checks whether clearance exists between core elements and the outer loop finds a new contact point.

2.2 Stress Analysis of Wrapper Tube

Based on the finite element creep analysis code FEMCRP [1], the finite element method (FEM) is applied to the stress analysis of a hexagonal wrapper tube, treating the tube as three-dimensional Rahmen finite elements. Since the details of FEM as applied to the structural mechanics can be found elsewhere, only an outline of the method will be given.

Consider a finite element g which lies between the nodal points i and j, as shown in Fig. 4. Neglecting the rotation around the Z-axis, the nodal displacement and force vectors, $\{\delta_g\}$ and $\{f_g\}$, are expressed by ten components,

$$\{\delta g\}^T = \{ w_i, u_i, v_i, \theta_{xi}, \theta_{yi}, w_j, u_j, v_j, \theta_{xj}, \theta_{yj} \} , \tag{10}$$

$$\{f_g\}^T = \{ Z_i, X_i, Y_i, M_{xi}, M_{yi}, Z_j, X_j, Y_j, M_{xj}, M_{yj} \} . \tag{11}$$

These vectors are related to each other by the following matrix equation,

$$[K_g] \{\delta_g\} = \{f_g\} . \tag{12}$$

The stiffness matrix $[K_g]$ is represented by the volume integral,

$$[K_g] = \iiint [C_g^{-1}]^T [B_g]^T E [B_g][C_g^{-1}] \, dxdydz, \tag{13}$$

where

$$[B_g] = [0, 0, 0, 0, 0, 1, -2x, -2y, -6zy, -6zx] , \tag{14}$$

$$[Cg^{-1}] = \begin{bmatrix} 1 & & & & & 0 & & & & \\ & 1 & & & 0 & & 0 & & & 0 \\ & & 1 & 0 & & & & 0 & 0 & \\ & & 0 & -1 & & & & 0 & 0 & \\ & 0 & & & 1 & & 0 & & & 0 \\ -1/\ell & & & & & 1/\ell & & & & \\ & -3/\ell^2 & & & -2/\ell & & 3/\ell^2 & & & -1/\ell \\ & & -3/\ell^2 & 2/\ell & & & & 3/\ell^2 & 1/\ell & \\ & & 2/\ell^3 & -1/\ell^2 & & & & -2/\ell^3 & -1/\ell^2 & \\ & 2/\ell^3 & & & 1/\ell^2 & & -2/\ell^3 & & & 1/\ell^2 \end{bmatrix}$$

The force vector $\{f_g\}$ can be calculated by means of thermal, swelling and creep strains, ε^t, ε^s and ε^c, and also external forces from adjacent elements,

$$\{f_g\} = \{f_g^t\} + \{f_g^s\} + \{f_g^c\} + \{f_g^a\} , \tag{15}$$

where

$$\{f_g^k\} = \iiint [C_g^{-1}]^T [B_g]^T E \varepsilon_x^k \, dxdydz, \tag{16}$$

$$(k = t, s, c)$$

and $\{f^a\}$ is the external force vector. Using eqs. (12) through (16) and the constraint conditions on a wrapper tube, the nodal displacement vector $\{\delta_g\}$ can be obtained.

The total axial strain ε_z is represented by

$$\varepsilon_z = [B_g][C_g^{-1}]\{\delta_g\} . \tag{17}$$

Hence the total strain, ε_{zi} and ε_{zj}, at the nodal points i and j can be calculated by the displacement vector $\{\delta_g\}$,

$$\varepsilon_{zi} = \{-\frac{1}{\ell}, \frac{6x}{\ell^2}, \frac{6y}{\ell^2}, -\frac{4y}{\ell}, \frac{4x}{\ell}, \frac{1}{\ell}, -\frac{6x}{\ell^2}, -\frac{6y}{\ell^2}, -\frac{2y}{\ell}, \frac{2x}{\ell}\}\{\delta_g\}, \tag{18}$$

$$\varepsilon_{zj} = \{-\frac{1}{\ell}, -\frac{6x}{\ell^2}, -\frac{6y}{\ell^2}, \frac{2y}{\ell}, -\frac{2x}{\ell}, \frac{1}{\ell}, \frac{6x}{\ell^2}, \frac{6y}{\ell^2}, \frac{4y}{\ell}, -\frac{4x}{\ell}\}\{\delta_g\}, \tag{19}$$

where ℓ is the length of the finite element g. The stresses σ_{zi} and σ_{zj}, which cause the material to creep at the nodal points i and j, respectively, can be obtained by Hooke's law of elasticity,

$$\begin{aligned} \sigma_{zi} &= E\,[\varepsilon_{zi} - (\varepsilon_{zi}^{t} + \varepsilon_{zi}^{s} + \varepsilon_{zi}^{c})] , \\ \sigma_{zj} &= E\,[\varepsilon_{zj} - (\varepsilon_{zj}^{t} + \varepsilon_{zj}^{s} + \varepsilon_{zj}^{c})] . \end{aligned} \tag{20}$$

3. Description of BEACON code

On the basis of the analytical method described in Section 2, a new computational code BEACON can be summarized as follows :

(1) It analyzes the whole core bowing behaviors,at various power levels during the reactor startup after refueling ; after a certain time of operation at full power; after cooling down to the refueling temperature with full power history during one refueling cycle and subsequent relaxation of the restraint system for fuel handling.

(2) Creep strain is calculated by proceeding in small constant-stress time increments. The additional amounts of bowing due to creep and swelling are calculated in several time increments.

(3) The following factors affecting the deformation of core elements are taken into account : radial expansions of core elements, support grid plates and core restraint rings ; free bowings of core elements ; forced bending of a core element caused by neighboring elements or the restraint system.

(4) The code can treat six kinds of core elements with different bending stiffnesses, and fuel assemblies with various burnup histories.

(5) It can handle 30°, 60°, 120° and 180° sectors of the whole core, assuming a boundary condition of rotational or reflective symmetry.

(6) Clearance gaps are included at the lower support points (e.g., nozzle receptacles) as well as at load pads, allowing the condition that each core element may be free to tilt within the clearance space.

4. Numerical Results

A typical LMFBR's core has been treated by BEACON for analyzing the core bowing history during one equilibrium cycle (123 full power days). Assuming the boundary condition of rotational symmetry, a 60° sector of the core is handled. Its cross-sectional view without the reflector region is shown in Fig. 5. The number of cycles each core element has stayed in the core at the begining of the considered cycle is also given. The major geometrical data of the core elements are given in Table I.

Figs. 6 and 7 show the axial shape of bowed core elements during reactor startup and after 123 full power days, respectively. These figures represent the components of core element displacement in the R direction shown in Fig. 5 , which are increased by a factor of 10 in comparison with the distance between the center lines of adjacent core elements. The figures also show the load from six neighboring elements or the outer restraint system.

From Fig. 6, it can be seen that core elements are bowed towards the center of the core with increasing power. This is due to both the increase of the radial temperature gradient and the outer constraint at the top of the outermost reflectors. The effects of swelling and creep on the core bowing can be found from Fig. 7. After 123 full power days, the maximum values of lateral displacement and contact force increase by 5% and 10%, respectively, compared with those at the beginning of the cycle. These values are changed by +28% and -74% after cooling down to the refueling temperature and relaxation of the restraint system.

During the equilibrium cycle, the maximum forces are small enough to prevent collapse of the wrapper tube, and to enable fuel assemblies to be discharged at refueling. The maximum stress and lateral displacement are also within the design limits.

The computational time in the present study is about 9 minutes on the IBM 370/158.

5. Conclusions

A new computational code BEACON has been developed for three-dimensional analysis of the bowing behavior of an LMFBR's core during reactor operation from startup to shutdown and refueling. The code is based on the following analytical methods :

(1) The mechanical interactions between core elements are analyzed in each of three directions, which form angles of 60° with one another.

(2) The finite element method is applied to the stress analysis of a hexagonal wrapper tubes, treating the tube as three-dimensional Rahmen finite elements.

Application of BEACON to a typical LMFBR's core confirms that the code is an effective tool for parametric studies as well as for detailed structural analysis. In particular, BEACON provides useful information on : effect of core restraint system (e.g., outer clamping system, core restraint rings, leaning post system) ; optinum design of core elements (e.g., axial height and thickness of load pads, core element arrangement pitch) ; and refueling conditions with particular emphasis on fuel assembly exchange.

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Table I Geometrical data of core elements

Number of core elements in 60° sector	120	
Core element length	3600	mm
Arrangement pitch	115.6	mm
Outer wall flat-to-flat distance	110.6	mm
Axial height of load pads	2340/3600	mm
Load pad thickness	2.0	mm
Flexural rigidity : fuel assembly, control rod element	0.30×10^{11}	$kg \cdot mm^2$
reflector	1.44×10^{11}	$kg \cdot mm^2$

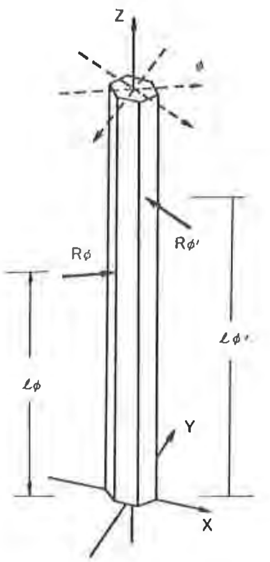

Fig.1 Model of a core element loaded by contact forces

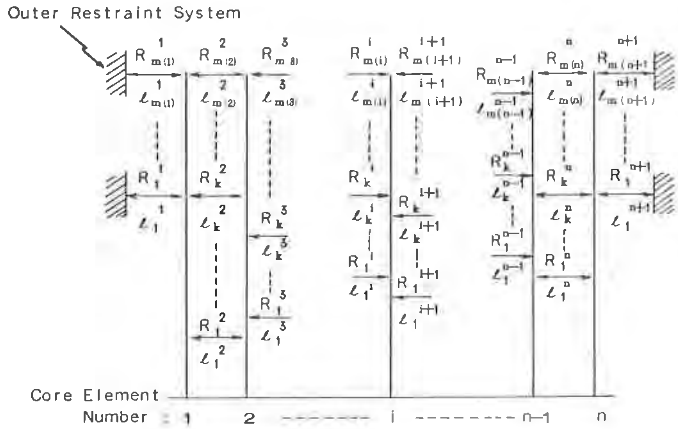

Fig.2 Contact force, R_j^i, and axial height of contact point, l_j^i, on core elements in a row

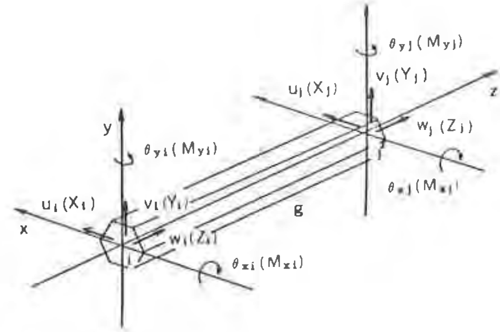

Fig.4 Nodal displacements and forces of a finite element g

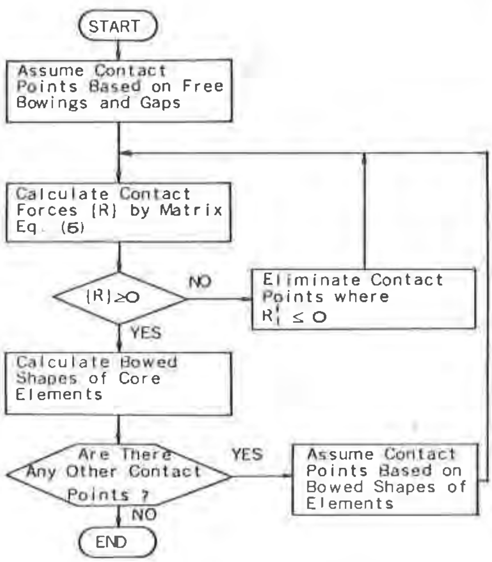

Fig.3 Flow chart for locating contact points on core elements

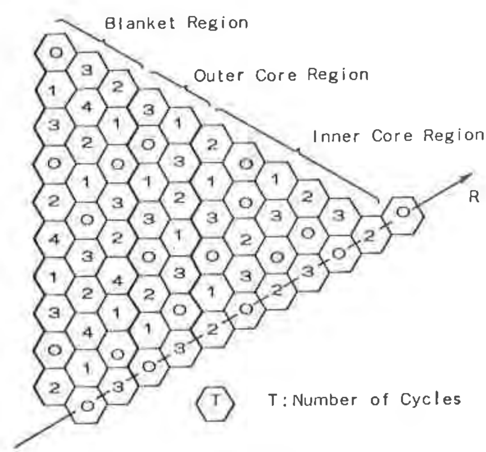

Fig.5 Cross-sectional view of 60^0 sector of core

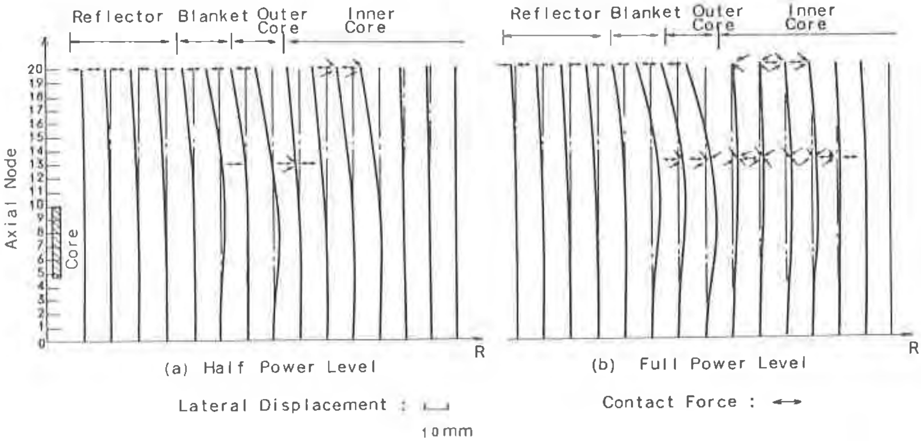

Fig.6 Axial shape of bowed core elements during reactor startup

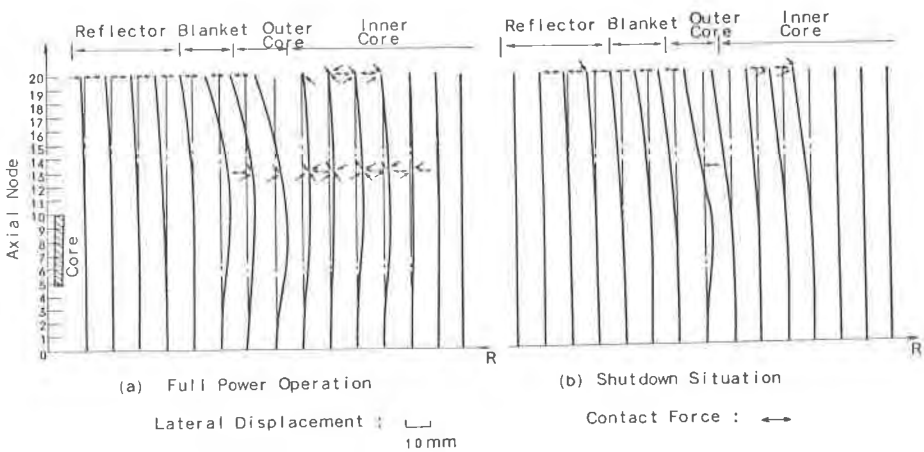

Fig.7 Axial shape of bowed core elements after 123 full power days