SMiRT–12 / K. Kussmaul (Editor)

K13/5

STATISTICAL METHOD TO COMPUTE FLOOR RESPONSE SPECTRA

F. Ben Jemaa[1], D. Brochard[1], F. Gantenbein[1] and R.J. Gibert[2]

[1]CEA - CE-Saclay DMT/SEMT/EMSI, F-91191 Gif-sur-Yvettte Cedex, France
[2]CEA - CE-Saclay, DRN, F-91191 Gif-sur-Yvette Cedex, France

1. INTRODUCTION

The seismic analysis of equipements needs the determination of floor response spectra at the points where the equipements are fixed on the supporting structure. The classical spectral methods (SRSS, CQC) do not allow this determination which can be done in using the time integration method ; this last approach needs to perform numerous calculations with various realizations of the seismic excitation (synthetic signals).

This paper aims to present a method allowing to calculate the response's statistical properties (mainly the variance) of a structure loaded by a transient excitation. This approach is based on an approximation of the structure's transient transfer function which validity is studied through comparisons with numerical (time integration) simulations. This method can be easily extended to the determination of the floor response spectra in including in this step, the peak factor concept allowing to calculate the mean maximal value of a process from its standard deviation.

2. STATISTICAL PROPERTIES OF A STRUCTURE'S RESPONSE LOADED BY A TRANSIENT EXCITATION

2.1. Background

The method proposed in this paper consists in characterizing the structure response through its statistical properties (i.e. the power spectral density (PSD) and consequently the standard deviation).

As usually in the seismic analysis, the structure, which is assumed to be linear, will be modelled through its eigen modes. In that case the equations governing the modal coefficients are uncoupled 1 degree of freedom (DOF) oscillator equations.

The first step to determine the structural response is then to characterize the response of a 1 DOF oscillator to a transient excitation. For that purpose an excitation model is needed, simple enough to allow analytical developments but representing the excitation transient aspect.

2.2. Excitation model

It is assumed that the seismic excitation accelorogramm γ_o (t) can be written in the following way (separable process) [1] and [2] :

$$\gamma_o(t) = E(t)\ \gamma(t) \tag{1}$$

Where : - $\gamma(t)$ is a stationnary random process characterized by its PSD $S_\gamma(f)$,
- $E(t)$ is a deterministic envelop function which variations are slow in comparison whith the fluctuations of $\gamma(t)$.

2.3. Analysis of a 1 DOF oscillator response

Let's consider a 1 DOF oscillator characterized by its unit mass, its frequency f_o ($\omega_o = 2\Pi f_o$) and its damping coefficient β_o, loaded by a transient excitation (separable process) $\gamma_o(t)$. The expression of the displacement instantaneous PSD S_x(f, to) is complicated but may be formally written as ([2] and [3]) :

$$S_x(f,to) = H_o(f,t_o)\ H_o^*(f,t_o)\ S_\gamma(f) \tag{2}$$

Where $H_o(f,t_o)$ is the instantaneous transfer function at time t_o (H_o^* is the conjugate of H_o). A simplified expression for $H_o(f,t_o)$ has been proposed in [3] in order to keep the general shape of a transfer function as in the stationnary case but in introducting an equivalent amplitude $C(t_o)$ and an equivalent damping $\beta(t_o)$ depending on the transient state of the oscillator :

$$H_o(f,t_o) = \frac{C(t_o)}{\omega_o^2 - \omega^2 + 2j\beta(t_o)\ \omega\ \omega_o} \tag{3}$$

$C(t_o)$ and $\beta(t_o)$ have been adjusted in order to get the right solution with two specific ($\gamma_o(t)$) excitations : a white noise and a sine at frequency equal to f_o.$C(t_o)$ and $\beta(t_o)$ may be written :

$$\beta(t_o) = \frac{2\int_0^{t_o} e^{-2\beta_o\omega_o\tau}\ E^2(t_o-\tau)\ d\tau}{\omega_o\left[\int_0^{t_o} e^{-\beta_o\omega_o\tau}\ E(t_o-\tau)\ d\tau\right]^2} \tag{4}$$

$$C(t_o) = \frac{2\int_0^{t_o} e^{-2\beta_o\omega_o\tau}\ E^2(t_o-\tau)\ d\tau}{\int_0^{t_o} e^{-\beta_o\omega_o\tau}\ E(t_o-\tau)\ d\tau} \tag{5}$$

The evolution of the displacement variance as a function of the time is derived by integrating $S_x(f,t_o)$ in the frequency domain :

$$\sigma^2(t_o) = \int_o^{\infty} S_x(f,t_o)\ df \qquad (6)$$

2.4. Case of a multi DOF structure

Using a modal representation of the structure, the motion equations are reduced to a set of uncoupled equations. The equation governing the ith modal coefficient α_i may be written :

$$\ddot{\alpha}_i + 2\ \beta_i\ \omega_i\ \dot{\alpha}_i + \omega_i^2\ \alpha_i = -\ b_i\ \gamma_o(t) \qquad (7)$$

ω_i, β_i, b_i being respectively the pulsation, the damping coefficient and the participation factor of the ith mode.

The displacement $X(P,t_o)$ at a point P is :

$$X(P,t_o) = \sum_{\text{modes } i} \alpha_i(t_o)\ \phi_i(P) \qquad (8)$$

$\phi_i(P)$ is the ith mode shape at the point P.
Proceeding in the same way as for the 1 DOF oscillator, and due to equation [8], the PSD of the motion at the point P and its variance may be determined in introducing the structure transfer function H_p :

$$H_P\ (f,t_o) = \sum_i b_i\ \phi_i(P)\ H_i(f,t_o) \qquad (9)$$

$H_i(f,t_o)$ is the transient transfer function for the ith modal coefficient. It can be approximated in the same way as in equation (3).

The equation 9 allows to characterize the motion of each point of the structure. It constitutes the first step for the determination of the floor response spectra.

It is also worth noticing that the proposed method allows a correct calculation for the PSD (and consequently for the variance) whatever the difference between the modes frequencies (even for closely spaced modes).

2.5. Validity of this approach - Comparison with numerical simulations

It is worth verifying the validity of this approach and especially the approximations proposed in the calcultions of the transfer functions (coefficients C(t) and $\beta(t)$). For that purpose comparisons with numerical simulations have been performed on both the PSD and the time evolution of the variance. In all calculations the envelope function E(t) has been choosen so that it increases linearly from 0 to 1 in 2 seconds, remains constant at 1 during 6 seconds and decreases linearily to 0 in 2 seconds. Different cases have been studied :

- the stationnary process $\gamma(t)$ is a white noise and the structure is reduced to a 1 DOF oscillator (frequency 3 Hz, damping 5 %),
- the stationnary process $\gamma(t)$ is a white noise and the structure is represented by 4 modes (table 1),
- the stationnary process $\gamma(t)$ is a white noise filtered by a 1 DOF oscillator (frequency 5 Hz, damping 5 %) and the structure is also a

1 DOF oscillator (frequency 3 Hz, damping 5 %).

Results (displacement variance and PSD) are presented on figures 1 to 3. It can be observed a good agreement with the numerical simulation for both PSD and variance. In the case of a 1 DOF oscillator excited by a filtered white noise, it has been also observed that the maximal variance is reached during the transient part of the response, when the frequency of the filtering oscillator is smaller than the frequency of the structure.

3. DETERMINATION OF THE FLOOR RESPONSE SPECTRA

To calculate the floor response spectra the following procedure is proposed :

- The floor motion is characterized through a separable process. The PSD of the associated stationnary process is choosen as the mean PSD (i.e. the time averaged PSD during the excitation) of the floor absolute acceleration during the seism and the envelope function has the shape (time evolution) of the variance of the floor motion. The method described in paragraph 2 is applied to get the PSD and the variance of the floor motion.
- The response statistical properties of a 1 DOF oscillator (characterized by a frequency f_0 and a damping β_0) clamped on the floor may be determined in using the method described in paragraph 2 from the separable process associated to the floor motion (as above proposed). This analysis allows to determine the standard deviation of this oscillator as a function of the time.
- The mean maximal response of this oscillator (i.e. the response spectrum value for the frequency f_0 and the damping β_0) is determined in introducing the peak factor concept. The peak factor of a process is the ratio between the maximal value and the standard deviation of this process. Numerous studies have been realised on the peak factors ;the method proposed in [4] has been applied but improved to take into account the transient aspect of the response as suggested in [2].

An example of floor response spectrum is presented on figure 4 ; it corresponds to a supporting structure reduced to a 1 DOF oscillator (15 Hz, 5 %) excited by a white noise wheighted by the envelope function defined in paragraph 2.5. The results show that the analytical approach overestimates slightly the simulations results (15 %). Work is in progress to improve these results, concerning mainly the peak factor when the PSD of the process presents 2 widely separated peaks, which is the case for an oscillator which frequency is great by comparison with the frequency of the excitation (asymptotic value of the spectrum).

4. CONCLUSION

The method proposed in this paper allows to calculate, without any time integration the response PSD and the variance of a structure loaded by a transient excitation. Comparisons with numerical simulations show a good agreement. Moreover this method operates correctly even for closely spaced modes.

This method may be applied to the determination of floor response spectra in introducing the peak factor concept. The first results show the interest of the method ; nevertheless some improvements are necessary especially for the calculation of the peak factors.

REFERENCES

[1] F. JEANPIERRE, F. ROULLIER (octobre 1979). "Caractéris-tiques des signaux sismiques pour l'analyse des structures". Conférence CEA-EDF sur les vibrations des structures dans le domaine industriel (Jouy, France).

[2] M. LIVOLANT, F. GANTENBEIN, R.J. GIBERT (octobre 1982). "Méthode statistique pour l'estimation de la réponse des structures aux séismes". Journal de Mécanique-Matériaux- Electricité, n° 394-395.

[3] R.J. GIBERT (1988). "Vibration des structures". Editions Eyrolles, Paris.

[4] E.H. VANMARCKE (1976). "Structural response to earthquakes (chap. 8). Journal of seismic risk and engineering decision.

Mode	Frequency (Hz)	Participation factor	Mode shape	Damping (%)
1	2,018	1,5718	0,5104	4,49
2	5,607	0,7351	0,6363	34,65
3	8,601	0,2725	- 1	19,91
4	19,344	0,0229	0,0270	20,45

TABLE 1 : Multi DOF structure eigen modes

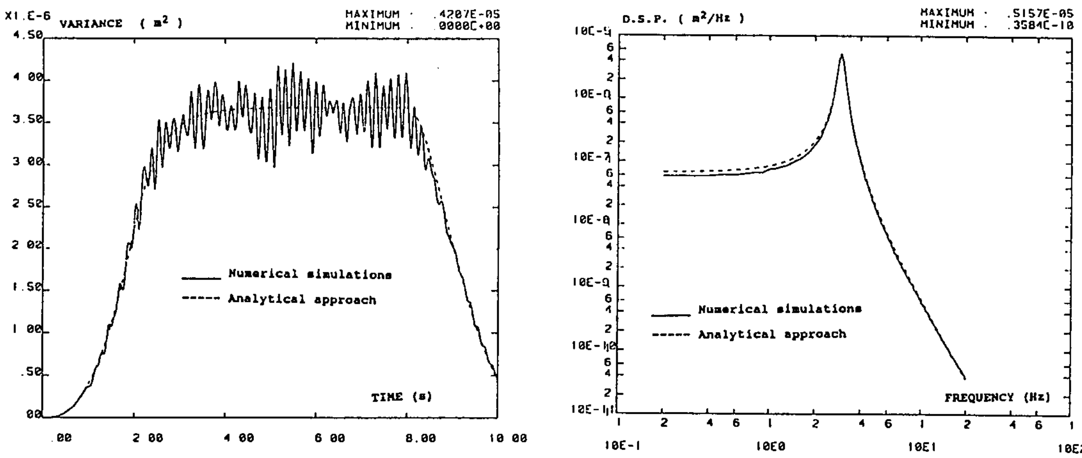
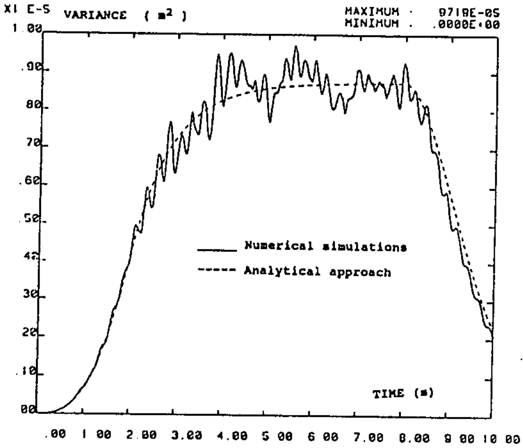


Figure 1 : 1 D.O.F. oscillator (3.Hz,5%) excited by a transient white noise



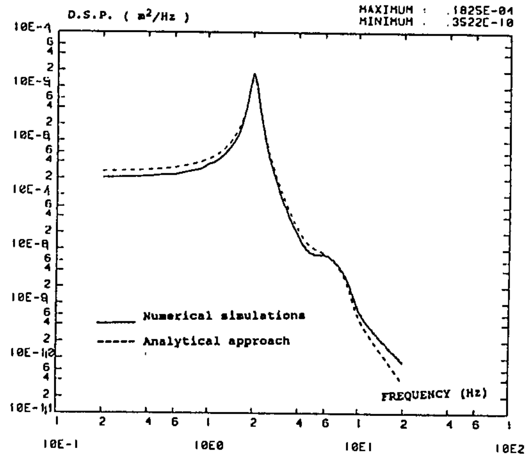
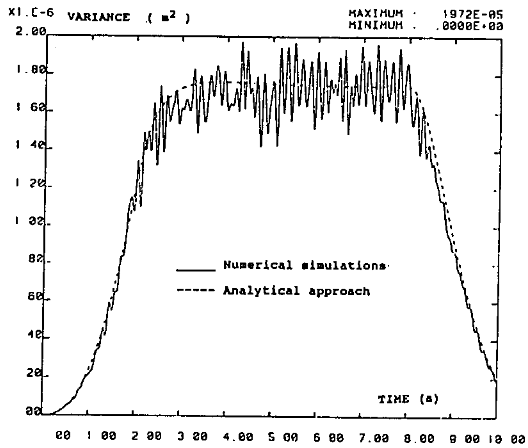


Figure 2 : Multi D.O.F. structure excited by a transient white noise.



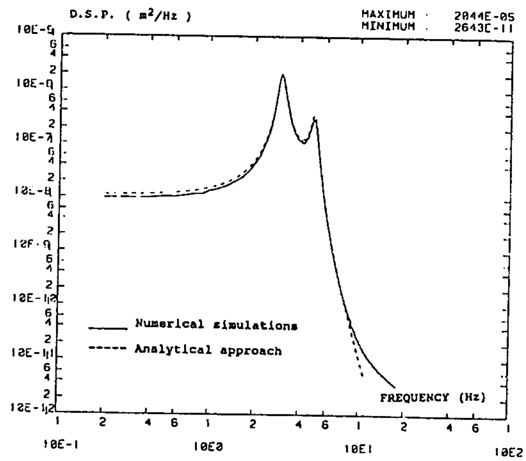


Figure 3 : 1 D.O.F. oscillator (3.Hz,5%) excited by a transient filtered (5.Hz,5%) white noise.

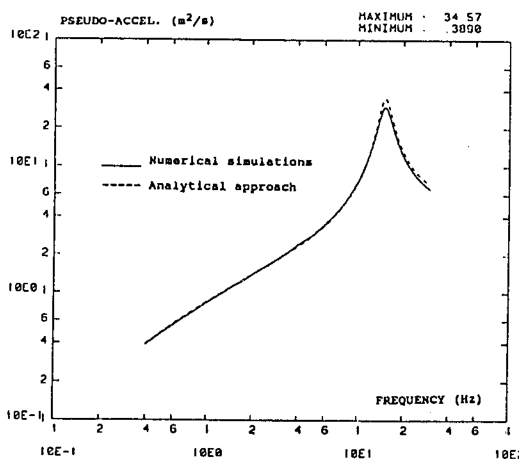


Figure 4 : Floor response spectrum (pseudo-accel. 5%). Structure 1 D.O.F. oscillator (15.Hz,5%) excited by a transient white noise.