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A THREE-DIMENSIONAL FINITE-ELEMENT FORMULATION FOR FLUID-STRUCTURE INTERACTION

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Summary

The safety evaluation of fast reactors often involves the modelling of various types of structural components which interact with surrounding fluid in three-dimensional space. In this paper, several key areas for performing three-dimensional fluid-structure interaction calculations are discussed. A new development is presented for a three-dimensional hexahedral hydrodynamic finite-element based upon a Quasi-Eulerian approach. By using trilinear shape functions and assuming a constant pressure field within the element, simple relations are obtained for internal nodal forces. The forms developed are easily coded and computationally efficient. Because the formulation is based upon a rate approach it is applicable to problems involving large displacements. Only adiabatic processes are considered.

A previously developed triangular plate-shell element is used for structural representation. The plate element is formulated using a corotational coordinate system which enables the use of simple strain-nodal displacement and nodal stress-force relations. This plate element can be subjected to linear in-plane and cubic transverse displacements. The orientation of lumped masses is described by unit vectors so that arbitrarily large rotations can be treated. The material response is taken to be elastoplastic. The integration of the elastoplastic constitutive equation employs a subincrementation procedure and a yield-surface return scheme so that the end-of-step stress lies on the yield surface.

The above elements are used in a three-dimensional fluid-structure interaction code named NEPTUNE. The current version uses an explicit temporal integrator. The code can economically solve large scale, nonlinear, fluid-structure interaction problems in three-dimensional space.

The difficult tasks (1) of input data preparation and checking and (2) the displaying of computed results are discussed. The use of automatic mesh generators, input data error analysis, and graphical display of the finite element meshes used to model reactor components are shown to ease the input task. In the final section, the above formulation is applied to the three-dimensional fluid-structure interactions involved in a current reactor safety problem.

1. Introduction

When assessing the integrity of three-dimensional structures which either contain or are embedded in a fluid it is oftentimes necessary to account for the fluid structure interactions which occur under transient dynamic loadings. For the class of problems in which both the structure and fluid undergo large displacements closed form analytical solutions are not generally available. The finite element method has been accepted as an analytical tool for nonlinear structural analysis and is currently gaining acceptance in the area of nonlinear fluid-structure interaction dynamics.

In this paper, a Quasi-Eulerian finite element formulation for three-dimensional fluid-structure interactions is presented. In this approach the kinematics of the material and the finite elements can be different. It should be noted that the Lagrangian and pure Eulerian formulations are special cases of the Quasi-Eulerian formulation. This method is similar to the Arbitrary Lagrangian Eulerian technique developed by finite difference researchers [1]. Two-dimensional Quasi-Eulerian finite element formulations have been reported [2,3].

In this paper, the forms for a three-dimensional Quasi-Eulerian formulation are presented. Because the formulation uses a velocity-strain tensor it is applicable to large deformation problems. Also, by using low order displacement fields in conjunction with one point quadrature in each element, simple relations are obtained for nodal forces. The finite element format enhances the versatility of the method because of the ease of coupling various types of elements, particularly fluid and structural elements. The fluid element developed here is an eight-node hexahedron. Within each element trilinear isoparametric interpolating functions are used to describe the unknown fields. This element is coupled to a three-dimensional plate-shell element for fluid-structure interaction calculations.

The semi-discrete equations of motion are numerically integrated with an explicit temporal integrator. Because the integration of these equations for undamped systems produces spurious oscillations, artificial viscosities are used here to produce numerical damping when needed.

2. Formulation

In this section, the finite-element equations for a single element are presented. The resulting forms can be used to represent a Lagrangian, Eulerian or Quasi-Eulerian hydrodynamic element. Since a formulation which is applicable to large deformations is desired, a suitable strain measure to use is the velocity strain tensor (rate-of-deformation tensor). The Cauchy stress and velocity strain tensors are split into hydrostatic and deviatoric components. In the finite element method, the velocity field for each element is approximated by

$$\dot{u}_i = \phi_I \dot{d}_{iI} \qquad (1)$$

where ϕ_I is the interpolation function for the Ith node and d_{iI} is the displacement of node J in the ith direction. A superposed dot is used to denote a temporal derivative.

When using the finite-element approach for solving fluid-structure interaction problems the governing differential equations of both the fluid and structure are transformed into a single set of spatially discretized equations in terms of nodal variables. Both the fluid and structural portions of the system contribute to these nodal equations which makes the finite-element approach ideal for blending, in a consistant manner, fluid elements with structural elements. The governing nodal equations for an element are

$$m_{IJ}\ddot{d}_{iJ} + f_{iI}^{int} = f_{iI}^{ext} \tag{2}$$

in which m_{IJ} is the nodal mass matrix and f_{iI}^{int}, f_{iI}^{ext} are the internal and external nodal forces, respectively.

The following three terms contribute to internal forces. The internal nodal force due to the hydrostatic material response and the deviatoric behavior are given, respectively, by

$$f_{kI}^{int,h} = \int_V p\phi_{I,k}dV \quad ; \quad f_{kI}^{int,d} = \int_V S_{ij}G_{Iijk}dV \tag{3}$$

where

$$G_{Iijk} = \frac{1}{2}(\phi_{I,j}\delta_{ik} + \phi_{I,i}\delta_{jk}) - \frac{1}{3}\phi_{I,k}\delta_{ij} \tag{4}$$

and δ_{ij} the Kronecker delta, p the hydrostatic pressure, and S_{ij} the deviatoric components of stress. The third contribution to internal nodal forces comes from the transport term which appears in both the Eulerian and Quasi-Eulerian formulations. With these approaches, the velocity of the material $\dot{u}_k^M$ is different from the velocity of the finite element grid $\dot{u}_k^G$. The approximation to the transport term is given by

$$f_{kI}^{int,t} = \int_V \rho\phi_I \frac{\partial \dot{u}_k^M}{\partial x_j} \dot{u}_j^D dV \tag{5}$$

where $\dot{u}_j^D$ is the difference between the material and grid velocities and ρ is the density. From (5) it is seen that with a Lagrangian approach the transport term is zero since the material and grid have the same kinematics.

The specific forms for the internal nodal forces for an 8-node hexahedral hydrodynamic element (Fig. 1) are developed below. The element interpolation functions ϕ_I used here are the trilinear functions described by Zienkiewicz [4]:

$$\phi_I = \frac{1}{8}(1 + \xi\xi_I)(1 + \eta\eta_I)(1 + \zeta\zeta_I) \tag{6}$$

where ξ, η, and ζ are the natural coordinates of the element.

The explicit evaluation of the above force integrals is a difficult task. An implicit evaluation can be performed by using numerical integration, say for example Gauss integration. However, this approach can be computationally expensive especially when higher order schemes, such as 3 x 3 x 3 point quadrature, are used. The initial philosophy followed here is that the use of a larger number of low order elements would be computationally more efficient than fewer higher order elements.

By assuming that the pressure field within the element is constant, it can be shown [3] that the hydrostatic internal nodal loads are given by the linear form

$$f_{kI}^{int,h} = pH_{kI} \tag{7}$$

where H_{kI} is defined in the rate-of-volme change relationship given by

$$\dot{V} = H_{kI}\dot{d}_{kI} \tag{8}$$

The form for H_{kI} is obtained by computing the volume of an element with a one point quadrature formula and then taking the temporal derivative. The above procedure gives the following form for the hydrostatic internal nodal forces

$$f_{iI}^{int,h} = \frac{p}{16}\left[X_{jPJ}\left(X_{kKM} - X_{kNL}\right) - X_{jKM}\left(X_{kPJ} - X_{kNL}\right) + X_{jNL}\left(X_{kPJ} - X_{kKM}\right)\right] \tag{9}$$

where, for example, X_{jPJ} is defined to be the difference between the jth component of nodes P and J. The appropriate node numbers are given in the permutation table (Table 1).

To determine the form for the deviatoric internal forces (equation 3) a point Gaussian quadrature is used, and the resulting form is given by

$$\begin{aligned} f_{iI}^{int} = \frac{1}{128J}\Bigg(&\left[X_{jPJ}\left(X_{kKM} - X_{kNL}\right) + X_{jKM}\left(X_{kJL} - X_{kPN}\right) + X_{jLN}\left(X_{kJP} - X_{kKM}\right)\right] S_{ii} \\ &+ \Big[X_{kPJ}\left(X_{iKM} - X_{iNL}\right) + X_{kKM}\left(X_{iJL} - X_{iPN}\right) \\ &+ X_{kLN}\left(X_{iJP} - X_{iKM}\right)\Big] S_{ij} + \Big[X_{iPJ}\left(X_{jKM} - X_{jNL}\right) \\ &+ X_{iKM}\left(X_{jJL} - X_{jPN}\right) + X_{iLN}\left(X_{jJP} - X_{jKM}\right)\Big] S_{ik}\Bigg) \end{aligned} \tag{10}$$

where J is the Jacobian of the isoparametric transformation. Repeated subscripts do not indicate a sum. Because of space limitations, detail forms for the remaining terms are not given.

For both the Eulerian and Quasi-Eulerian approaches the mass of each finite element changes with time. The global principle of conservation of mass is used to update the element densities and nodal masses. The principle applied at the element level gives

$$\frac{\partial}{\partial t}\int_V \rho dV = -\int_S \rho \dot{u}_i^D n_i dS \tag{11}$$

where n_i is the outward unit normal. By assuming that the density is constant within the element and across the surface and using a one point quadrature rule on the integral on the right side, the rate-of-change of density is approximated by

$$\frac{\partial \rho}{\partial t} = -\frac{W^2}{V}\rho \bar{v}_i^D N_i - \frac{\rho}{V} H_{kI} \dot{d}_{kI}^G \tag{12}$$

where

$$N_i = \frac{1}{8}\left(X_{kLJ} X_{jKI} - X_{kKI} X_{jLJ}\right) \tag{13}$$

and W is a weighing factor and $\bar{v}_i^D$ is the average velocity in the ith direction on the surface. In equation (13) the subscripts (i,j,k) take the permuted values of (x,y,z), and the subscripts (I,J,K,L) are the four node numbers of each face of the hexahedron which allow mass influx. The node subscripts are consecutively numbered counterclockwise about an outward normal to the surface. An element adjacency table is used to determine if mass transfer occurs between elements with a common face. With the density change in each element determined, the nodal masses can accordingly be modified.

For the class of problems which is of interest here, the fluid can be considered to be Newtonian and isotropic. In terms of dilatational and deviatoric components the Newtonian constitutive equations are

$$\bar{p} = p - \kappa\dot{\varepsilon}_{ii} \quad ; \quad S_{ij} = 2\mu\dot{e}_{ij} \tag{14}$$

where $\dot{e}_{ij}$ are the deviatoric components, κ is the bulk viscosity and μ the dynamic viscosity.

The numerical solution of certain transient problems result in high frequency oscillation which are not present in the analytical solution. Therefore, in order to reduce or eliminate spurious oscillations when they occur artificial viscous stresses, a form of numerical damping, are introduced. Specifically, the forms used for the hydrostatic and viscous stresses are given respectively by

$$p^v = -\frac{1}{3}\sigma^v_{ii} = -d_L\,\rho\,h\,(C_1 - \frac{1}{3}C_2)\,\dot{\varepsilon}_{ii} \quad ; \quad S^v_{ij} = 2d_L\,\rho\,h\,C_2\,\dot{e}_{ij} \tag{15}$$

where h is a characteristic length, C_1 and C_2 are the dilatational and shear wave speeds, respectively, and d_L a fraction of critical damping for the highest mode. The characteristic length here was taken to be the length of the smallest side of the hexahedron.

In addition to the above type of viscosity there is need for another type called antihourglass viscosity. The form incorporated in our computations was proposed in reference [5]. The antihourglass nodal force for the Ith node in the ith direction is given by

$$f^a_{iI} = \frac{\rho V C_a}{24\Delta t}\,[2\,(\dot{d}_{iJ} + \dot{d}_{iM} + \dot{d}_{iL}) - (\dot{d}_{iN} + \dot{d}_{iP} + \dot{d}_{iK}) - 3\dot{d}_{iI}] \tag{16}$$

in which ρ and V are the current density and volume, respectively, of the element, C_a is a constant with a recommended value of 20 and Δt is the computational time step.

A previously developed triangular plate-shell element [6] is used for structural representation.

4. Solution of Equations

The Newmark $-\beta$ temporal integrator is used to integrate the equations of motion at time t. By using an explicit form ($\beta = 0$) of the integrator in conjunction with a diagonal mass matrix, an efficient solution to the uncoupled equations is achieved. Because the transport nodal forces depend upon the velocity at time t which is not available these velocities are extrapolated from the previous values according to the relations proposed in [2]. For each time step the computations proceed in the following manner. First, the nodal displacements and extrapolated nodal velocities are computed at time t. Next, the nodal forces are evaluated and the nodal masses updated. The nodal accelerations are then computed from equation (1) and the nodal velocities and displacements determined from Newmark's relations. The above computation cycle continues until the final problem time is reached. For the Lagrangian element the extrapolation, mass updating and transport force calculation are bypassed.

5. Overview of Program Structure

In the safety evaluation of fast reactors the three-dimensional solution of fluid-structural interaction systems often leads to large, complex finite element models which can take a considerable amount of computer time to solve. To economically solve these problems it is advantageous to divide the programming effort into the development of three separate but complementary codes: a preprocessor code, a computational code, and a postprocessor code. This was the philosophy followed when the above formulation was transformed into a computer code. The computational code is named NEPTUNE which is a quasi-acronym obtained from Nonlinear Elastic Plastic Three-Dimensional Fluid Structure Interaction Code. The preprocessor and post-processor codes are named PRENEP and POSTNEP, respectively.

The purpose of the preprocessor is to preprocess input data. This includes automatic data generation, data-error analysis, and graphical display of the finite-element meshes used to model reactor components. The inclusion of as many options as possible allows the user to generate a mesh with the least amount of error. Options which should be included are: (1) the capability to generate elements within a region by specifying a minimal number of points on its bounding surface, (2) the capacity to generate both hydrodynamic and structural elements within the region and on the boundary, and (3) the capability to obtain a mirror image of a previously generated mesh by reflecting the original about a plane. A three-dimensional mesh plot should be used to quickly check the geometric data and boundary conditions.

During the computations NEPTUNE generates several tape files for (1) restarting a problem run from a normal ending, (2) restarting a run from an abnormal ending such as a system breakdown, and (3) postprocessing computed results.

Once the computations are completed, the analyst faces the task of data interpretation. This task becomes tractable by postprocessing the computed results obtained from a tape file. Element and nodal time histories can be displayed digitly or graphically for interpretation. The systems configuration can be displayed at various instants during the deformation process in several hard copy forms such as CALCOMP plots or 35mm output. Also, the continuously changing system configuration can be viewed with a 16mm movie.

6. Results

During the verification phase of this work several wave propagation problems in cartesian, cylindrical and spherical coordinates were run. Some of these problems involved fluid-structure interactions in which the structure responded elastoplastically. Because of space limitations these problems will not be discussed here, but they will be available for presentation at the conference.

The following brief discussion is presented to illustrate the application of the above formulation to a realistic reactor problem. This problem is of current interest in the safety evaluation of a pool LMFBR during a hypothetical core disruptive accident (HCDA). During this accident, the expanding core creates pressure waves which propagate through the sodium pool and load the embedded structures (e.g. intermediate heat exchanges and primary sodium pumps) and primary tank. The details of a preliminary study of this problem are reported by Kulak [7]. Here a brief description of the model is presented along with some of the results. A model based upon a 12° repeated symmetry of the system is shown in Fig. 2. It includes a 12° sector of the sodium pool, one half of an in-tank component, and a 12° sector of the primary tank. This model was subjected to an expanding core gas bubble, which was modelled as a pressurized cavity, and the resulting fluid-structure interaction studied. Figure 3 shows the finite element mesh of the model in the deformed configuration 50ms after the start of the accident.

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I	J	K	L	M	N	P
1	2	3	4	5	6	8
2	3	4	1	6	7	5
3	4	1	2	7	8	6
4	1	2	3	8	5	7
5	8	7	6	1	4	2
6	5	8	7	2	1	3
7	6	5	8	3	2	4
8	7	6	5	4	3	1

Table I. Permutation Table

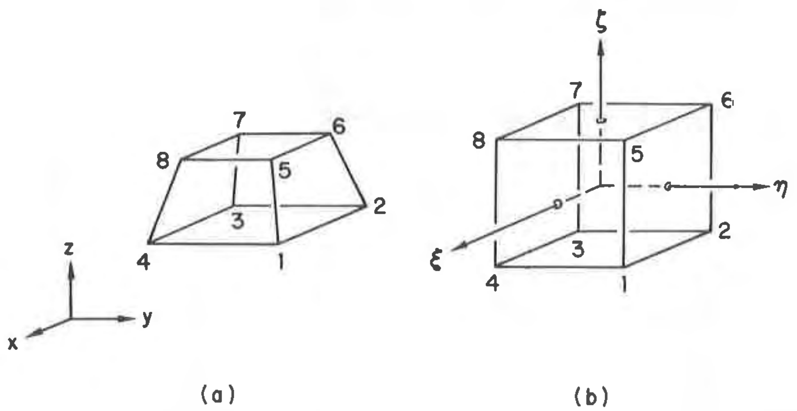

Fig. 1. Eight-Node Hexahedron. (a) Arbitrary Hexahedron in Global (x,y,z) Coordinate System. (b) Hexahedron in Natural (ξ,η,ζ) Coordinate System.

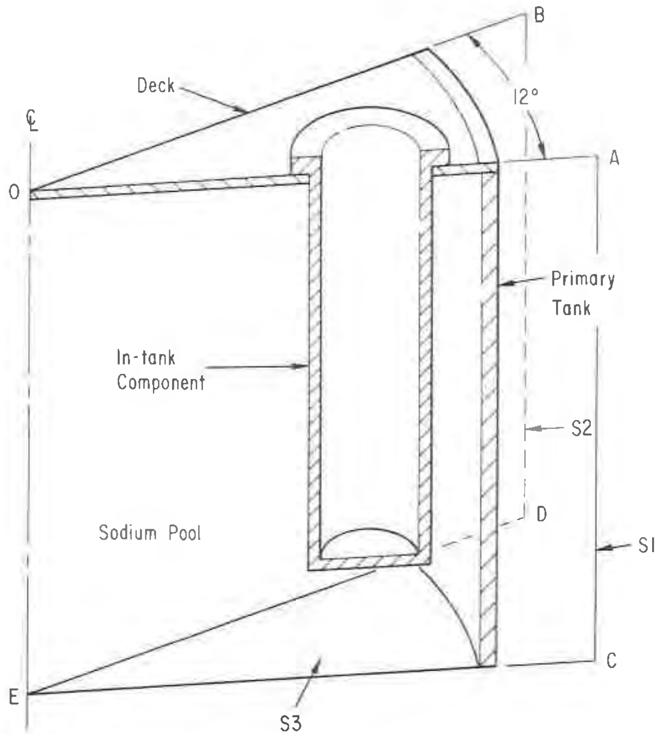

Fig. 2. Sector Model for Primary Tank, Deck, Sodium Pool and In-Tank Component.

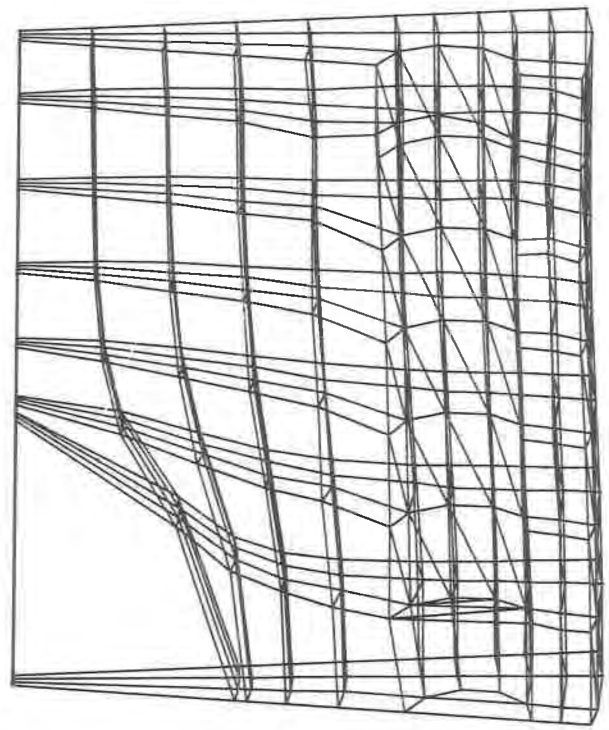

Fig. 3. Deformed Mesh of Sector Model at 50 ms After Accident Initiation.