Transactions of the 13th International Conference on Structural Mechanics in Reactor Technology (SMiRT 13), Escola de Engenharia - Universidade Federal do Rio Grande do Sul, Porto Alegre, Brazil, August 13-18, 1995

Closed-form equations for response of secondary systems

George, Z., Gupta, A.K.
Center for Nuclear Power Plant Structures, Equipment and Piping, North Carolina State University, Raleigh, NC, U.S.A.

ABSTRACT: A common method of evaluating the coupled response of secondary systems is the instructure response spectrum(IRS) method. An IRS is a curve of the maximum responses of a series of single-degree-of-freedom(SDOF) oscillators with varying frequencies attached to an instructure location on the primary system. Closed-form equations to evaluate the IRS are presented. Separate formulas are derived for tuned or nearly tuned and detuned conditions. Matching is performed to obtain equations for all conditions ranging from tuned to detuned. The results from the closed-form equations for coupled analysis are shown to be in good agreement with those from a more rigorous IRS method. These closed-form equations are used to obtain response of multiply connected multi-degree-of-freedom(MDOF) secondary systems.

1 INTRODUCTION

It is often impractical to perform a dynamic finite element analysis of the coupled primary(Building) and secondary(HVAC, piping, equipment, etc.) system subjected to earthquake ground motion. In the conventional method, a time history compatible with the design response spectrum at the base of the building is evaluated(a non-unique process). The primary system is analyzed subject to this time history. The IRS at various locations where the secondary system would be supported are determined. These IRS are then used to analyze the secondary system. If the secondary system is supported at several floors, a spectrum that envelops the spectra at various floors is used in the analysis(introducing conservatism). To account for the effect of relative support motions a worst-case static analysis is performed(adding to the conservatism). The method ignores mass interaction between the secondary and primary systems leading to peaks at resonant frequencies that are too high(further adding to the conservatism). Responses evaluated from this method can be up to an order of magnitude too high. A literature review of recent methods for performing accurate coupled analysis are given in (Gupta 1990; George and Gupta 1994a; George and Gupta 1994b). These methods are implemented using a computer program. In the present research approximate closed-form equations are developed that can be used in hand calculations to determine the IRS.

Formulas are first developed for SDOF-SDOF coupled systems. Similar formulas are then developed for a SDOF oscillator attached to a MDOF primary system. The general approach is to obtain separate formulas for tuned or nearly tuned and detuned conditions. Matching is then performed so as to obtain general formulas that can be applied to all frequency ranges. The instructure spectral values from these closed-form equations are utilized to determine response of multiply connected secondary systems. Results from using the closed-form equations are compared with those from a more rigorous method given in (Gupta 1990; Gupta et al. 1991; Megahed and Gupta 1992).

2 COUPLED SDOF-SDOF SECONDARY PRIMARY SYSTEM

A coupled SDOF-SDOF secondary-primary system is shown in Figure 1(a). The terms m, c, and k, represent mass, damping, and stiffness values, respectively; subscripts p and s denote primary and secondary system properties. Displacements of the primary and secondary system masses relative to the support are u_p and u_s. Let us define the following parameters (George and Gupta 1994a; George and Gupta 1994b):

$$(1)\quad \beta = \frac{\omega_p^2 - \omega_s^2}{\omega_p^2 + \omega_s^2}, \qquad \omega^2 = \frac{\omega_p^2 + \omega_s^2}{2}, \qquad \zeta = \frac{\omega_p\zeta_p + \omega_s\zeta_s}{2\omega},$$

$$\zeta_d = \frac{\omega_p\zeta_p - \omega_s\zeta_s}{\omega} \qquad \text{and} \qquad \overline{r} = r(1-|\beta|)$$

in which ω_p and ω_s are the circular frequencies of the uncoupled primary and secondary system, respectively; ζ_p and ζ_s the corresponding damping ratios, and $r = m_s/m_p$ is the mass ratio. Parameter β is a measure of the difference in natural frequencies of the secondary and the primary systems and ζ_d that in the damping ratios. Parameters ω and ζ are approximate measures of averages of the primary and secondary system frequencies and damping ratios, respectively. The coupled frequencies and damping ratios are given by (George and Gupta 1994a; George and Gupta 1994b):

$$(2)\quad (\omega_1^*)^2 = \omega^2(1+\overline{\beta}), \qquad (\omega_2^*)^2 = \omega^2(1-\overline{\beta}),$$

$$\omega_1^*\zeta_1^* = \omega(\zeta + \frac{\overline{\zeta}_d}{2}), \qquad \omega_2^*\zeta_2^* = \omega(\zeta - \frac{\overline{\zeta}_d}{2}),$$

$$\overline{\beta}, \overline{\zeta}_d = sgn(\beta,\zeta_d)\sqrt{\frac{\pm(\beta^2 - \zeta_d^2 + \overline{r}) + \sqrt{(\beta^2 - \zeta_d^2 + \overline{r})^2 + 4\beta^2\zeta_d^2}}{2}}$$

in which $sgn(\beta,\zeta_d)$ is read as "sign of β or ζ_d".

The instructure spectral acceleration values, $S_{A_i}^{*d,v}$, in each coupled mode i can be expressed as a multiple of the input spectral acceleration values, $S_{A_i}^{d,v}$ as:

$$(3)\quad S_{A_i}^{*d,v} = R_i^{d,v} S_{A_i}^{d,v}$$

where the superscripts d and v represent relative displacement and velocity spectra values, respectively. The response ratios, $R_i^{d,v}$, can be derived separately for cases when primary and secondary system modes are tuned or nearly tuned($\beta \approx 0$) and when they are detuned($\beta = \pm 1$). In order to find general formulas that asymptotically match the respective formulas for both these conditions, the applicability of the tuned condition formulas under detuned condition is examined. This process, known as matching, yields (George and Gupta 1994a; George and Gupta 1994b):

$$(4) \quad R_1^d = \left\{\frac{1}{2} - \frac{X}{Den}(X^2 + Y^2 + \overline{r})\right\}, \qquad R_2^d = 1 - R_1^d$$

$$R_1^v = -\frac{Y}{Den}(X^2 + Y^2 - \overline{r})(1 - |\beta|), \qquad R_2^v = -R_1^v$$

$$(5) \quad X = \beta + \overline{\beta}, \quad Y = \zeta_d + \overline{\zeta}_d, \quad Den = 4X^2Y^2 + (X^2 - Y^2 + \overline{r})^2$$

The combined value of the instructure spectral acceleration, S_A^*, can be obtained from,

$$(6) \quad (S_A^*)^2 = \sum_i^N \sum_j^N \left[\overline{\varepsilon}_{ij}^d S_{A_i}^{*d} S_{A_j}^{*d} + \overline{\varepsilon}_{ij}^v S_{A_i}^{*v} S_{A_j}^{*v}\right]$$

in which the subscripts i and j denote the modes of the coupled system, N the total number of modes. A factor representing cross-correlation between the relative displacement and velocity spectra values, $\overline{\mu}_{ij}$, which is relatively small, is neglected in Equation 6. $\overline{\varepsilon}_{ij}^d$ and $\overline{\varepsilon}_{ij}^v$ are the correlation coefficients defined below: (Gupta 1990; George and Gupta 1994a; George and Gupta 1994b)

$$(7) \quad \overline{\varepsilon}_{ij}^a = \alpha_i^a \alpha_j^a + \left\{\sqrt{[1 - (\alpha_i^a)^2][1 - (\alpha_j^a)^2]}\right\} \varepsilon_{ij}, \quad 0.0 \le \overline{\varepsilon}_{ij}^a \le 1.0, \quad a = d \text{ or } v,$$

$$\varepsilon_{ij} = \left[1 + \left(\frac{\omega_i - \omega_j}{\omega_i\zeta_i + \omega_j\zeta_j}\right)^2\right]^{-1} \left(\frac{2\sqrt{\zeta_i\zeta_j}}{\zeta_i + \zeta_j}\right),$$

$$\alpha_i^a = \frac{\log(f_i/f_1^a)}{\log(f_2^a/f_1^a)}, \qquad 0.0 \le \alpha_i^a \le 1.0$$

where the key frequencies $f_{1,2}^a$ are defined using empirical expressions (Megahed and Gupta 1992).

3 COUPLED SDOF-MDOF SECONDARY-PRIMARY SYSTEM

Response in each mode of the secondary system can be viewed as the response of a SDOF oscillator attached to an appropriate primary system connecting DOF denoted by c (Gupta 1990) . For the MDOF primary system we use ω_{pi} and ζ_{pi} to denote the circular frequency and damping ratio, respectively, of the ith uncoupled mode. For an SDOF oscillator we denote the modal frequency by ω_s, the modal damping ratio by ζ_s and the energy mass ratio by r_i. Interaction between the primary and secondary systems will be most significant with the primary system mode, I, whose frequency, ω_{pI}, is closest to the oscillator frequency, ω_s and the specified mass ratio is assumed to be between the oscillator and the primary system mode I (Gupta 1990).

The values of β, ω, ζ, ζ_d and $\overline{r}$ needed for evaluating the coupled properties can be obtained from Equation 1 by replacing ω_p, ζ_p and r with ω_{pI}, ζ_{pI} and r_I, respectively. Parameters $\overline{\beta}$ and $\overline{\zeta}_d$ can then be calculated from Equation 2. The response ratios can be obtained from (George and Gupta 1994a; George and Gupta 1994b),

$$
\begin{aligned}
(8)\quad R_i^d &= \frac{-\gamma_{pi}\phi_{ci}\omega_s^2}{\omega_{pi}^2-\omega_s^2}, \qquad R_i^v = 0 \quad \text{for } (i \neq I) \\
R_I^d &= \gamma_{pI}\phi_{cI}\left\{\frac{1}{2}-\frac{X}{Den}(X^2+Y^2+\overline{r})\right\}, \quad R_s^d = 1-\sum_{\text{all } i} R_i^d \\
R_I^v &= -\frac{\gamma_{pI}\phi_{cI}Y}{Den}(X^2+Y^2-\overline{r})(1-|\beta|), \quad R_s^v = -R_I^v
\end{aligned}
$$

where γ_{pi} is the participation factor and ϕ_{ci} is the mode shape value at the connecting DOF, c, in the ith uncoupled primary system mode. X, Y and Den are the same as those defined in Equation 5 for SDOF-SDOF secondary-primary systems. The modal values of the instructure spectral acceleration are calculated using Equations 3 and 8. They can be combined to obtain the instructure spectral acceleration, S_A^*, using Equation 6.

3.1 *Examples: IRS spectra*

IRS are generated on the top floor of the six story shear building in Figure 1(b) for various cases as shown in Figures 2(a) to 2(d). Details of the uncoupled mode shapes and frequencies of the primary system are given in (George and Gupta 1994a; George and Gupta 1994b). The comparison is between IRS obtained from the closed-form equations and from the more rigorous program, CREST-IRS (Gupta et al. 1991). It is observed that the IRS from the closed-form equations for values of $r \leq 0.01$ are in excellent agreement with those from CREST-IRS.

3.2 *Examples: Response of MDOF secondary systems*

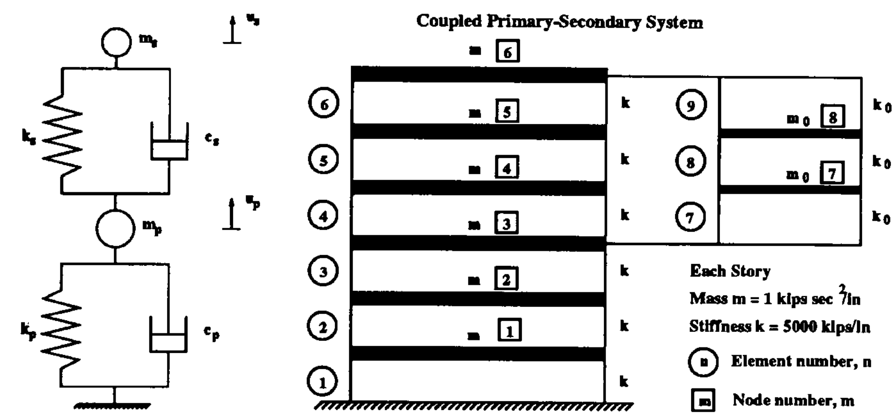
The response of the secondary system shown in Figure 1(b) can be obtained by using the IRS values from the closed-form equations. The primary system is the same as that used in the previous section. The mass, m_0, and stiffness, k_0, of the secondary system are varied to obtain six different cases. Displacement and spring forces of the secondary system can be calculated (George and Gupta 1994a) and the results are shown in Tables 1(a) and 1(b). The maximum error in the response calculated using the closed-form equations is 13%, which is in a spring force in Case 3.

4 CONCLUSIONS

The closed-from equations that were developed can be used in hand calculations to evaluate points on the IRS. These IRS values can be used to determine the response of multiply connected MDOF secondary systems. Very small errors in almost all the displacements and only about 13% error in one of the spring forces indicate that the approximate closed-form equations give acceptably accurate response values.

ACKNOWLEDGEMENTS

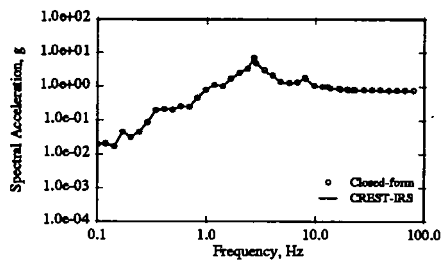
This research was partially supported by the Center for Nuclear Power Plant Structures, Equipment and Piping, North Carolina State University, U.S.A.



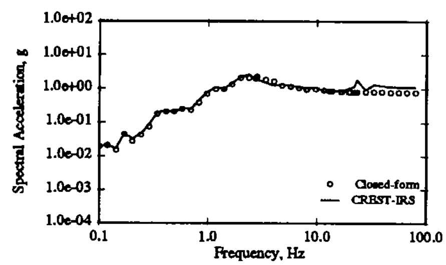
(a) SDOF coupled system

(b) MDOF coupled system

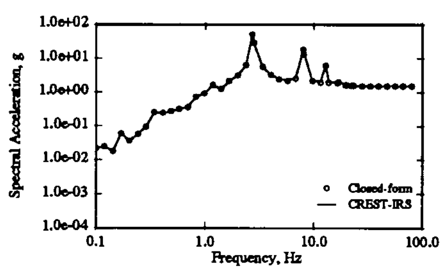
Figure 1: Coupled systems



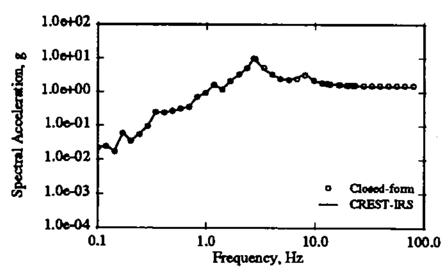
(a) Top floor $\zeta_{pi} = 0.07$, $\zeta_s = 0.02$, $r = 0$



(b) Top floor $\zeta_{pi} = 0.07$, $\zeta_s = 0.02$, $r = 0.1$



(c) Top floor $\zeta_{pi} = 0.01$, $\zeta_s = 0.005$, $r = 0$



(d) Top floor $\zeta_{pi} = 0.01$, $\zeta_s = 0.005$, $r = 0.01$

Figure 2: Instructure response spectra for the El Centro(S00E, 1940) ground motion

Case	Displacement, (inch)			
	DOF m	CREST-IRS(1)	Closed-Form(2)	Error $\frac{(2)-(1)}{(1)} * 100$
1	7	1.28	1.28	0.0
	8	1.39	1.39	0.0
2	7	5.50	5.26	-4.4
	8	5.49	5.24	-4.7
3	7	0.87	0.87	0.0
	8	0.97	0.97	0.0
4	7	0.80	0.80	0.0
	8	0.91	0.91	0.0
5	7	0.78	0.78	0.0
	8	0.89	0.89	0.0
6	7	0.78	0.78	0.0
	8	0.89	0.88	0.0

(a) Nodal Displacements

Case	Spring Force, (kips)			
	Element n	CREST-IRS(1)	Closed-Form(2)	Error $\frac{(2)-(1)}{(1)} * 100$
1	7	10.06	9.93	-1.3
	8	3.43	3.32	-3.2
	9	7.46	7.46	0.0
2	7	32.36	31.31	-3.2
	8	1.10	1.10	0.0
	9	32.54	31.57	-3.0
3	7	26.43	26.37	-0.2
	8	14.01	14.00	0.0
	9	3.08	2.68	-13.0
4	7	46.24	46.21	0.0
	8	34.92	34.91	0.0
	9	22.30	22.20	-0.4
5	7	35.55	35.55	0.0
	8	30.04	30.04	0.0
	9	23.87	23.80	0.0
6	7	67.49	66.58	-1.4
	8	58.85	58.85	0.0
	9	49.57	49.99	0.9

(b) Spring Forces

Table 1: Secondary system response

REFERENCES

George, Z. and Gupta, A. K. (1994a). Closed-form equations for coupled seismic response of equipment and piping. In Gupta, A. K., editor, *Fifth Symposium on Current Issues related to Nuclear Power Plant Structures, Equipment and Piping, Orlando, Florida*, VI/4–1 to VI/4–24. Sponsored by Center for Nuclear Power Plant Structures, Equipment and Piping, North Carolina State University.

George, Z. and Gupta, A. K. (1994b). Simplified closed-form equations for evaluating instructure response spectra. Technical report, Center for Nuclear Power Plant Structures, Equipment and Piping, North Carolina State University, Raleigh, North Carolina, U.S.A.

Gupta, A. K. (1990). *Response Spectrum Method in Seismic Analysis and Design of Structures.* Blackwell Scientific Publications.

Gupta, A. K., Jaw, J. W., and Megahed, H. (1991). *CREST-IRS, a Computer Program for Generating Instructure Response Spectra.* Center for Nuclear Power Plant Structures, Equipment and Piping, North Carolina State University, Raleigh, North Carolina, U.S.A. User's manual.

Megahed, H. A. and Gupta, A. K. (1992). Topics in seismic response of nonclassically damped systems. Technical report, Center for Nuclear Power Plant Structures, Equipment and Piping, North Carolina State University, Raleigh, North Carolina, U.S.A.