Reliability Analysis of Class I Structures

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Abstract

The present paper strives for the development of a reliability analysis method which can be used for the reliability estimation of Class I Structures, taking into consideration their possibly severely nonlinear responses. Such an analysis method is urgently needed in order to perform credible seismic probabilistic risk assessments (SPRA) for structures subjected to earthquake acceleration. Specifically, this paper focuses on stochastic equivalent linearization techniques and intends to develop a technique that can be utilized for the finite element response analysis of nonlinear structures. The linearization technique to be developed takes advantage of a particular analytical approach which uses auxiliary variables in order to describe nonlinear constitutive equations in the finite element formulation. The technique proposed here is believed to be eminently suitable for the eventual reliability analysis of nonlinear structures under random excitations.

1. Stochastic Equivalent Linearization and Finite Element Analysis

Since the present study strives for the development of stochastic equivalent linearization techniques applicable in the finite element formulation of structural reliability analysis, only those approaches that allow one to deal with structures having a large number of degrees of freedom can be utilized here. The case in point is a nonlinear model first proposed by Bouc [1] and generalized by Wen [2]. This model, when applied to a single degree of freedom system, characterizes its dynamic behavior in terms of the following equations involving the displacement u and auxiliary variable z

$$m\ddot{u} + c\dot{u} + q(u,z) = f(t) \tag{1}$$

$$q(u,z) = \alpha ku + (1-\alpha)kz \tag{2}$$

$$\dot{z} = [A\dot{u} - \theta(\beta|\dot{u}||z|^{n-1} z + \gamma\dot{u}|z|^{n})]/\eta \tag{3}$$

where m = mass, c = coefficient of linear viscous damping and f(t) = random excitation force. Equation (3) is a nonlinear auxiliary equation involving the auxiliary variable z, and it implicitly describes the hysteretic nature of the system, and the restoring force $q(u,z)$ given in eq. (2) can be interpreted as consisting of two parallel elements, a linear spring element with stiffness αk and hysteretic component $(1-\alpha)kz$. As well documented by Baber and Wen [3], the adjustment of the values of the six parameters A, β, γ, θ, n and η appearing in eq. (3) provides the model with sufficient versatility.

One of the major benefits accruing from such a linearization is that, once linearized, the result of the response analysis can then be utilized for the system safety analysis following such an approach as that recently developed by Shinozuka [4]. In this sense, it is most encouraging to observe the success demonstrated by Baber and Wen [3] and by Sues et al [5]: With the aid of a stochastic linearization technique, they succeeded in evaluating the response statistics (response covariance matrix) of multi-degrees-of-freedom systems.

In the present study, the stochastic equivalent linearization method used in conjunction with the auxiliary variable model will be extended so as to accommodate the finite element formulation with a significantly large number of variables. However, the present study will concentrate on plane stress and plane strain problems under mean-zero stationary Gaussian base motion.

For the state of plane stress, for example, the stress vector $\underline{\tau} = [\tau_x \ \tau_y \ \tau_{xy}]^T$, strain vector $\underline{\varepsilon} = [\varepsilon_x \ \varepsilon_y \ \varepsilon_{xy}]^T$ and auxiliary variable vector $\underline{z} = [z_x \ z_y \ z_{xy}]^T$ may be related as follows.

$$(1-\nu^2)\ \tau_a = E[\alpha_a\varepsilon_a + (1-\alpha_a)z_a] + E\nu[\alpha_b\varepsilon_b + (1-\alpha_b)z_b] \qquad (a=x,b=y \text{ or } a=y,b=x) \tag{4}$$

$$\tau_{xy} = G[\alpha_{xy}\varepsilon_{xy} + (1-\alpha_{xy})z_{xy}] \tag{5}$$

$$\dot{z}_a = H(\dot{\varepsilon}_a, z_a) = A_a\dot{\varepsilon}_a - \beta_a|\dot{\varepsilon}_a||z_a|^{n_a-1}z_a - \gamma_a\dot{\varepsilon}_a|z_a|^{n_a} \qquad (a=x,y,xy) \tag{6}$$

where eq. (6) represents a set of auxiliary nonlinear equations involving the auxiliary variables, E = Young's modulus, G = shear modulus and ν = Poisson's ratio and n_a, A_a, α_a, β_a, γ_a (a=x,y,xy) are the parameters of the auxiliary variable model. These constitutive equations could be interpreted as representing a type of nonlinear viscoelastic material. Indeed, their linearized version represents those of a standard solid.

Stochastic linearization will then be applied to eq. (6) resulting in

$$\dot{z}_a = c_a\dot{\varepsilon}_a + k_a z_a \qquad (a=x,y,xy) \tag{7}$$

The coefficients c_a and k_a can be obtained from

$$\frac{\partial}{\partial b_a} E[\{H(\dot{\varepsilon}_a, z_a) - c_a\dot{\varepsilon}_a - k_a z_a\}^2] = 0 \qquad (a=x,y,xy;\ b=c,k) \tag{8}$$

where $E[\cdot]$ denotes the expectation. The explicit expressions for c_a and k_a can be derived under the assumption that $\dot{\varepsilon}_a$ and z_a are mean-zero and jointly Gaussian.

The linearized constitutive equations are then written as

$$\underline{\tau} = \underline{D}_L\underline{\varepsilon} + \underline{D}_N\underline{z} \qquad \dot{\underline{z}} = \underline{C}_e\dot{\underline{\varepsilon}} + \underline{K}_e\underline{z} \tag{9}$$

where

$$\underline{D}_L = \frac{E}{1-\nu^2}\begin{bmatrix} \alpha_x & \alpha_y\nu & 0 \\ \alpha_x\nu & \alpha_y & 0 \\ 0 & 0 & \alpha_{xy}(1-\nu)/2 \end{bmatrix} \qquad \underline{C}_e = \begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & c_{xy} \end{bmatrix} \tag{10}$$

$$\underline{D}_N = \frac{E}{1-\nu^2}\begin{bmatrix} 1-\alpha_x & (1-\alpha_y)\nu & 0 \\ (1-\alpha_x)\nu & 1-\alpha_y & 0 \\ 0 & 0 & (1-\alpha_{xy})(1-\nu)/2 \end{bmatrix} \qquad \underline{K}_e = \begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_{xy} \end{bmatrix} \tag{11}$$

It is noted here that the stochastic linearization must be performed element by element and the iterative procedure to update the covariance matrix may require some computational effort.

Within a finite element (e), the strain-displacement and strain rate-displacement-rate relationships can be written as

$$\underline{\varepsilon}^{(e)} = \underline{B}\,\underline{\delta}^{(e)} \qquad \dot{\underline{\varepsilon}}^{(e)} = \underline{B}\,\dot{\underline{\delta}}^{(e)} \tag{12}$$

where $\underline{B}$ = gradient matrix and $\underline{\delta}^{(e)}$ = nodal displacement vector of the element. The Galerkin method, when applied to a linearized system with the inertial effects taken into consideration, produces the following finite element equation of motion:

$$\underline{M}\,\ddot{\underline{\delta}} + \underline{C}\,\dot{\underline{\delta}} + \underline{K}\,\underline{\delta} + \underline{G}\,\underline{Z} = \underline{P} \qquad \dot{\underline{Z}} = \underline{H}\,\dot{\underline{\delta}} + \underline{K}_Z\,\underline{Z} \tag{13}$$

where $\underline{\delta}$ = system nodal displacement vector, $\underline{Z}$ = system auxiliary variable vector, $\underline{P}$ = externally applied Gaussian nodal force vector, $\underline{M}$ = system mass matrix, $\underline{C}$ = system linear viscous damping coefficient matrix, $\underline{K}$ = system stiffness matrix assembled from element stiffness matrices involving $\underline{D}_L$ and $\underline{B}$, $\underline{G}$ = system matrix assembled from corresponding element matrices involving $\underline{D}_N$ and $\underline{B}$, $\underline{H}$ = system matrix assembled from corresponding element matrices involving $\underline{C}_e$ and $\underline{B}$ and $\underline{K}_Z$ = system matrix assembled from corresponding element matrices involving $\underline{K}_e$.

2. Complex Modal Analysis

Instead of utilizing the differential equation approach for the solution of the covariance matrix as employed in Refs. 2, 3 and 5, the complex modal analysis method will be used in this paper. To this end, eq. (13) is rewritten in the following form:

$$\underline{A}\,\dot{\underline{X}} + \underline{J}\,\underline{X} = \underline{P}_0 \tag{14}$$

in which $\underline{X} = [\underline{\delta}^T\ \dot{\underline{\delta}}^T\ \underline{Z}^T]^T$, $\underline{P}_0 = [\underline{P}\ \underline{0}\ \underline{0}]^T$, while $\underline{A}$ is a symmetric matrix and $\underline{J}$ is an asymmetric matrix. The matrices $\underline{A}$ and $\underline{J}$ consist of $\underline{M}$, $\underline{C}$, $\underline{K}$, $\underline{H}$ and $\underline{K}_Z$. Then, the eigenvalue problems

$$\underline{A}\,\dot{\underline{X}} + \underline{J}\,\underline{X} = \underline{0} \qquad \underline{A}^T\,\dot{\underline{X}} + \underline{J}^T\,\underline{X} = \underline{0} \tag{15}$$

produce the complex eigenvalue matrix $\underline{\lambda}$ = diagonal $[\lambda_1\ \lambda_2\ \cdots\ \lambda_M]$ and respective complex modal matrices $\underline{\Phi}$ and $\underline{\Psi}$ (both M×M where M is the dimension of the $\underline{X}$ vector).

These modal matrices satisfy the orthogonality condition

$$\underline{\Psi}^T\,\underline{A}\,\underline{\Phi} = \underline{I} \qquad\qquad -\underline{\lambda} = \underline{\Psi}^T\,\underline{J}\,\underline{\Phi} \tag{16}$$

The response matrix $\underline{X}$ of eq. (14) can then be expanded into $\underline{X} = \underline{\Phi}\,\underline{Q}$ where $\underline{Q}$ = generalized coordinate vector, which can be evaluated from

$$\dot{\underline{Q}} - \underline{\lambda}\,\underline{Q} = \underline{\Psi}^T\,\underline{P}_0 \tag{17}$$

If $\underline{\delta}$ represents the relative displacement vector under a mean-zero stationary Gaussian base acceleration $\ddot{x}_g$ (say, in the x-direction), $\underline{P}_0$ is written as

$$\underline{P}_0 = -\underline{M}_0\,\tilde{\underline{I}}\,\ddot{x}_g \tag{18}$$

where $\underline{M}_0$ = extended mass matrix consistent with $\underline{X}$ and $\tilde{\underline{I}}$ = extended influence vector. A shear wall under horizontal base motion within its plane represents an example. It is important to recognize that the components of the vector $\underline{\Psi}^T\,\underline{M}_0\,\tilde{\underline{I}}$ can serve the same purpose as the participation factor in classical modal analysis. Indeed, in evaluating the components, one can choose only those modes that contribute significantly to the response.

The analysis that remains for evaluation of the covariance matrix of X is straightforward, since $\underline{Q}$ can be explicitly evaluated from eq. (17) as a function of $\underline{P}_0$ and hence of $\ddot{x}_g$.

3. Numerical Example

A two-dimensional structure shown in Fig. 1 and subjected to in-plane horizontal ground acceleration $\ddot{x}_g(t) = f(t)\ddot{x}_0(t)$ is considered, when $f(t)$ is a deterministic envelope function defined by $f(t) = c_0(e^{-at} - e^{-bt})$ with c_0 being a normalizing factor to make the maximum value of $f(t)$ equal to unity; $f(t)$ is then depicted in Fig. 2 for the first six seconds with a = 0.25/sec and b = 0.50/sec. The quantity $\ddot{x}_0(t)$ represents a mean-zero, stationary Gaussian process with the well-known Kanai-Tajimi spectrum;

$$G(\omega) = S_0 \frac{1 + 4\zeta_g^2(\omega/\omega_g)^2}{\{1-(\omega/\omega_g)^2\}^2 + 4\zeta_g^2(\omega/\omega_g)^2} \tag{19}$$

The parameter values used for the numerical analysis are:

$S_0 = 0.1$ ft²/sec³, $\omega_g = 15.56$ rad/sec, $\zeta_g = 0.64$,
$E = 5.184 \times 10^5$ kips/ft², $G = 2.16 \times 10^5$ kips/ft², $\nu = 0.2$, $A_x = A_y = A_{xy} = 1$,
$\eta_x = \eta_y = \eta_{xy} = 1$, $\beta_x = \beta_y = \beta_{xy} = 237$, $\gamma_x = \gamma_y = \gamma_{xy} = 237$,
$\rho = 46.7$ kips·sec²/ft², $\alpha = 0.217$ (1/sec), $\beta = 0.0107$ (sec).

where α and β are such that the linear viscous damping matrix $\underline{C}$ in eq. (13) is $\underline{C} = \alpha\underline{M} + \beta\underline{K}$. The value of ρ is assumed to be much larger than those found for the usual engineering materials for the purpose of producing responses clearly in the nonlinear range.

In the present study, the termination of the iterative procedure with respect to upgrading the covariance matrix takes place when

$$\frac{1}{NN}\sum_{n=1}^{NN}\left|\frac{\sigma_{\delta_n}(k) - \sigma_{\delta_n}(k-1)}{\sigma_{\delta_n}(k-1)}\right| < \varepsilon_0 \tag{20}$$

is satisfied. In the present example, NN = total number of non-zero displacement components = 4, $\sigma_{\delta_n}(k)$ = standard deviation of the n-th displacement component δ_n after the k-th iterative upgrading and $\varepsilon_0 = 0.05$.

Some of the results of the numerical analysis are shown in Figs. 3 and 4, where the standard deviations of $\tau_{xy}^{(3)}$ and $\varepsilon_{xy}^{(3)}$ (τ_{xy} and ε_{xy} in element 3) are shown as functions of time, respectively.

4. Conclusions

The auxiliary variable method is applied to introduce nonlinear characteristics into the stress-strain relationship. A stochastic equivalent linearization technique is utilized to linearize the constitutive equations by minimization of the mean square errors. On the basis of the linearized constitutive equations, the finite element equations are derived and the state space approach is employed to combine these equations into a single equation. A complex modal analysis will be performed to find the state space vector explicitly. Under the condition that the excitation represents a mean-zero stationary Gaussian base acceleration, significant modes are identified on the basis of the participation factor and only these modes are used in constructing the state space vector, thus reducing the number of degrees of freedom and making the analysis tractable. The covariance matrix of the state vector thus constructed can then be expressed in terms of the covariance of the base acceleration. This covariance matrix is then used in the stochastic linearization of the system in its iterative updating and at the same time produces the desired response statistics. The technique developed here possibly provides a much more practical alternative, particularly within the framework of the finite element method, to the approach in which the differential equation for the covariance matrix is solved repeatedly in the process of iterative upgrading.

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4. References

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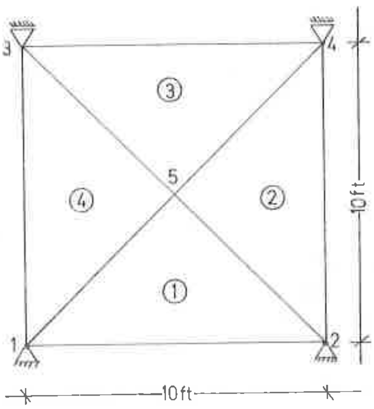

Figure 1 Two-Dimensional Structure (Mesh Configuration)

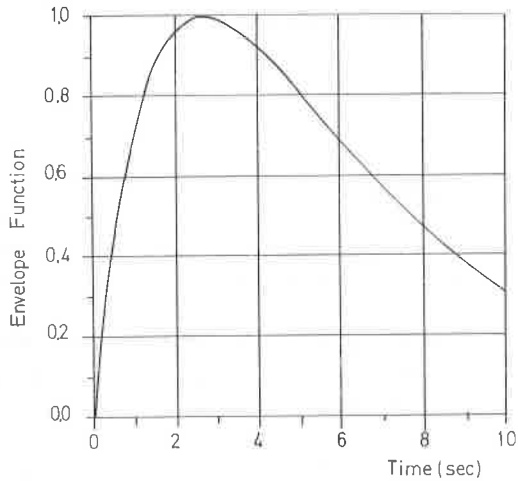

Figure 2 Envelope Function

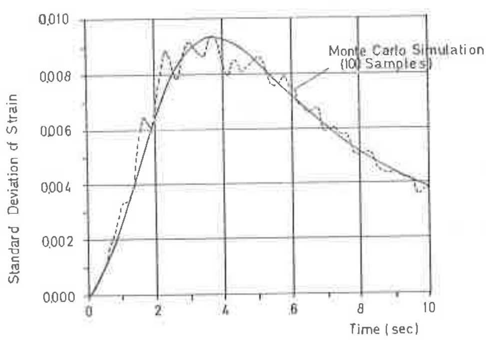

Figure 3 Standard Deviation of Strain $\varepsilon_{xy}^{(3)}$

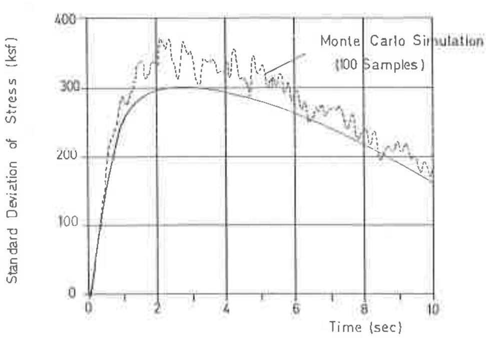

Figure 4 Standard Deviation of Stress $\tau_{xy}^{(3)}$

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