STATIC, STABILITY, AND DYNAMIC ANALYSES OF SHELLS OF REVOLUTION BY NUMERICAL INTEGRATION — A COMPARISON

V. SVALBONAS

Grumman Aerospace Corporation, Bethpage, New York 11714, U.S.A.

J. KEY

Astronautics Laboratory, NASA Marshall Space Flight Center, Huntsville, Alabama 35812, U.S.A.

SUMMARY

The use of numerical integration for the analysis of practical shell of revolution structures was documented almost simultaneously in the United States by three independent groups of researchers (Cohen, Kalnins, Mason *et al.*). These early efforts have been refined, reformulated, and increased in scope and applicability to become major program systems (SRA, Kalnins, STARS). While all three programs utilize basically the same mathematical formulation for integrating the shell differential equations, the matrix solution procedures from this point are basically different. The purpose of this paper is twofold as follows:

(1) present the differences in solution procedures of the largest system (the Grumman – NASA "STARS") from the other two, and point out the inherent advantages of this approach;
(2) compare the numerical integration procedure, as utilized in the "STARS", with finite difference and finite element procedures, noting the relative advantages of each in the analysis of shells of revolution for static, buckling, and dynamic loadings.

To fulfill the above purpose, a brief review of the numerical integration procedure for the analysis of shells of revolution is presented, and the matrix solution procedures of the SRA, Kalnins, and STARS programs are contrasted. The limitations imposed by the relative procedures are discussed. The unique formulation utilized by STARS for the solution of stability and vibration problems, and its advantages, are discussed in detail.

The STARS program's analytical capabilities, capacity, and user options are compared with those of other major systems utilizing either finite differences or finite elements for the analysis of shells of revolution. Comparisons are made in terms of program size, program accuracy, number of degrees of freedom required for analysis, ease of idealization and user inputs, limitations imposed on analysis capability or output, running time, and so forth.

All advantages and differences are demonstrated by use of solutions for realistic shell problems in the areas of statics, stability (including dead and live load distributions), vibrations, and dynamic response of shells subjected to time-dependent loadings.

1. Introduction

Recent innovations in digital computer technology have enabled engineers to analyze shell structures of complex configurations without unduly restrictive approximations. The basic techniques making this possible in shell of revolution analysis may be classified as finite difference methods, numerical integration methods, and finite element methods. A number of versatile computer programs, based on the various methods of analysis are presently available to the engineer. Many of these programs are checked out, documented, and of sufficient size and scope in analysis capability, capacity, and user options, so as to be readily useable outside the originating organization. The purpose of this paper is to compare the various programs now generally available from the point of view of the advantages of the relative techniques utilized, as well as the programmed state of the art. Much of the comparisons are based upon sample problems solved by the STARS-2 system of programs. These examples indicate both the structural detail which can be analyzed by, and the analytical capabilities available in the numerical shell-of-revolution programs.

2. The Numerical Integration Method

2.1 The Theory

The basic numerical integration procedure for shells of revolution was first presented by Goldberg [1,2] for conical shells. The application of this technique to the analysis of general shells of revolution was documented almost simultaneously by three independent groups: Cohen [3] , Kalnins [4] , and Mason et. al. [5] . Since that time the latter three efforts have been expanded and reformulated into major program systems: SRA [6] , Kalnins [7] , and STARS-2 [8,9] .

All three of the above systems utilize the same basic approach to the numerical integration procedure. The shell equations are first cast (after Fourier expansion in the θ coordinate) in the form of eight coupled first-order equations of the form

$$\left\{Y^{(n)}(\varphi)\right\}_{,\varphi} = \left[D^{(n)}(\varphi)\right] \left\{Y^{(n)}(\varphi)\right\} \qquad (1)$$

where φ, θ are the meridional and hoop coordinates respectively, and n signifies the Fourier harmonic. These equations are then integrated by using some standard procedure such as the Runge-Kutta method. Eight influence coefficient solutions are thus obtained corresponding to eight independent initial condition vectors, and a particular solution for zero initial variables may be obtained for each independent loading pattern to be investigated. Due to the numerical difficulties caused by exponential growth or decay of the influence coefficient solutions, the shell must be segmented into pieces of limited size to obtain a satisfactory solution. Thus for each shell segment a set of (8+P) solutions is obtained, where P is the number of individual load patterns being considered. The total sets are then combined to satisfy the segment continuity conditions, and the overall boundary conditions for the shell, thus yielding P solutions for stresses and displacements in the case of a static analysis.

The SRA program formulation presently utilizes a refinement of the above procedure due Zarghamee and Robinson [10] . In this technique only four independent vectors are utilized in the homogeneous integrations. The vectors are chosen so as to satisfy automatically four conditions, such as continuity conditions at the shell segment ends. However this technique will not analyze different load patterns simultaneously, nor is it capable of handling shells with topologies including closed branches.

2.2 Different Matrix Formulations

It is the requirement of routine analysis capability for shells with arbitrary branching, that first led to a different matrix formulation for the STARS program [5] as opposed to the other numerical integration programs. Both the SRA and the Kalnins programs utilize influence coefficient matrices in the formulation, and Gaussian elimination in the solution of the matrix problem. The STARS approach is to form stiffness and load matrices for the shell segments, and thence utilize a finite element direct stiffness procedure for solution. The immediate advantages of this procedure are twofold. First, even with the most recent updates for branching analysis made to the other programs, the STARS is still the most comprehensive in this area, as illustrated in Fig. 1. Second, the procedure allows for the use of STARS as a very accurate stiffness matrix generator in problems where shells of revolution must be coupled to other structures [11]. Less obvious advantages revolve around the fact that in keeping STARS compatible with finite element technology, all the methods developed for finite element matrix procedures, both past and present (such as Guyan reduction [12], for example) become directly applicable.

In the area of stability or vibration analysis of shells of revolution each of the numerical integration programs utilize different procedures. The Kalnins program uses a determinant plotting procedure. The determinant of the equations describing the prestressed shell is evaluated for different assumed values of the prestress, until a zero determinant value is found indicating that the assumed prestress is a root of the equation system. There are several important drawbacks to this procedure. In analyzing a complex shell of revolution the prestress load (or frequency) must be stepped in small increments, from zero, using substantial computer time. Even so, closely spaced roots may be skipped, and thus no guarantee can be made that the lowest root has been found. The SRA program, on the other hand, utilizes a Stodola-type iteration procedure equivalent to the power method [13] favored by large finite element programs. In this procedure, the iteration is performed upon the shape of the eigenvector, and there is a guaranteed convergence to the lowest eigenvalue. Basically, the homogeneous equations resulting from stability (vibrations) analysis are converted into a series of nonhomogeneous equations by assuming a buckled (vibration) shape and, thus, creating nonhomogeneous terms from the prestress (accelerations). This reduces the problem solution to a series of static problems, each in turn, providing a better estimate for the eigenvector. The drawback in this procedure is concerned mainly with finding more than one eigenvalues, such as the many frequencies which may be of interest in a vibrations problem. To obtain higher frequencies with any accuracy or speed, all the lower roots must be swept out, or origin shifts used [14]. In addition, the whole procedure may encounter slow convergence depending upon the initial choice of the three displacement eigenvectors u, v, w for a complex-geometry shell [14, 15].

To eliminate the above drawbacks, and in order to take advantage of the fact that the numerical integration procedure allows the reduction of a large shell problem into a small matrix problem, a different eigenvalue solution procedure was adopted [16] for the

STARS program. It is first recognized that the stability or vibrations problem for the shell is actually transcendental in the eigenvalue. Then, in successive passes through the program, the elastic stiffness matrix of the shell, $\left[K_0\right]$, and the prestressed shell stiffness matrix, $\left[K_P(\lambda)\right]$, for an assumed prestress value, are calculated. The two matrices are subtracted to isolate a matrix containing the effect of the prestress

$$\left[K_I(\lambda)\right] = \left[K_P(\lambda)\right] - \left[K_0\right] \tag{2}$$

and a linear eigenvalue approximation is formulated

$$\left[K_0\right] + \frac{\lambda_i}{\lambda_{i-1}} \left[K_I(\lambda_{i-1})\right] \left[\Delta\right] = \left[0\right] \tag{3}$$

where $\left[\Delta\right]$ is the deflection vector. As evident from the notation, eq. (3) is solved by a series of iterations, with convergence being signaled by $\lambda_i/\lambda_{i-1} \rightarrow 1$. In this fashion the nonlinearity of the eigenvalue problem is retained. The iteration procedure has been found to exhibit quick convergence. A single iteration is equivalent to the process utilized in most finite element programs, whereas further passes include the refinements of accounting for the nonlinear dependence of the incremental stiffness matrix upon the eigenvalue. The formulation of eq. (3) includes the effects of all "consistent" [17] nonlinear terms, including predeformation.

In staying compatible with finite element theory, the STARS stability and vibrations programs are able to utilize matrix reduction schemes prior to solving eigenvalue problems. This allows the use of a fast in-core matrix algorithm such as the Householder-Givens [13] technique for eigenvalue extraction. Furthermore, the nonlinear nature of eq. (3) allows for the solution of many eigenvalues without the use of many degrees of freedom, by merely varying prestress or frequency estimates to get different matrices. A demonstration of the procedure is provided in Table I which documents the overall stability analysis of the reinforced cylinder shown in Figure 2. The first guess for the critical compression load provides an eigenvalue multiplier of 6.01777. Adjusting the load by this multiplier provides the second multiplier of .9706. The load is then adjusted by the cumulative product of the multipliers in future passes through the program. It is important to note that although the only buckling load accurately extracted is the first or critical load, the very same computer passes provide estimates to higher buckling loads. The accuracy of these estimates is noted in Table I in comparison with solutions of Refs. [18, 19]. Thus in a vibrations problem it may take three or four program passes to establish the first frequency. However, the accuracy of the estimates to the higher frequencies established in these same passes insure that only one additional pass will be satisfactory for the extraction of each higher frequency.

2.3 Comparison of the Numerical Integration Programs

The present capabilities of the STARS-2 system of programs are presented in Table II, and compared to the SRA and Kalnins programs in Table III. In Table III the comparisons are split into three groupings: general capacity and capability, eigenvalue programs, and special capabilities of each system. It is clear that the STARS system holds the advantage in both theoretical scope, and completeness for engineering user requirements.

3. Numerical Integration versus Finite Differences and Finite Elements

3.1 General Comparisons

The theoretical comparison between the three basic numerical methods is presented in Table IV. It appears that the numerical integration method holds most of the theoretical advantages in this comparison. However, "theoretical" should be emphasized, since these advantages may not be apparent to an engineering program user, if through programming the advantages are lost or are not able to be utilized.

In recent years there have been several comparisons [20, 21] made between finite difference and numerical integration procedures. The drawbacks in these comparisons arise from the fact that in the discussions the theoretical advantages are confused with actual programmed procedures of the programs under comparison, i.e. SRA for numerical integration versus SALORS [20] and BOSOR [22] for finite differences. One such confusion involves the term "segment". The original definition, coined in numerical integration terminology, is the length for which numerical integration can proceed along the shell before encountering numerical inaccuracies. There is no dependance on wall crossection changes except insofar as they influence this length. In finite difference terminology the same term is identified with a discontinuity in wall geometry, or material properties, or mesh size. The STARS program segment will accept tabular input at up to 30 points. This is sufficient to describe up to 28 different "slope changes in wall geometry, or up to 14 points of discontinuity <u>within</u> one segment. The so called segment in <u>all</u> other programs (including SRA and Kalnins) can only accept a two point input for the segment. Thus in a comparison for very complex problems, the STARS segment capacity may still be multiplied by 14 for a one to one correspondance. Furthermore, the thermal dependence of material properties is accepted in STARS in a separate tabular form, and the program automatically interpolates for the properties as a function of the applied thermal loading. In finite difference programs, such as BOSOR, the material properties are set for a "segment". Thus a shell with a complex thermal loading variation along the meridian will require many finite difference "segments" while the STARS idealization remains unaffected in such problems.

Another point of confusion relates to the conclusion [21] that in the finite difference solution of an eigenvalue problem, detailed prestress and predeformation information is available at every difference station, whereas this is not so in numerical integration. This conclusion actually results from comparing a self-contained finite difference eigenvalue program with the SRA numerical integration program, which is not self-contained. The SRA (and the Kalnins) program requires a computer run with the static analysis program to provide prestress card input, at 100 points, for the stability analysis program. The STARS eigenvalue programs, however, are self-contained and automatically supply prestress and predeformation information at every integration point used, and thus are as consistent in accuracy as the self-contained finite difference programs.

Figure 3 illustrates the comparison of numerical integration segments with revolved finite elements. The segment, as built and utilized within STARS, can result in a massive reduction of the degrees of freedom which would be necessary to solve the same problem, with equivalent accuracy, using either revolved finite elements or finite differences. Unfortunately computer running time is not reduced in proportion, due to the extra time needed to first form the segment stiffness matrices.

3.2 Comparisons of Specific Programs

A comparison of the STARS-2 system with the best representative of finite difference programs, BOSOR4 [22], is presented in Table V. As can be seen from the comparison, the BOSOR4 program is substantially closer to the STARS-2 system capability than the other numerical integration programs listed in Table III. A comparison with finite element programs is somewhat more difficult. The most logical candidate for comparison is NASTRAN. However, being a general purpose program, NASTRAN has substantially greater capabilities than STARS-2 for general analysis, while its shell of revolution capability is somewhat limited (considering the revolved elements alone). Comparisons of analyses with shells idealized by flat triangles or quadrilaterals are not altogether fair as will be shown in some examples. The most analytically extensive finite element shell of revolution programs are those of Texas A & M University [23], however these are small special purpose programs in spite of their analytical sophistication. Thus again comparisons as functions of capabilities or capacities would not be valid.

It appears that here is no program based on revolved finite element technology which contains a comparative capacity in size and options to BOSOR4 or STARS-2. The finite element analysts prefer more general elements and capabilities, rather than the specialized programs for shells of revolution. While this point of view has many merits, it results in higher costs when such a program is used to analyze a specialty structure such as a shell of revolution. An illustration of this is presented in Figure 4 and Tables VI and VII. Fig. 4 presents the analysis of a model motor casing using various programs. The pertinent facts from the point of view of economy in computer time are presented in Table VI. As can be seen, NASTRAN, an extremely fast general purpose finite element program, is still not competitive with a specialty program like BOSOR4, for shell of revolution structures. The MARC program, based on sophisticated curved triangular finite elements, is totally non-competitive even using an unrealistic coarse idealization.

The 1900 node idealization in NASTRAN could not be run for stability or vibration analysis without the use of reduction procedures. Table VII compares NASTRAN and STARS-2 running times for eigenvalue problems. Again the specialty program is much faster. In addition, it should be noted that matrix reduction in NASTRAN requires a substantial amount of time compared with the actual eigenvalue extraction. The actual comparison in Table VII is shown in a favorable nature to NASTRAN, since the program was allowed to use the results of STARS in order to determine the search ranges for the eigenvalues. When denied this information in the initial run, the search terminated without finding an eigenvalue.

In all fairness it should be noted that by removing one element we can create the problem of a shell of revolution with a cutout, which cannot be solved by a specialty program. On the other hand even such a problem may be solved more efficiently as follows. First the problem (without the cutout) is solved utilizing a shell of revolution program. Second the local cutout area is idealized with a fine grid using a program such as NASTRAN, and using the shell of revolution analysis results as specified displacement inputs sufficiently far away from the cutout. In this combined procedure the discrete element program can be utilized in a more efficient fashion to analyze the local cutout problem for which it is best suited.

References

[1] Goldberg, J., Bogdanoff, J., and Marcus, L., "On the Calculation of the Axisymmetric Modes and Frequencies of Conical Shells", J. Accoust. Soc. Am., Vol. 32, pp. 738 - 742 (1960).

[2] Goldberg, J., "Computer Analysis of Shells", in Proceedings of Symposium on the Theory of Shells, Ed. by D. Muster, Univ. of Houston, pp. 5-22 (1966).

[3] Cohen, G., "Computer Analysis of Asymmetrical Deformation of Orthotropic Shells of Revolution", AIAA J., Vol. 2, No. 5, pp. 932-934 (1964).

[4] Kalnins, A., "Analysis of Shells of Revolution Subjected to Symmetrical and Nonsymmetrical Loads", Trans. ASME, Series E, Vol. 31, pp. 467-476 (1964).

[5] Mason, P., Rung, R., Rosenbaum, J., and Ebrus, R., "Nonlinear Numerical Analysis of Axisymmetrically Loaded Arbitrary Shells of Revolution", paper No. 64-439, AIAA Annual Meeting, 1964.

[6] Cohen, G., "Computer Analysis of Ring-Stiffened Shells of Revolution", NASA CR-2085 (1973).

[7] Kalnins, A., "Static, Free Vibration, and Stability Analysis of Thin, Elastic Shells of Revolution", AFFDL-TR-68-144 (1969).

[8] Svalbonas, V., "Numerical Analysis of Shells", NASA CR-61299 (1969).

[9] Svalbonas, V., "Numerical Analysis of Stiffened Shells of Revolution", NASA CRs to be issued.

[10] Zarghamee, M., and Robinson, A., "A Numerical Method for Analysis of the Vibrations of Spherical Shells", AIAA J., Vol. 5, No. 7, pp. 1256-1261 (1967).

[11] Rung, R., Petricone, R., and Mahoney, J., "Submarine Structural Design System", Applied Technology Assoc. Rpt. No. 132.R.1.70 (1970).

[12] Guyan, R., "Reduction of Stiffness and Mass Matrices", AIAA J., Vol. 3, No. 2, (1965).

[13] Wilkinson, J., The Algebraic Eigenvalue Problem, Clarendon Press, Oxford, 1965.

[14] Hurty, W., and Rubinstein, M., Dynamics of Structures, Prentice-Hall Inc., N.J., Second Printing, 1965.

[15] Pestel, E., and Leckie, F., Matrix Methods in Elastomechanics, McGraw-Hill Book Co. Inc., N.Y., 1963.

[16] Svalbonas, V., and Balderes, T., "Buckling and Vibration Analysis of Stiffened Orthotropic Shells of Revolution", AIAA J., Vol. 10, No. 7, (1972).

[17] Kempner, J., "Unified Thin Shell Theory", Polytech. Inst. Brooklyn PIBAL No. 566, (1960).

[18] Dickson, J., and Brolliar, R., "The General Instability of Eccentrically Stiffened Cylindrical Shells under Axial Compression and Lateral Pressure", NASA CR-1280, 1969.

[19] Block, D., Card, M., and Mikulas, M. Jr., "Buckling of Eccentrically Stiffened Orthotropic Cylinders", NASA TND-2960, 1965.

[20] Anderson, M., Fulton, R., Heard, W. Jr., and Walz, J., "Stress, Buckling, and Vibration Analysis of Shells of Revolution", Computers & Structures, Vol. 1, pp. 157-192 (1971).

[21] Fulton, R., "Numerical Analysis of Shells", Proc. of Symp. of Int. Union of Theo. and Appl. Mech., Vol. 61, pp. 293-319 (1971).

[22] Bushnell, D., "Stress, Stability, and Vibration of Complex Branched Shells of Revolution: Analysis and User's Manual for BOSOR4", NASA CR-2116, (1972).

[23] Stricklin, J., et al, "Large Deflection Elastic-Plastic Dynamic Response of Stiffened Shells of Revolution", Report No. TEES-RPT-72-25 (1972).

Table I Over-All Buckling of Reinforced Cylinder (n = 0)

Root No	m	NASA TND 2960	NASA CR 1280	Current Method (20 segments)			
				Trial 1 [illegible]	Trial 2 [illegible]	Trial 3 [illegible]	Trial 4 [illegible]
1 = crit	12	6048.16*	6048.0	6017.77	6040.0	6040.3	6048.01
2	14	6077.07	6072.2	6073.60	6071.1	6070.0	6079.22
3	12	6074.00	6068.4	6120.70	6072.4	6070.0	6070.56
4	16	6032.12	6075.6	6264.84	6033.4	6043.1	6042.75
5	10	6290.96	6201.0	6445.00	6308.4	6315.4	6316.07
6	11	6301.05	6293.0	6600.61	6310.3	6310.0	6310.13
7	17	6627.3	6624.0	6971.63	6603.3	[illegible]	6605.42
8	10	6810.31	6860.4	7040.00	6923.0	6920.0	6920.60
9	18	7060.56	7044.1	7440.06	7140.0	7162.0	7102.07
10	19	7542.30	7530.0	7640.50	7901.5	7704.7	7704.21
11	9	7075.04	7581.1	8033.70	7923.0	7920.1	7937.03
12	20	8100.50	8070.0	8574.50	8030.6	8043.2	8041.50
13	21	8710.03		9021.53	9100.4	9214.4	9213.05
14	22	9343.32		10100.41	9403.0	9400.4	9400.24
15	9	9414.05	9393.2	11002.38	10005.7	10030.0	10030.53
16	23	[illegible].1		12005.57	10049.3	10080.0	10085.00
17	24	10003.3		12010.31	11050.2	11062.7	11062.57
18	25	11074.00		13167.73	12009.9	12041.4	12049.73

*1 lbf./in. x 175.1 = 1 N/m

Roots 1–11: Av Dif = [illegible]%, Max Dif = 2.0%

Roots 11–18: Av Dif = 2.72%, Max Dif = 10.0%

Table II Present STARS System Capability

STARS-2S	STARS-2B	STARS-2V	STARS-2P
1. Linear analysis for axisymmetric and unsymmetric loads 2. Fourier harmonics automatically summed 3. Thermal stress analysis included 4. Up to 10 simultaneous loadings allowed 5. Many forms of stiffening (both discrete and smeared allowed) 6. Arbitrary branching allowed 7. Nonlinear (geometric) analysis for axisymmetric loads.	1. Linear axisymmetric prebuckling state including predeformation 2. Live pressure fields included 3. Both fixed and eigenvalue loads accepted 4. Many forms of stiffening allowed 5. Program is self-contained (will calculate prestress state) 6. Changes of boundary conditions in harmonic search allowed 7. Arbitrary branching allowed 8. Thermal buckling analysis included	1. Linear axisymmetric prestress state including predeformation. 2. Critical speeds of rotating shells calculated 3. Many forms of stiffening allowed 4. Program is self-contained 5. Changes of boundary conditions in harmonic search allowed 6. Arbitrary branching allowed 7. Thermal prestress included	1. Nonlinear (geometric and material) analysis of axisymmetrically loaded shells. 2. Cyclic loading included. 3. Kinematic or isotropic hardening available 4. Bauschinger effect included 5. Uneven orthotropic yield surface. (Reduces to Hills and Von Mises yield criteria.) 6. Arbitrary branching allowed 7. Live pressure allowed 8. Many forms of stiffening considered 9. Thermal loads included
STARS-2D: Experimental (Marching/Transient Dynamics) Version			

Table III

Comparison of Numerical Integration Programs

STARS-2	Kalnins	SRA
100 Segments/100 Rings	100 Segments/20 Parts/No Rings	33 Segments/34 Rings/100 Input Pts.
10 Simultaneous Load Cases	1	1
No Limits	No Internal Restraints Allowed	No Limits
Arbitrary Branching	"Y" Branches Only (3 Max.)	1 Closed, 7 Open Branch Pts.
Data Debugging System Available	None	None
Many Wall Reinforcings	No Reinforcing	Smeared Stringers
Surface Moment Loads Allowed	Not Available	Not Available
Thermal Interpolation of Properties	Material Properties Set for Part	Thermal Interpolation
Eigenvalue Solution: 1. Transcendental Iteration 2. Determinant Plotting	Determinant Plotting	Sturm Iteration
No Set Limit	No Set Limit (Time Problem)	4 Eigenvalues/Harmonic
Live Pressure Available	No	Live Pressure Available
Eigenvalue Program Self Contained	Both Static and Eigenvalue Programs Must be Used	Both Static and Eigenvalue Programs Must be Used
Boundary Conditions Variable in Harmonic Search	Fixed Set	Fixed Set
Not Available	Not Available	Nonlinear Prebuckling Option in Eigenvalue Program
In Programming	Limited	Unsymmetric Load Eigenvalue Formulation
Critical Speed Analysis	Not Available	Not Available
Inelastic Analysis	Not Available	Not Available
Transient Dynamic Analysis	Not Available	Not Available

Table IV

Relative Accuracy of Three Numerical Methods

Numerical Integration	Finite Differences	Finite Elements
Runge-Kutta truncation error of order of $(spacing)^5$	a) Finite difference truncation error of order of $(spacing)^2$ b) Finite differences taken of geometric and stiffness variables	Finite element accuracy dependent upon a) displacement assumption b) geometric representation accuracy
One Analysis shows accuracy of mesh	Two analyses needed for convergence trends	Two analyses needed for convergence trends
Variable mesh automatically set based on uniform accuracy	Mesh set by user: need apriori knowledge of stress state	Mesh set by user: need apriori knowledge of stress state
No equivalent problems	Worse accuracy at boundaries or segment ends	Displacements better accuracy than stresses
Single precision programs	Double precision programs	Single precision programs
Necessity of segmentation of long shells	Instability of fine mesh sizes	No equivalent problems

Table V STARS-2 System Versus BOSOR4 System

STARS-2	BOSOR4
900 Segments/800 Rings/No Limit on Integration Points	25 Segments/50 Rings/450 Mesh Points
10 Simultaneous Load Cases	1
No Limits	50 Constraints
Arbitrary Branching	Many Branching Options
Data Debugging System Available	None
Many Wall Reinforcings	Many Wall Reinforcings
Surface Moment Loads Allowed	Not Available
Thermal Interpolation of Properties	Material Properties Set for Segment
Eigenvalue Solution: 1. Transcendental Iteration 2. Determinant Plotting	1. Power Method 2. Determinant Plotting
No Set Limit	20 Eigenvalues/Harmonic >5 Not Recommended
Live Pressure Available	Yes
Eigenvalue Programs Self-Contained	Yes
Boundary Conditions Variable in Harmonic Search	Yes
Mechanical and Thermal Loads = Simultaneous Eigenvalue	Mechanical and Thermal Loads ≠ Simultaneous Eigenvalue
Not Available	Nonlinear Prebuckling Option in Eigenvalue Program
Critical Speed Analysis	Not Available
Inelastic Analysis	Not Available
Transient Dynamic Analysis	Not Available
Not Available	Buckling Imperfection Sensitivity (Not Checked Out)

Table VI Comparison of Analytical Results for Model Motor Case

Program	Idealization	Run Time	Max. Deflection
$NASTRAN_1$ (Version 12)	855 Nodes/6 DOF per Node	60 Min	10.16 cm.
$NASTRAN_2$ (Version 15)	1900 Nodes/6 DOF per Node	70 Min	~33. cm.
BOSOR4	346 Mesh Pts/4 DOF per Pt./20 Harmonic Runs	3 Min	32.766 cm.
MARC	80 Nodes/9 (?) DOF per Node	~360 Min	8.61 cm.
STARS-2S	65 Segments/4 DOF per Pt./51P per Segment/20 Harmonic Runs	8 Min (IBM 360)	32.9 cm.
STARS-2SF (Special Fast Version for Shells Involving Long Cylinders)	As Above	1.6 Min (IBM 360)	32.9

Table VII Comparison of Frequency Analysis for Free Cylinder

Program	Idealization	Analytical Results	Run Time
STARS-2V n = 0, 1	40 Segments/10IP per Segment	8 Frequencies; 4 Mode Shapes; 160 Frequency Estimates	2.7 Min
n = 2 - 4	40 Segments/10IP per Segment	30 Frequencies; 6 Mode Shapes; 222 Frequency Estimates	3.93 Min
NASTRAN (Version 15.5)	252 Nodes/6 DOF per Node	1 Frequency; 1 Mode Shape (Search in Range of n = 2 Lowest Root Indicated by STARS-2)	21.3 Min (With Reduction) 7.7 Min (No Reduction)
	378 Nodes/6 DOF per Node	1 Frequency; 1 Mode Shape (Search in n = 2 Range)	— 12 6 Min (No Reduction)
	252 Nodes/6 DOF per Node	No Results - Search in Range Set Prior to STARS-2 Results	30 Min Cutoff Exceeded

Structure Analyzed:

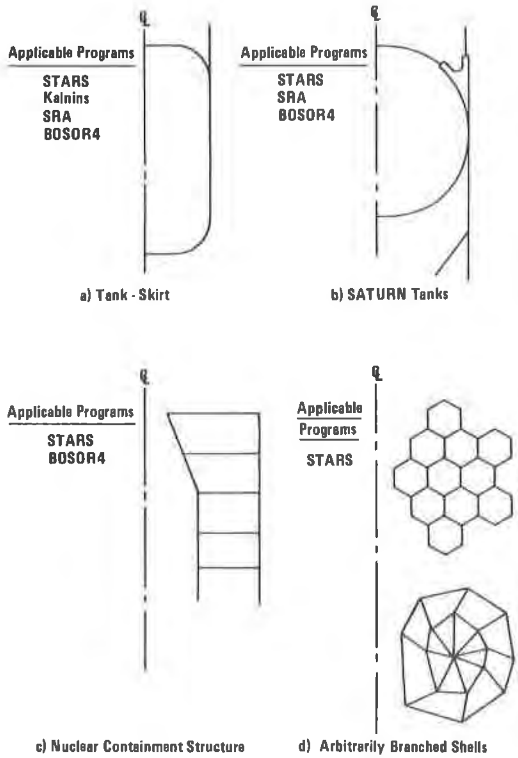

Fig. 1 Shell Program Branching Capabilities

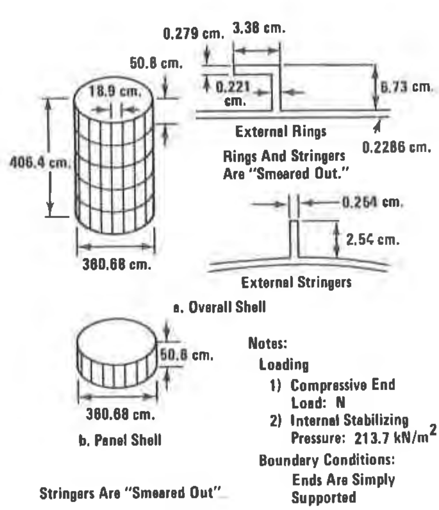

Fig. 2 Stiffened Cylinder

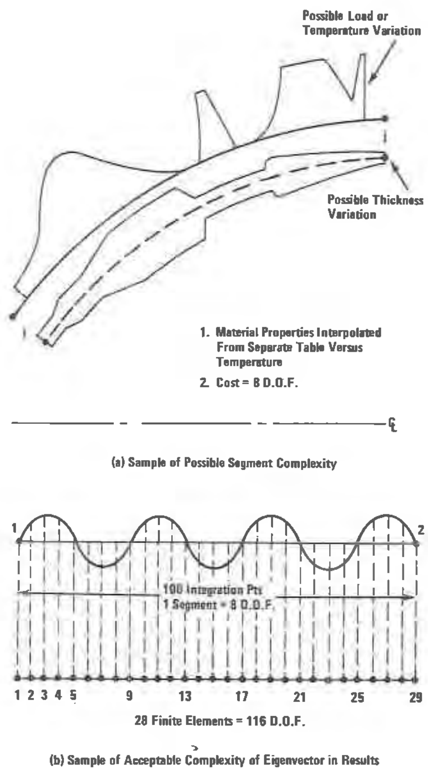

Fig. 3 STARS Segment = "Super" Finite Element With Coarse Mesh Option

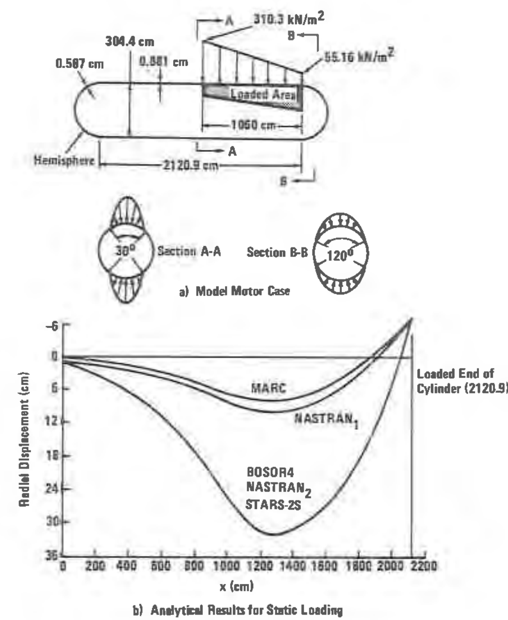

Fig. 4 Analysis of Model Motor Case

G 1/