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THE INTERACTION OF RATE AND HISTORY-DEPENDENT EFFECTS AND ITS SIGNIFICANCE FOR SLOW CYCLIC INELASTIC ANALYSIS AT ELEVATED TEMPERATURE

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SUMMARY

Life prediction for creep and low-cycle fatigue interaction and the analysis of ratcheting phenomena are of great importance in the design of future high-temperature nuclear reactors. These problems involve slow cyclic load application, inelastic deformation and are inherently non linear so that analytical closed form solutions are only possible in isolated cases. Fortunately, finite element computer programs with their time-following and non linear capabilities are now available for the efficient solution of these complicated problems. The task is now to ensure that a proper representation of the material deformation behavior, the constitutive equation, is used in these computer programs.

In this paper a qualitative comparison is made between the deformation behavior of metals observed in slow cyclic tests at high homologous temperature and the prediction obtained if various material idealizations (linear and non linear viscoelasticity, plasticity, creep theory, equation of state) are employed in the analytical simulation of the same experiments.

A systematic study of the deformation behavior of metals exhibited in slow cyclic laboratory tests such as cyclic creep, cyclic relaxation, low-cycle fatigue with and without hold-time shows that rate- and history-dependent effects interact. Surprisingly, history dependence was found for repeated creep as well as relaxation tests.

Because of the interaction of rate- and history-dependent effects, an additive use of elastic, plastic and creep strains is not realistic for cyclic conditions. Non linear viscoelasticity laws are incapable of reproducing history dependence and cannot be recommended for cyclic loading. It appears that a realistic constitutive equation for low-cycle fatigue creep interaction and ratcheting must be developed.

Various possibilities of modeling the interaction of rate and history dependence are discussed. The concept of a non fading or partially fading memory of discrete past events, such as the maximum stress tensor seen in the past, is introduced. A discontinuous growth law is specified for this parameter which can be incoprorated into a non linear, differential or integral constitutive equation.

1. Introduction

"The designer of critical vessels is limited, or will soon be limited by his knowledge of material behavior".

"By critical vessels, I mean those which require extraordinary care in design because of size, complexity or severity of operating conditions" [1].

Since this writing the ASME Boiler and Pressure Vessel Code for Nuclear Vessels has been augmented by Code Cases 1331-1 through 6. In these additions the finite life of critically stressed components has been recognized. Moreover, creep-fatigue interaction and ratcheting are considered and design rules against these failure modes are now incorporated.

Stresses and strains leading to finite life, failure due to creep-low-cycle fatigue interaction and excessive deformation due to ratcheting are invariably such that inelastic deformation occurs. While there are exceptions, such as linear anelastic behavior, inelastic deformation in metals is highly nonlinear. Further plastic (time independent) and rate (time dependent) deformation occur at elevated temperature. It is therefore clear that stress-analysis performed in these regimes is highly complicated.

The above loadings are of interest since they can lead either to failure of the component by initiating a crack (creep, low-cycle fatigue fracture) or by excessive deformation (ratchet). In the former case a separate failure criterion has to be introduced to estimate the time or the number of cycles at which cracking will occur. The usual creep-rupture or the Coffin-Manson and related criteria [2] are nonlinear and cycle or time dependent.

Before the advent of finite element computer programs, these problems were intractable for design calculations under a general state of stress. Therefore great emphasis was placed on the simplification of the problem. For components which are not critically stressed such simplified calculations are still the most useful and economical analysis tool. For critical components a detailed calculation using the nonlinear and time-following capability of the finite element method is not only economically justified but also mandatory from a safety standpoint. To be comparable to the capabilities of the computer the material deformation behavior must now be represented as accurate as possible in analytical form, the constitutive equation. Further failure laws compatible with computer applications are necessary. For design analyses aimed at these critical components the emphasis is no longer on a very simple representation but rather on a very accurate description of real metallic material deformation and failure behavior. Such a description has to be commensurate with the capabilities of the computational techniques.

The adequate representation of inelastic material, especially for cyclic applications, has received considerable attention recently. It has been stated that "progress in analytical methods has far outstripped our knowledge of the material to be analyzed" [3]. Sometimes the verification of computed results vis a vis actual material deformation behavior may become necessary. "For computer codes that deal with situations in which the theory is not highly developed or clear, the theory must be qualified for its proposed usage. This is true, for example, for problems in which the structure is loaded mechanically and thermally so that time-independent plasticity and viscoelasticity take place with various cyclic combinations thereof. Although computer codes have been developed which can deliver numbers, the theory on which the codes are based may be sufficiently different from the real material behavior for the time, number of cycles, and history of loading involved, that totally erroneous numbers result. In other words, the program may have been verified so as to correctly manipulate the numbers, but it may give a completely false answer to the engineering problem", p.119 of [4].

The objective of the present paper is to examine key features of metallic material deformation behavior under slow cyclic loading, such as low cycle fatigue, cyclic creep, cyclic relaxation. A qualitative comparison is then made between the actual material behavior and the behavior predicted by some "ideal materials" which are presently being used. A quantitative comparison on the basis of numerical results is not contemplated.

2. Significance of Cyclic Inelastic Loading for Stress Analysis

The significance of cyclic inelastic loading of metals lies in the permanent changes caused in the material due to slip during deformations prior to the present time. Strictly speaking, a new material can exist after every loading and unloading. The characterization of the deformation induced material property changes becomes therefore of great importance.

Prior to the advent of cyclic inelastic loading in engineering structures, these property changes were unimportant for design analysis. The analyst used either the stress-strain diagram of the annealed or of the cold-worked version of the same material and pretended that they were just different materials. Such an approach is fully justified if the service loadings are such that no significant changes in the material properties occur.

In creep-low-cycle fatigue loading and ratcheting such changes may occur at every cycle. For the analysis of critical components such changes must be accounted for in the constitutive equation. This is indeed a formidable challenge and gives cyclic inelastic loading a special status and importance.

At low homologous temperature T (defined as test temperature over melting temperature both measured in degree Kelvin) where time-independent behavior predominates, shake-down theorems exist for elastic-perfectly plastic solids, see the discussion in [5]. In this idealization the material properties are essentially cycle independent [5], and useful design information can be extracted from such an idealization.

There is also a possibility of using a cyclic stress-strain diagram to represent cyclic material properties and to use it in stress analysis [6] at low homologous temperature, such avoiding the assumption of an elastic-perfectly plastic material. Such a method is valid only if the imposed loadings are such that a cyclic steady state can be reached in the material. At high homologous temperature in the presence of hold-times a cyclic stress-strain diagram can be defined for every hold-time [7].

In these last two approaches [6,7] the cyclic steady state is given a special status and these methods can be advantageously used in design if the existence of cyclic steady state can indeed be assumed.

A generally useful constitutive equation for cyclic loading should be able to predict the cyclic steady state from the knowledge of the virgin (annealed) material properties and the history of loadings imposed on the material. For the purpose of this study no use is made of the stable cyclic properties of the material.

From this short discussion it is clear that the deformation induced material property changes, which are unimportant in the absence of load reversal, become significant for cyclic inelastic loading. While general theories exist which would permit the inclusion of such property changes [5], to the knowledge of this author, no definite constitutive equation has been worked out for cyclic inelastic deformation.

3. Salient Features of Inelastic Deformation Behavior of Metallic Materials

To form a basis on which the predictive capabilities of constitutive equations can be compared, it is necessary to delineate key features of inelastic metal deformation behavior. These properties are

a) aging
b) rate (time) dependence
c) history dependence.

By aging we mean the change in mechanical properties in the absence of mechanical loads due to exposure to the environment alone. A material is said to have aged in a certain time period if the same test with two identical specimens yields different results. One of them was exposed to a certain environment for a certain time period before testing whereas the other one was tested immediately.

Rate dependence identifies rate sensitivity in a tensile test, certain forms of "time dependent" behavior such as creep or relaxation and the aftereffect. These properties are sometimes referred to as anelasticity. Under rate dependence we mean "viscous type" behavior which is not associated with a change in the structure of the metal.

History dependence is used here in the sense of classical plasticity. A metal is said to show history dependence if the response of an annealed and a deformed specimen to the same test loading are different. To delineate clearly the effect of prior deformation, the environment has to be such that no aging takes place. It should be noted that this definition of history dependence does not require a yield surface.

The physical reason for history dependence is the deformation induced structure change during prior deformation. In many cases slip is the predominant factor. While these structure changes are permanent at low homologous temperature, thermal activation (diffusion processes) may partially or completely anneal these deformation induced changes.

These annealing processes together with rate dependence which occur simultaneously make the modeling of metallic material behavior extremely complex. An indication of the possible mechanisms which may be operative is given in Table 1. The information given is qualitative only and conceptual in nature.

For a nonaging metallic material the influence of prior deformation is such that only changes in degree but not in kind of the deformation behavior are observed. A creep or relaxation curve has always a characteristic shape no matter what the prior deformation. Figure 1 is an attempt to list the properties invariant under prior deformation. Since only a limited number of multiaxial experiments are available at elevated temperature, the expressions related to multiaxial effects are considered to be tentative.

Ferritic and austenitic low strength materials are usually employed in the intermediate homologous temperature range. They are made that the influence of aging is minimized. If we concentrate on this temperature regime the relevant constitutive equations must at least have a repository for rate dependence and history dependence. To obtain further requirements specific examples have to be cited.

4. Interaction of Rate and History Dependence

An example of such an interaction which is considered to be typical by this author is shown in Fig.2 [8]. The diagram depicts the behavior of a Cr - Mo - V steel in a creep test at 1000° F. The specimen was subjected to a creep loading for about 24 hrs which was followed by a rest period at zero load for about 310 hrs before it was loaded again to the same creep stress. It should be noted that total strain is plotted in the diagram.

The following observations are of importance.

Although all the initial loading is elastic (as indicated by the authors of [8]) a permanent set exists at the end of the rest period at zero load. We have to conclude that creep has introduced a permanent set. Upon reloading a creep curve different from the original one is obtained. Prior deformation must have changed the creep property of the Cr - Mo - V steel. (Aging is not considered to be a problem at this temperature for this material.) We therefore have a history dependence in creep.

Other examples of interaction of rate and history dependence are shown in [9], see Figs.9, 11 and 12. Further evidence of history dependence in creep are given in [10] where the secondary creep rate after a step-down test was invariably lower than for a virgin material, (Fig.1 in [10]). Further prior relaxation is shown to have an effect on the subsequent creep behavior, see [11] Fig.3.

The possible influence of aging was shown not to exist in experiments on 1100 Aluminum [12]. These results confirm history dependence in creep as previously discussed.

The above examples appear to justify the conclusion that history dependence in the sense of plasticity can be found in creep and that an interaction with rate (time) dependence takes place. Further the examples tend to indicate that history dependence may occur under any kind of loading and is not restricted to the "time independent" behavior of materials.

5. Representation of Elevated Temperature Inelastic Deformation Behavior in Engineering Calculations

Inelastic stress analysis of structures usually combines elastic, plastic and creep strains, e.g.

$$\varepsilon_{ij} = \varepsilon^{e\ell}_{ij} + \int_0^{e_{ij}} de_{ij} + p_{ij} \tag{1}$$

where

ε_{ij} - total strain
e_{ij} - plastic strain
p_{ij} - creep strain
} tensor
$\varepsilon^{e\ell}_{ij}$ - elastic strain

Various forms of this equation exist in the literature, e.g., [15]. Common to these approaches is the following. The repository for history dependence rests with the plastic strains e_{ij} and the repository for "time dependence" rests with the creep strains p_{ij}.

The plastic stress-strain relations for a strain-hardening material are given by [16, p.369]

$$de_{ij} = \hat{G} \frac{\partial f}{\partial S_{ij}} \frac{\partial f}{\partial S_{k\ell}} dS_{k\ell} \tag{2}$$

where $\hat{G}$ is a scalar factor, $S_{k\ell}$ is the stress deviator tensor and f represents the yield surface.

The creep strains p_{ij} are usually given by

$$p_{ij} = f(S_{ij}, \eta)\, S_{ij} \tag{3}$$

Various forms exist, such as the time or strain hardening version or the definition of a creep potential. They all will reproduce, when specialized for the uniaxial case, to the creep curves measured in a regular uninterrupted creep test. Therefore, no possibility exists to account for history dependence in Eq.(3). Equation (3) states that the creep strain p_{ij} depends on the present deviatoric stress S_{ij} and the time variable η which is, by convention, zero at the start of the creep test.

Suppose we apply Eq.(1) through (3) to the prediction of the test results in Fig.2. As indicated by the authors of [8] no plastic deformation takes place during loading. The stresses have not"reached the yield surface", no plastic strain is accumulated. During creep $dS_{k\ell}$ is zero so that the plastic strain is zero throughout the test shown in Fig.2. To apply Eq.(3) two possibilities exist:

a) The time η is set zero at the origin of the time axis and (3) is evaluated only for constant S_{ij}. Then Eq.(3) would not predict any primary creep for the second loading. (Secondary creep was already reached during the first loading.
b) The time η is set zero at the start of the first and the second loading, respectively. If this is done the predicted creep curve during the initial and the secondary loading would be identical.

The predictions of the commonly used theories are therefore at variance with experimental results since the theory has no repository for history dependence in creep.

The above argument was entirely based on a qualitative comparison. In certain cases the influence of prior history may be small so that the procedures centering around Eq.(1) can be satisfactorily used for engineering calculations. This success in particular isolated cases cannot be considered to be a proof for the validity of the conventional theory.

In cyclic tests the material properties become strongly cycle dependent [2]. It is there that the limitations of the theory become most apparent. Striking examples of strongly cycle dependent creep properties are given in [17], where considerable cycle dependence of the creep curves of Cr - Mo - V and 2 1/4 Cr - 1Mo steels are reported.

6. Need for New Theories of Inelastic Material Behavior for Cyclic Loading

Historically, plasticity was conceived for carbon steels which usually exhibit a definite yield point [18] and the concept of the elastic-perfectly plastic material was introduced. Work-hardening was included later [19] when it became technologically necessary. Engineering creep theories were started in the 1930's [20] when they were technologically required.

To combine these theories,as outlined in Eq.(1),is expedient as is their application to cyclic loading. In doing so one has to realize that cyclic loading was of no concern when these theories were originally conceived. If they are extended to cyclic inelastic analysis correspondence between theory and experiment cannot always be expected.

The above arguments have shown the shortcomings of the presently used constitutive equations on a qualitative basis. These shortcomings are severe indeed. It is therefore desirable to make a quantitative comparison between the results of finite element computer calculations and experiments involving critical tests on simple specimens. It should be stressed that not any arbitrary test can be used, rather those that exhibit significant history dependence have to be chosen. A low-cycle fatigue test with load or displacement control would be such a test [2,17]. It is unfortunate that the engineering community is very busy doing cyclic finite element calculations on complicated structures. There is presently very little work towards the verification of the computed results against simple laboratory experiments as suggested on p.119 of [4].

For an accurate inelastic stress analysis under slow cyclic loading new theories of material deformation behavior are necessary. The additive composition of strains, Eq.(1), appears to be too restricted. Continuum mechanics does not appear to make it necessary to postulate separate constitute equations for additively composed strains. In an experiment the permanent or plastic strain is obtained by unloading to zero stress as indicated in Fig.2. (This procedure is only useful if the state of stress is homogeneous.) Consequently, a constitutive equation must predict a permanent or plastic strain if the same experiment is simulated mathematically. An a priori additive decomposition of strains as suggested in Eq.(1) does not appear to be necessary.

Recently some new theories have been proposed. Starting from the experimental evidence in low-cycle fatigue tests the author has introduced the concept of a nonfading or partially fading memory of discrete past events [9]. The memory parameters were incorporated into a first order differential form containing time derivatives of stress and strain [9]. A three-dimensional formulation was given in [21]. It was shown that the interaction of rate and history dependence can easily be accommodated within this theory [9,21]. Specific examples are given in [9,21].

Another way of modeling history dependence in the plasticity sense is to postulate an intrinsic time which measures the path length in strain space [22,23,24]. The intrinsic time is suitably combined with real time so that plasticity and aging effects can be modeled.

These relatively new concepts should be developed further so that they ultimately may be used in engineering calculations for cyclic inelastic analysis using finite element techniques. It does not appear to be fruitful to continue working on the traditional concepts of "time independent" and "time dependent" behavior in the area of elevated temperature deformation behavior of structural metals where history and rate dependence show a strong interaction.

7. Acknowledgement

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TABLE 1

PHYSICAL PROCESSES IN INELASTIC METAL DEFORMATION

Homologous Temperature $T = \frac{T_{test}}{T_{melt}}$ in °K	Rate Dependence. "Viscous" Effects.	Deformation Induced Structure Change. History Dependence	Property Changes due to Exposure to the Environment Alone "Aging"	Material Idealization
Low ~ < .3	Slight	Permanent	None	Plasticity. Rate Independent Materials
Intermediate ~ 0.3 ≤ T ≤ .55	Pronounced	None	None	Linear and nonlinear viscoelasticity. Certain integral representations. Kernel depends on $(t-\tau)$*; differential equations.
	Pronounced	None	Exist	Linear and nonlinear viscoelasticity. Certain integral representations. Kernel depends on $(t-\tau, t_e)$, differential equations.
	Pronounced	Present, may be partially fading out due to thermal activation	None	Need to be Developed.
	Pronounced	Present, may be partially fading out due to thermal activation	Exist	
High T > .55	Pronounced	Completely fading	Exist	

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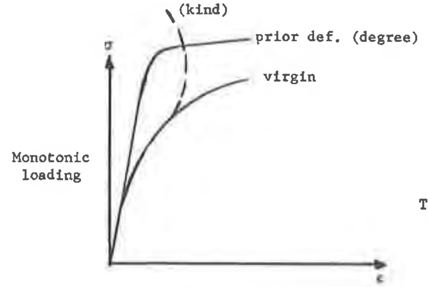
* t - present time; τ - time variable; t_e - designates the time on a clock that starts running when the specimen is exposed to the environment.

- Effect of prior deformation history, nonaging materials.

  Material deformation behavior can change in degree but not in kind

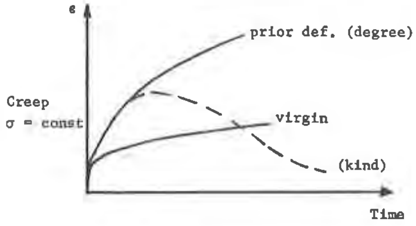


$0 \le \frac{d\sigma}{d\varepsilon} \le E$ always

$\frac{d\underset{\sim}{\sigma}}{d\underset{\sim}{\varepsilon}} = \underset{\sim}{\beta}\,(\underset{\sim}{\sigma})$

Tentative

$\underset{\sim}{T} \cdot \underset{\sim}{\beta} \cdot \underset{\sim}{T} \ge 0$ for every $\underset{\sim}{T}^T = \underset{\sim}{T}$



$\frac{d\varepsilon}{d\tau} = p \ge 0$ for $\varepsilon > 0$

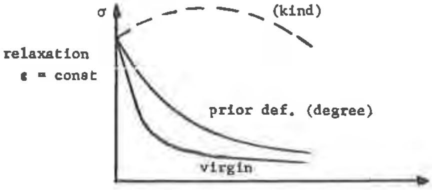
$= p \le 0$ for $\varepsilon < 0$

Tentative

$\underset{\sim}{u} \cdot \underset{\sim}{p} \cdot \underset{\sim}{u} \ge 0$ for $\underset{\sim}{u} \cdot \underset{\sim}{\varepsilon} \cdot \underset{\sim}{u} \ge 0$

$\underset{\sim}{u} \cdot \underset{\sim}{p} \cdot \underset{\sim}{u} \le 0$ for $\underset{\sim}{u} \cdot \underset{\sim}{\varepsilon} \cdot \underset{\sim}{u} < 0$

for every nonzero vector $\underset{\sim}{u}$



$\frac{d\sigma}{d\tau} = \dot{\sigma} \le 0$ for $\sigma > 0$

$= \dot{\sigma} \ge 0$ for $\sigma < 0$

Tentative

$\underset{\sim}{u} \cdot \underset{\sim}{\dot{\sigma}} \cdot \underset{\sim}{u} \le 0$ for $\underset{\sim}{u} \cdot \underset{\sim}{\sigma} \cdot \underset{\sim}{u} > 0$

$\underset{\sim}{u} \cdot \underset{\sim}{\dot{\sigma}} \cdot \underset{\sim}{u} \ge 0$ for $\underset{\sim}{u} \cdot \underset{\sim}{\sigma} \cdot \underset{\sim}{u} < 0$

Multiaxial, rate independent. Yield surface may translate, rotate and change shape, but will stay convex [13,14]

Figure 1 Invariance Properties of Nonaging Metals

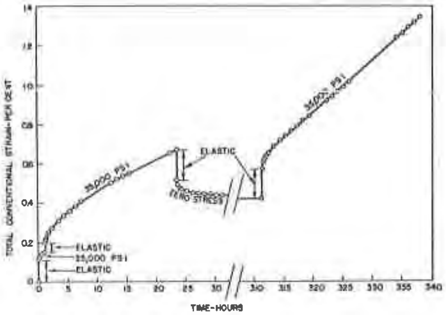


Figure 2 The Behavior of a Cr - Mo - V Steel at 1000°F under a Repeated Creep Test, from [8]