Effect of internal wall thickness on convective heat transfer in a multi-passage pipe with heat generation

M.A.Ebadian & H.C.Topakoglu
Mechanical Engineering Department, Southern University, Baton Rouge, La., USA

ABSTRACT

In this paper convective heat transfer analysis in a multi-passage tube subjected to a uniform external heat flux is extended to include the effect of the internal separating wall thickness. The analytical expressions for Nusselt numbers corresponding to the outer and inner walls are obtained with inner and outer region fluids having independent arbitrary heat generation densities. The curves representing the outer and inner Nusselt numbers including the effect of wall thickness considering Molten Salt Breeder Reactor (MSBR) operation are plotted against the dimensionless radius of separation surface ω. Also, to show the effect of wall thickness, the rate of change of Nusselt numbers with the wall thickness are plotted against the dimensionless radius of separation. The significance of these results are discussed.

1 INTRODUCTION

Recently, some interest including Ebadian et al. (1986-A) and (1986-B) has arisen in investigating the convective heat transfer characteristics of a multi-passage tube using an analysis based on exact temperature distributions rather than the conventional, lumped-parameter approach (Shah and London, 1978). However, these references (Ebadian et al. 1986-A and 1986-B and Shah and London, 1978), and a literature survey which includes (Eckert et al. 1985), show that the effect of conductive thermal resistance at the inner separating surface has not been included. In ordinary heat exchange devices using multi-passage conduit, the significance of this effect especially for long operation periods, becomes important. In addition, this effect becomes an important factor when one or both of the fluids are replaced by liquid metal.

In this paper the external surface of the multi-passage pipe is assumed to be maintained under uniform heat flux. Along the length of the conduit, both flows are assumed to be hydrodynamically and thermally fully developed. In order to maintain generality, both flows are assumed to have independent arbitrary heat generation densities. It was shown in Ebadian et al. (1986-A) that for a multi-

passage conduit, Figure 1, the dimensionless heat transfer coefficients (Nusselt numbers) for the outer and inner surfaces can be defined in three different ways (i.e., Case I, Case II, Case III). In the analysis of this paper, the first definition (Case I) is used. According to this definition the Nusselt numbers are defined relative to the mixed, combined bulk temperature of the inner and outer region flows.

Numerical results are produced by considering an example of the primary and secondary flow cycles of a Molten Salt Fast Breeder Reactor (MSBR). The data needed for the calculations are chosen such that the inner flow of the multi-passage flow fluid is assumed to be liquid sodium, having a heat generation number $\gamma = 3.00$ (Ebadian et al. 1986-A). The selection of this number is based on the data given by Foster (1967) and it corresponds approximately to the internal heat generation density of the fluid in the primary cycle of a MSBR. The outer flow region fluid is considered as water with no heat generation, $\gamma = 0$. The numerical values of the heat exchanger number η are selected as 1.02, 10.2,-1.02,-10.02. The first two values correspond to parallel flow arrangements, whereas the last two correspond to counter-flow arrangements.

A steel dividing wall having a thickness equal to 5% of the outer surface radius was selected to separate the two flow regions. Using the derived Nusselt number expressions corresponding to this case, the outer surface Nusselt number (Nu_o) and the internal surface Nusselt number (Nu_i) are plotted against the dimensionless radius (ω) of the internal surface. In each of these figures, four curves are presented with heat exchanger numbers $\eta = 1.02, 10.2, -1.02, -10.2$, respectively. In order to evaluate the degree of influence of the separating wall thickness on the heat transfer characteristics, the rate of change of Nusselt numbers with wall thickness are plotted against ω.

2 TEMPERATURE DISTRIBUTION

At sufficient lengths from the entrances of the inner and outer flows, where temperatures are fully developed, the temperature distributions in each flow regions due to the convective heat transfer caused by the externally applied uniform heat flux, including the effect of the finite thickness of the internal separation wall will have the following functional form:

$$(1) \qquad T_o = CZ + E_o(X,Y), \quad T_i = CZ + \Delta E + E_i(X,Y),$$

where

- C: The uniform external temperature gradient along the pipe
- E_o and E_i: The excess temperatures of the outer and inner flows, respectively
- E_{in}: The interface temperature at the outer surface of the inner separating wall
- ΔE: The temperature change on the inner surface of the separating wall relative to the temperature at the outer surface of the inner wall

The other notations used in (1) are the same as those given by Ebadian et al. (1986-A). The effects of the thickness of the separating wall is taken into account by considering the thermal resistance of this wall. Otherwise for simplicity, in the field itself, this thickness is neglected. This is equivalent to a situation that only a finite temperature change, equal to ΔE, on the internal wall is considered in the analysis. Under these assumptions the boundary conditions of excess temperatures are:

$$(2)\quad \begin{cases} E_o = 0 & \text{on the outer periphery} \\ E_o = E_{in} & \text{on the outer surface of the interface} \\ E_i = 0 & \text{on the inner surface of the interface} \end{cases}$$

The energy equation to be satisfied in each flow region, including a uniform heat generation density H is

$$(3)\quad \vec{V} \cdot \text{grad } T = \alpha(\Delta^2 T + \frac{H}{k}) \ .$$

It must be noted that this equation must be used in each flow region.

The dimensionless thermal variables are defined by the following relations:

$$(4)\quad \begin{cases} H_o = \frac{F}{L} h_o, \quad C = \frac{F}{k_o} c, \quad E_o = \frac{LF}{k_o} e_o \\ H_i = \frac{F}{L} k_i, \quad E_{in} = \frac{LF}{k_o} e_{in}, \quad E_i = \frac{LF}{k_o} e_i \\ \Delta E = \frac{LF}{k_o} (\Delta e) \end{cases}$$

where H_o, H_i and h_o, h_i are dimensional and dimensionless heat generation densities for the outer and inner flow passages, respectively. F denotes an arbitrary reference heat flux. L is the radius of the multi-passage tube.

One can note that, with the notations and assumptions used in this paper, the excess temperature distributions remain the same as those given by Ebadian et al. (1986-A). Because of their physical significances, only three parameters are repeated here. These are:

$$(5)\quad \begin{cases} \gamma_o = \frac{h_o}{G_o}, \quad \gamma_i = \frac{h_i}{G_i} \\ \text{and} \qquad \eta = \frac{k_i}{k_o} \frac{(Pe)_i}{(Pe)_o} = k_k \, k_{pe} \\ \text{where} \qquad G_o = c(Pe)_o \ , \ G_i = c\omega^3 (Pe)_i \end{cases}$$

$(Pe)_o$, $(Pe)_i$ are the Peclet numbers for the outer and inner flows, respectively, η is the heat exchanger number of the multi-passage tube. γ_o and γ_i represent the heat generation numbers of the outer and inner flow fluids, respectively, as defined by Ebadian et al. (1986-A).

3 HEAT FLUXES AND HEAT TRANSFER COEFFICIENTS

The rate of heat flows per unit length of the pipe through the outer and inner surfaces, considered positive when flowing in the radial direction are [Topakoglu and Arnas, 1974]:

$$(6) \qquad \begin{aligned} U_o &= 2\pi LFG_o\left[\frac{\beta}{\ell n\omega} + e'_{oo}(1)\right] \\ U_i &= -\,2\pi LFG_o\left[\frac{\beta}{\ell n\omega} + \omega\, e'_{oo}(\omega)\right] \end{aligned}$$

where β is the alternate dimensionless interface temperature on the outer surface of the internal separating wall. The expression of β as well as the functions $e'_{oo}(1)$ and $e'_{oo}(\omega)$ are given by Ebadian et al. (1986-A).

The mixed mean temperature of the combined flow is

$$(7) \qquad T_m = \left(C_{po}\,\rho_o \int_{Ao} W_o T_o dA + C_{pi}\,\rho_i \int_{Ai} W_i T_i dA\right)/\left(C_{po}Q_o + C_{pi}Q_i\right)$$

where ρ_o, A_o, C_{po}, Q_o and ρ_i, A_i, C_{pi}, Q_i are the densities, cross sectional areas, specific heats and the mass rate of flows for the outer and inner passages, respectively. Substituting T_o and T_i from (1), this reduces to

$$(8) \qquad T_m = CZ + \left[E_{mo} + k_c\lambda E_{in} + k_c\lambda(\Delta E) + k_c\lambda E_{mi}\right]\left(\frac{1}{1+k_c\lambda}\right)$$

where $$E_{mo} = \frac{\rho_o}{Q_o}\int_{Ao} W_o E_o dA \ , \quad E_{mi} = \frac{\rho_i}{Q_i}\int_{Ai} W_i E_i dA$$

$$k_c = \frac{C_{pi}}{C_{pi}}$$

$$(9) \qquad \begin{cases} \lambda = \dfrac{Q_i}{Q_o} = \dfrac{1}{4}\dfrac{\mu_i}{\mu_o}\dfrac{(Re)_i}{(Re)_o}\dfrac{\omega^4}{I_{oo}} \\ I_{oo} = \dfrac{1}{4}(1-\omega)^2(1+\omega^2-u) \\ u = -(1-\omega^2)/\ell n\omega \end{cases}$$

where μ_o and μ_i are the coefficients of viscosities, and $(Re)_o$ and $(Re)_i$ represent the Reynolds numbers for the outer and inner flows

respectively. These Reynolds numbers are defined as the Reynolds numbers for a simple circular pipe of radius L filled with the corresponding fluid and maintained under the corresponding pressure gradient in each passage [e.g. Topakoglu and Arnas (1974)].

The heat transfer coefficients ($\bar{h}_o$ and $\bar{h}_i$) at the outer wall and at the inner separating wall are defined on the basis of the overall bulk temperature of the entire flow by the relations

$$(10) \qquad U_o = (CZ-T_m)2\pi L\bar{h}_o \ , \ U_i = (CZ+E_{in}+\Delta E-T_m)2\pi\omega L\bar{h}_i .$$

Selecting the fluid in the outer passage as the reference fluid, the dimensionless heat transfer coefficients (the Nusselt numbers) on each surface based on their individual diameters are defined as

$$(11) \qquad (Nu)_o = \frac{2L}{k_o}\bar{h}_o \quad \text{and} \quad (Nu)_i = \frac{2\omega L}{k_o}\bar{h}_i \ .$$

Finally, one can obtain the following expressions for $(Nu)_o$ and $(Nu)_i$ as:

$$(12) \begin{cases} 4I_{oo} + \eta\omega^4 + 2\gamma_o(\omega^2 - 1) - 2\gamma_i\omega^2 = \dfrac{8}{4I_{oo} + \eta\omega^4}\left[\dfrac{J_o}{G_o}\right. \\ \qquad\qquad \left. + \dfrac{1}{4}\eta\omega^4\left(\dfrac{4}{\omega^4}\dfrac{J_i}{G_o} - \beta - \dfrac{\Delta e}{G_o}\right)(Nu)_o \right. \\ \text{and} \quad \eta\omega^4 - 2\gamma_i\omega^2 = -\dfrac{8}{4I_{oo} + \eta\omega^4}\left(\dfrac{J_o + J_i}{G_o} + \beta I_{oo} + \dfrac{\Delta e}{G_o}\right)(Nu)_i \end{cases}$$

where

$$(13) \begin{cases} \Delta e = \dfrac{1}{2\pi k}\dfrac{U_i}{2\pi\omega}\ell n\dfrac{\omega + \frac{\Delta r}{2}}{\omega - \frac{\Delta r}{2}} \\ \dfrac{1}{2\pi}\dfrac{U_i}{G_o} = -\dfrac{1}{4}(\eta\omega^2 - 2\gamma_i)\omega^2 \\ \Delta r = \text{The dimensionless wall thickness of the separating surface} \end{cases}$$

The expressions of $\frac{J_o}{G_o}$, $\frac{J_i}{G_o}$ and β involving in (12) are the same as those given by Ebadian et al. (1986-A).

4 DISCUSSION OF NUMERICAL RESULTS AND CONCLUSIONS

A representative case is selected corresponding to the heat exchange operation between the primary and secondary cycles of a Molten Salt Breeder Reactor (MSBR). The inner flow of the multi-passage flow is assumed to be liquid sodium with a heat generation number $\gamma = 3.00$. This numerical value corresponds approximately to the internal heat generation density of the fluid in the primary cycle of a MSBR. The outer flow region fluid is considered to be water with no heat generation, $\gamma = 0$. The numerical values selected for η are 1.02, 10.2, -1.02, -10.2 for which the positive values correspond to a parallel flow arrangement and the negative values correspond to a counter-flow arrangement.

The inner separating wall material is selected as 1% chrome steel and its wall thickness is assumed as 5% of the outer surface radius of the multi-passage pipe.

Four figures are presented, and their significances are discussed below.

4.1 Figure 2

The curves shown in this figure depict the variation of the outer wall Nusselt number, Nu_o, with the dimensionless radius of the separation wall ω, for four different η values as indicated in the figure. In the special case of $\omega = 0$, the multi-passage pipe reduces to a simple pipe flow of a non heat generating fluid containing a central wire. This central wire acts as a heat source line or a heat sink line depending on the constant value of the heat exchanger number η. For this reason, the indicated Nusselt number of 3.2350 deviates from the classical value of 48/11. In the special case of $\omega = 1$, the multi-passage pipe reduces to a simple pipe flow of a heat generating fluid having an outer wall possessing a finite thermal resistance. Again, because of the effects of η, the finite wall thermal resistance and the selected internal heat generation, a Nusselt number of 158.5000 is produced as shown in the figure. The phenomenon of infinite Nusselt numbers and the sign changes have been explained by Ebadian et al. (1986-A) and the same reasoning remains valid here.

4.2 Figure 3

In this figure, the variation of the Nusselt number on the inner separation wall with the dimensionless radius of the separating wall is presented. In the limiting case of $\omega = 1$, the outer flow disappears for parallel and counter-flow arrangements and the system reduces to that of a simple pipe flow having a Nusselt number at its outer wall equal to that on the inner surface value. This number has been found infinite because of the existence of a thermal resistance at the surface.

3 Figure 4

The curves presented illustrate the variation of the rate of change of outer wall Nusselt number with the dimensionless radius of the separation wall, ω. It can be seen that there are two vertical asymptotes at $\omega = 0.5000$ and $\omega = 0.6900$. The locations of these asymptotes correspond to the locations indicated in Figure 2.

4.4 Figure 5

The curves presented illustrate the variation of the rate of change of the Nusselt number on the inner separation wall with the dimensionless radius of the separation wall, ω. It must be noted that for ω = 0.8100 the curves have a vertical asymptote. Again this value corresponds to the position of the vertical asymptote in Figure 3.

5 ACKNOWLEDGEMENT

The results presented in this paper were obtained in the course of research sponsored by the National Science Foundation, Washington, D.C., under Grant R11-8305297 to Southern University and its precursors.

The authors appreciate the contributions made by Mr. Lee Chiang and Mr. Chaouki Helal to the numerical computations and the graphical work involved in this paper.

REFERENCE

Ebadian, M. A., Topakoglu, H. C. and Arnas, O. A., (1986-A), "The Effect of Heat Generation of Convective Heat Transfer for Laminar Flows in a Multi-Passage Circular Pipe Subjected to an External Uniform Heat Flux", AIAA/ASME Thermophysics and Heat Transfer Conference, (86-HT-4).

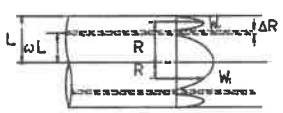
Ebadian, M. A., Topakoglu, H. C. and Arnas, O. A., (1986-B), "Convective Heat Transfer for Laminar Flows in a Multi-Passage Circular Pipe Subjected to an External Uniform Heat Flux", Int. J. Heat and Mass Transfer, V.29, pp. 107-117,.

Eckert, E. R. G., Goldstein, R. J., Pfender, E., Ibele, W. E., Ramsey, J. W., Simon, T. W., Deckor, N. A., Kuehn, T. H., Lee, H. O. and Girshick, S. L., (1985), "Heat Transfer - A Review of the 1985 Literature", Int. J. Heat and Mass Transfer, V.29, pp. 1767-1842.

Foster, A. R., (1967), Basic Nuclear Engineering, Allyn and Bacon Book Company, pp. 346,.

Shah, R. K. and London, A. L., (1978), Laminar Flow Forced Convection in Dusts, Academic Press Book Company.

Topakoglu, H. C. and Arnas, O. A., (1974), "Convective Heat Transfer for Steady Laminar Flow Between Two Confocal Elliptic Pipes with Longitudinal Uniform Wall Temperature Gradient," International Journal of Heat and Mass Transfer, 17, pp. 1487-1498.



L : Outer radius
ωL : Radius of interface
R : Radial coordinate for a particle located at the inner or outer region
W_o, W_i : Velocities of the outer and inner flows respectively
ΔR : Thickness of separating wall

FIGURE 1. ANNULAR GEOMETRY

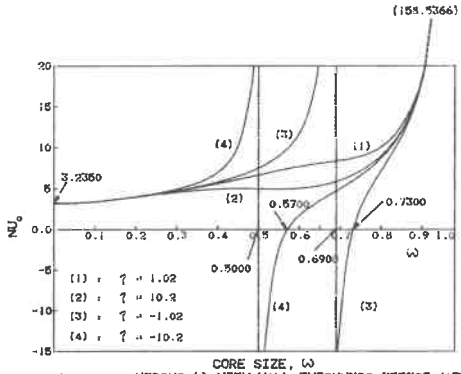


FIG 2. Nu_o VERSUS ω WITH WALL THICKNESS EFFECT AND HEAT GENERATION

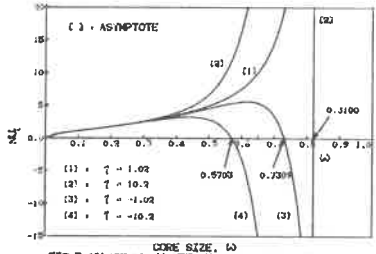


FIG 3. Nu_i VERSUS ω WITH WALL THICKNESS EFFECT AND HEAT GENERATION

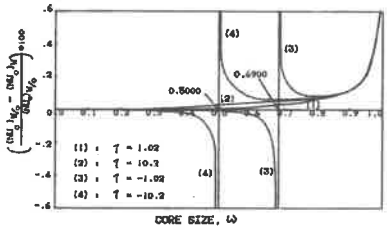


FIG 4. RATE OF CHANGE OF Nu_i VERSUS ω WITH WALL THICKNESS EFFECT AND HEAT GENERATION

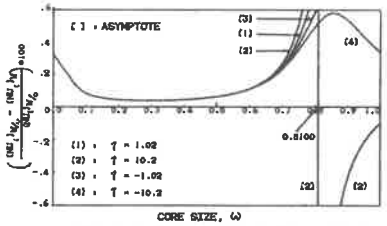


FIG 5. RATE OF CHANGE OF Nu_i VERSUS ω WITH WALL THICKNESS EFFECT AND HEAT GENERATION