

2D FLUID FLOW IN THE DOWNCOMER AND DYNAMIC RESPONSE OF THE CORE BARREL DURING PWR BLOWDOWN

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SUMMARY

As a part of the HDR program, methods for coupled fluid/structural dynamics are being developed. On the fluid side the 2D finite difference code YAQUI has been modified and adapted to describe the fluid dynamics in the downcomer of PWR's. On the structural side for determination of the dynamic core barrel response the code CYLDY2 has been developed. In this code the core barrel is treated as a thin cylindrical shell fixed at the upper end and ring stiffened at the lower end. The mass of the lower end ring also simulates a part of the core mass. Both models have been successfully tested. Coupling has been achieved for a simplified structural model proving the correctness of the coupling procedure.

YAQUIR is a significantly modified version of the code YAQUI originally developed at LASL. A finite difference scheme is used to solve the fluid dynamics equations for mass, momentum and energy transport on a two-dimensional grid. For analysis of the blowdown the downcomer has been represented by two different grids: one consisting of rectangular meshes and one of so-called "potential meshes", in which the orthogonal grid lines follow the expected lines of liquid flow. Both models show the axial and azimuthal wave propagation and the pressure transients when approaching the quasistationary flow.

The structural model CYLDY2 is based on Flügge's shell equations and uses variational principles. The solution is a superposition of steady-state and transient eigenfunctions. Results indicate that for the relatively thin-walled core barrel of the HDR-experiments in most cases the local deformations are somewhat higher than the global deformation (beam mode). In comparison to well known multipurpose codes which were also used, the code CYLDY2 yields very detailed results but requires only small computer effort, which is essential for coupling with the fluid dynamics. These advantages are due to the orthogonality of the eigenfunctions which leads to several uncoupled equation systems instead of one large system.

The coupling of YAQUIR and CYLDY2 is performed by imbedding the structural model in the fluid model. Fluid velocities are parallel to the fluid/structure interface. The structure displacements define the time and space dependent thickness of the two-dimensional fluid layer (2½-dimensional model). The general procedure for integration over one time step from t_1 to t_2 is as follows:

Phase 1: Determine fluid accelerations using the t_1 pressure field; determine structural acceleration using the t_1 pressure field; integrate fluid and structure velocities from t_1 to t_2 .

Phase 2: Iterate pressures and fluid and structural velocities until continuity equation is satisfied for the pressure field at t_2 .

Phase 3: Use iterated velocities to integrate displacements in the structural model.

While coupling of the complete CYLDY2 model with YAQUIR is still underway, results have been obtained with a simple axisymmetric structural model. For an axisymmetric test case three forms of pressure fluctuations have been observed: 1) radial oscillations dominated by the local compressibility of the water, 2) axial compression/expansion waves in the water considerably different from those obtained for a rigid barrel, 3) bulk axial water oscillations dominated by the global compressibility of the core barrel.

1. Introduction

In the HDR (a former test reactor which was in operation only for a short time) large scale experiments simulating main inlet pipe ruptures of a PWR (socalled blowdown) will be performed. These tests will be used to verify both state of the art codes for analysis of blowdown fluid dynamics and the structural response of vessel internals as well as newly developed best estimate codes which take into account the effect that even small deformations of the reactor structure have a significant influence upon the pressure field. The HDR experiments and the accompanying theoretical program is more fully explained in another paper of this conference [1]. In this paper, a code is described for coupled fluid structure analysis. The fluid dynamics part of the code is based on YAQUI [2], the structural dynamics part on CYLDY2 as explained below.

2. Modification of YAQUI for Fluid Calculations

YAQUI has originally been developed at LASL [2]. The code description [3] was used to program a REGENT subsystem YAQUIR. REGENT [4] is a PL/1 based software system which especially supports development and use of large application programs. As one of its main characteristics it provides powerful support for program and data management and a flexible "problem oriented language" for input.

After the first successful test calculations with YAQUIR significant improvements have been achieved in the areas of

- equation of state formulation
any algorithmic form of the function $p = p(\rho, e)$ can be used
- boundary condition
curved boundaries are now implemented. For outflow and inflow regions an arbitrary state may be set for the fluid just outside the control volume
- plot output representation.

With YAQUIR the twodimensional pressure field in the downcomer of the HDR has been analysed. In YAQUIR the mass, momentum and energy equations are integrated in a spatial finite difference net over time in three phases for each time step. In phase 1 a Lagrangian model is used to predict velocities, energies and densities for the advanced time; in phase 2 the pressure is iterated so as to satisfy the implicit form of the continuity equation; the momentum equation is also treated implicitly except for the convective terms. In phase 3 the Lagrangian model is convected to a Eulerian model on the basis of time advanced velocities and pressures.

Since the geometry underlying the YAQUI code is twodimensional, the complete fluid system consisting of the blowdown nozzle, the downcomer, the lower plenum, the region inside the core barrel and the upper plenum, cannot be represented altogether. However, since two-dimensional effects will be dominant in the downcomer, only this region has been modelled with YAQUIR. The downcomer has been unwrapped in order to fit into the 2D representation. This is possible, because no significant centrifugal accelerations are expected. Because of symmetry only a 180° section of the downcomer is used.

The basic models used for the finite difference mesh are shown in fig. 1. Fig. 1a shows a rectangular arrangement of the grid points while the grid in fig. 1b is derived from potential flow theory [5]. Fig. 2 shows the integrated force exerted on the core barrel in the direction of the nozzle for a rectangular and a potential flow mesh. The difference is most likely due to the few large mesh cells (in the upper left corner of fig. 1b) of the potential flow mesh, which induce a tilting of pressure waves in almost the same order of magnitude as the physical process investigated. Thus we conclude that the potential mesh overpredicts the barrel load. More results obtained with YAQUIR are presented in another paper of this conference [1]. Comparison of YAQUIR results with results of other codes and with shallow water simulation experiments has demonstrated the applicability of the code to the phenomena under investigation.

3. Structural Dynamics Code CYLDY2

3.1 Mathematical Model

The behaviour of the core barrel under transient loading is simulated by the code CYLDY2. This program represents a semi-analytical approach for determination of the dynamic response of a circular cylindrical shell, which is clamped at the upper edge and stiffened by a rigid circular ring at the lower end. This means that the lower edge can undergo rigid body movements but no deformations are allowed. Spatial load distributions, and the shell deformations, are symmetrical with respect to a plane which is defined by the axis of the shell and the blowdown nozzle. The load variation with time has been approximated by a step-type sequence of time - independent loadings. Thus the problem is essentially reduced to the analysis of the shell under a single load step with arbitrary initial conditions.

For solution it is assumed that the displacements may be described by the following expressions containing products of modal shape functions:

$$\begin{aligned}
 u(x, \theta, t) &= \sum_{m=1}^M \sum_{n=0}^N A_{mn}(t) \cdot F'_{mn}(x) \cdot \cos n\theta \\
 v(x, \theta, t) &= \sum_{m=1}^M \sum_{n=0}^N B_{mn}(t) \cdot F_{mn}(x) \cdot \sin n\theta \\
 w(x, \theta, t) &= \sum_{m=1}^M \sum_{n=0}^N C_{mn}(t) \cdot F_{mn}(x) \cdot \cos n\theta
 \end{aligned} \tag{1}$$

Here u , v and w are the displacements in axial, azimuthal and radial direction, respectively, x is the axial shell coordinate ($0 \leq x \leq d$), θ is the azimuthal coordinate ($0 \leq \theta \leq 2\pi$), t denotes the time.

The azimuthal shape functions $\cos n\theta$ and $\sin n\theta$ are obvious because of the period 2π in θ -direction and due to the symmetry mentioned above. The axial shape functions $F_{mn}(x)$ and $F'_{mn}(x)$ (the second being the derivative of the first with respect to x) must be chosen in accordance with the kinematic boundary conditions at the upper and lower edge. These may be characterized as follows:

- rigid clamping for all modes at $x = 0$ (upper edge);
- free end conditions for the azimuthal mode $n = 1$ at $x = d$ (lower edge);

- rigid clamping for azimuthal modes with $n \neq 1$ at $x = d$.

In CYLDY2 these boundary conditions are satisfied by establishing the shape functions $F_{mn}(x)$ in accordance with the deflection curves due to free vibrations of simple beams clamped at one end, and free or clamped at the other end. Thus $F_{mn}(x)$ generally reads

$$F_{mn}(x) = \cosh \alpha_{mn} x - \cos \alpha_{mn} x - a_{mn} (\sinh \alpha_{mn} x - \sin \alpha_{mn} x) \quad (2)$$

where the "characteristic numbers" α_{mn} and a_{mn} settle the end conditions at $x = d$.

The "modal amplitudes" $A_{mn}(t)$, $B_{mn}(t)$ and $C_{mn}(t)$ (eq. (1)) are evaluated in the following way. Using the strain-displacement relations for thin circular cylindrical shells derived by Flügge [6], the strain energy and the kinetic energy of the shell are expressed in terms of the displacements u , v and w . Substituting u , v , w by eq. (1) and applying Hamilton's principle yields Lagrange's equations of motions for the free vibration of the shell as well as a system of linear equations for the static response of the shell due to the time independent loading. As far as the free vibration problem is concerned, a similar approach was done by Sharma and Johns [7].

Due to the well known orthogonality of the trigonometric functions the azimuthal modes n can be treated strictly separated. But also the axial shape functions are found to be orthogonal. Thus the big eigenvalue problem as well as the big system for the static response of the whole shell is split up into small systems for each mode m, n . Only for the free vibrations in the azimuthal mode $n=1$, all axial modes m must be treated together due to the coupling by the end ring which has a finite amount of inertia. This separation of the large equation systems into several uncoupled systems represents one of the major advantages of CYLDY2, because it reduces considerably the computer efforts in comparison to well known multipurpose codes which were also used during preparation of the HDR-experiments.

Finally the static response and the free vibrations are superimposed. Hereby the amplitudes of the latter are determined in such a way that the initial conditions for the displacements u , v , and w and for their derivatives with respect to time, u^t , v^t and w^t , are satisfied at the beginning of the considered time step.

A detailed description of the mathematical model on which CYLDY2 is based, is given by Ludwig and Krieg [8].

3.2 First results from CYLDY2

CYLDY2 was tested successfully using simple load distributions in space. Here, in fig. 3 the response of the HDR-core barrel due to blowdown loading, as expected by fluid dynamic calculations to be typical for 4 msec after blowdown initiation, is shown.

Due to the high rotational inertia of the lower part of the barrel, the bottom of the core barrel moves first remarkably away from the blowdown nozzle and returns to its initial position after about 5 msec.

4. Fluid Structure Coupling

The combination of YAQUIR and CYLDY2 to a coupled code requires modifications to both. First let us consider the fluid model. An important variable which must be added to the model, is the height of the fluid region at each grid point (corresponding to the local downcomer width). Since this height and its time derivative is the only quantity dealing with the third dimension in the modified YAQUIR model, we call it 2 1/2-dimensional rather than threedimensional. We assume that one wall of the fluid region (at height = 0) is fixed while the other can move. It was found that by proper interpretation of the radius which was included in the original YAQUI version for cylindrical geometry, almost no changes are required. The mass conservation equation shall be used to illustrate this. The original equation reads (slightly simplified):

$$\int_F \frac{d(\rho R)}{dt} dF + \int_S (\rho R) \bar{u} \bar{n} dS = 0 \quad (3)$$

Here F is the area of a single mesh and S is its periphery.

If we just replace the node radius R by the node height H, we obtain the corresponding equation for the 2 1/2-dimensional model. The same technique applies to energy and momentum conservation except for the following modifications:

- a) in the momentum equation a term $\int_F p \cdot \text{grad } H \cdot dF$ must be added to account for the tilting of the movable wall,
- b) in the energy equation a term $\int_F p H \cdot \dot{H} \cdot dF$ must be added to account for the mechanical work of the wall movement.

Thus the modifications in phase 1 of the YAQUI integration method were simple. Some more work was needed for the pressure iteration in phase 2.

In a Newton-Raphson iteration the advanced time pressures for all mesh cells are iterated according to

$$\delta p \left[\frac{-1}{c^2} \left(\frac{1}{\delta t} + D + \frac{\bar{H}}{H} \right) + 2 \delta t \left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} \right) + \frac{\rho \delta t}{H} \cdot \frac{\partial \bar{H}}{\partial p} \right] = -\omega \left[\frac{-1}{\delta t} (\rho^{-n-1} \rho) + \rho \cdot \left(D + \frac{\bar{H}}{H} \right) \right] \quad (4)$$

This equation is identical to the original equation in [3] except for the terms

$$\frac{\bar{H}}{H} \quad \text{and} \quad \frac{\rho \delta t}{H} \cdot \frac{\partial \bar{H}}{\partial p}$$

The first term is related to the fluid model alone; it adds the third dimension to the divergence term D. The second term, however, is related to the structural model; the change in local wall acceleration due to local pressure changes is the meaning of this term.

Whether the iteration according to eq. (4) converges generally, is an open point at this time. A mathematical proof appears too complicated. Hence, a heuristic approach was taken. As a "worst" case one can assume all $\frac{\partial \bar{H}_i}{\partial p_K} = \text{const}$. This is the case for the so-called vessel model described below. Since convergence was good for this case, no convergence problems are expected in other practical applications.

Finally phase 3 of the YAQUI integration scheme must be considered. No modifications were needed besides the exchange of H for R to obtain the 2 1/2-dimensional model from the cylindrical model. The whole program obtained by the above described modification of YAQUIR is called STRUYA.

Let us now consider the structural model. The intention of the development of STRUYA was to provide the capability of coupling many different structural models with the fluid model. Thus, the following list of requirements describes a class of structural models which can be coupled to STRUYA. The structural model must

- be based on linear theory (this restriction is to be removed in more advanced versions)
- compute end of timestep values of deformation and its time derivative for a field of constant pressure loads during the timestep
- compute changes of these results due to small changes in the pressure field
- deliver values of H and \dot{H} to STRUYA
- deliver a vector of local influence factors $\frac{\partial H}{\partial p}$ to STRUYA.

Thus STRUYA provides the capability to couple a great number of structural models. However, two modifications to this structural models are still necessary:

- modification of the structural mass
- modification of the pressure load.

Both these modifications stem from the fact that for structural computations one assumes that the pressure load is given for the structural surface. This is not the case for STRUYA coupled models. On one side of the structure the average pressure in the fluid mesh is given. On the reverse side another fluid may or may not be present. If there is fluid on the reverse side (as is true for the core barrel) the pressure field here will depend on the wall movement. The following approach is taken to solve this problem. We consider a control volume as shown in fig. 4 to derive a momentum equation in the z-direction. If the gradient of H is small, we may neglect the effect of momentum transport and obtain:

$$\frac{d}{dt} (\mu \dot{H} + \int_{\tilde{H}}^H \rho q dz) - \int_{\tilde{H}}^H \rho q^2 dz - (p - p_R') + f(\sigma) = 0 \quad (5)$$

Here \tilde{H} is the particular value of z where we expect the local pressure to be equal to the average pressure, which is computed by the fluid model, p_R' is the pressure immediately on the reverse side. $f(\sigma)$ is the effect of the stresses in the structure. We find \tilde{H} , if we assume that ρ is constant over H and the velocity w varies linearly over z:

$$\tilde{H} = \frac{1}{\sqrt{3}} H \quad (6)$$

On the reverse side let us assume an acoustic model:

$$p_R' = p_R + \rho_R C_R \dot{H} \quad (7)$$

Here, p_R is the pressure which would exist there if the wall did not move. Remember, that in YAQUIR the momentum equation is treated implicitly. Hence, we can write finally for eq. (5)

$$\left(\mu + \frac{1}{3} \rho H + \rho_R C_R dt\right)^{n+1} H - \left({}^{n+1}P - P_R - \rho_R C_R \dot{H}\right) + {}^{n+1}f(\sigma) = 0 \quad (8)$$

If we compose this equation with the case of a structure in vacuum ($\rho = 0$; $\rho_R = 0$), we see that for coupling of a structural model to STRUYA

- the shell density must be increased to account for one third of the liquid layer and the acoustic damping on the reverse side and
- the pressure difference across the wall must be modified according to the acoustic behaviour.

As an additional simplification, because CYLDY2 requires constant shell density, the added mass $\frac{1}{3} \rho H$ is assumed to be independent of the space coordinates.

5. First Results of Fluid-Structure Coupling

In order to test the method described in chapter 4 for the coupling of STRUYA and different structural models, two simple one degree of freedom models for the core barrel were established. The "vessel" model assumes that the whole core barrel can expand and contract radially only in an uniform manner. The "beam" model assumes that the barrel behaves like a rigid lever supported at the top by a rotational spring. While these models have no physical significance, they are useful in demonstrating the effect of a coupled solution as compared to an uncoupled fluid dynamics analysis followed by a calculation of the structural response.

With the vessel model a hypothetical axisymmetric blowdown was simulated by assuming that the pressure at the lower end of the downcomer is suddenly reduced. Fig.5a shows the deviation X_S of the downcomer width from the initial value and its acceleration \dot{X}_S . During the first five milliseconds we notice a rapidly damped vibration at about 510 cps. This corresponds to a pure radial vessel vibration against the stiffness of the compressible water. No significant axial water movement occurs in this time period. The subsequent behaviour is characterized by the superposition of two vibrations. With a little simplification these can be explained as follows. In a longterm vibration (20 cps) the almost incompressible water moves up and down as the vessel contracts and expands. In addition, an axial pressure wave travels through the water (typical frequency 120 cps). This frequency is 4.5 times the frequency expected for axial acoustic waves in the water with a rigid wall. The increased wave speed is due to the fact that a pressure reduction at the lower end via a constant barrel expansion over the whole length induces a pressure rise at the top (see fig.5b). This additional pressure gradient leads to faster accelerations. Similar, far reaching influences (which are physically unrealistic) must be expected for all structural models using a superposition of global shape functions, if the number of modes used is not sufficient.

With the beam model a HDR blowdown test has been simulated. Fig. 6 shows the computed movement of the lower end of the core barrel for both the coupled and uncoupled structural model. Although the model is rather crude in a way that it will overemphasize the coupling effect, the reduction in amplitude and the frequency decrease are at least plausible.

Coupling of CYLDY2 with STRUYA is underway. By the end of January 1977 only the phase 2 part of the structural model was yet missing.

6. References

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7. List of Symbols

- c sound velocity
- d length of the cylinder
- D two-dimensional divergence of the velocity field
- e specific internal energy
- F area of a fluid mesh
- H height of a fluid mesh at a grid point
- \bar{H} average height of a fluid mesh
- \bar{n} normal vector on S
- p pressure
- q local velocity in a mesh in direction z
- R radius
- S periphery of a fluid mesh
- t time
- \bar{u} fluid velocity vector

u, v, w local displacements in axial, azimuthal and radial direction of the cylinder
 u^t, v^t, w^t derivative of u, v, w versus time
 x, θ, z axial, azimuthal and radial coordinates of the cylinder
 $\Delta x, \Delta y$ average sizes of a single mesh
 XS radial displacement of the "vessel" model
 μ mass per unit area of cylindrical shell
 ρ density
 ρ_i^n density at time step n in mesh i
 ω overrelaxation factor
subscript R reverse side of the cylinder

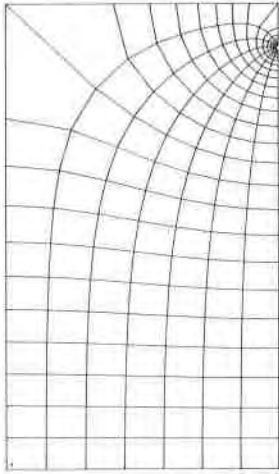


Fig. 1a: Rectangular grid used for modelling of the unwrapped downcomer

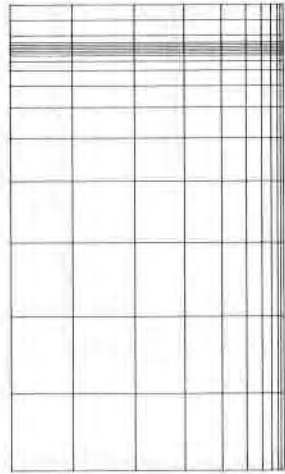


Fig. 1b: Potential flow grid used for modelling of the unwrapped downcomer

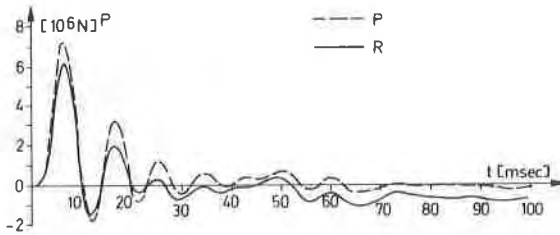


Fig. 2: Integrated force of the blowdown pressure reduction on the core barrel for rectangular grid (R) and potential flow grid (P)

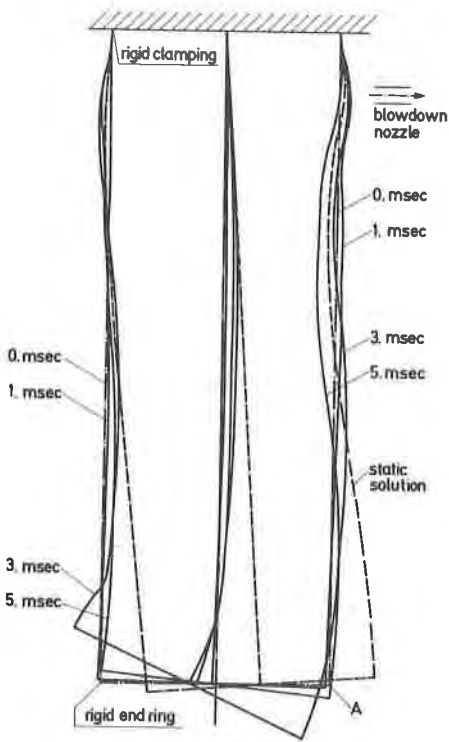


Fig. 3a: Core barrel response to a steptype load (corresponding to the blowdown load at 4 msec after start of blowdown) according to uncoupled YAQUIR-CYLDY2 calculations. Snapshot at times 0, 1, 3 and 5 msec and static solution

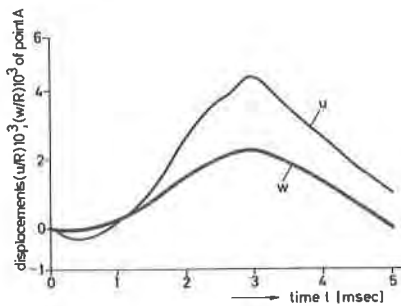


Fig. 3b: Displacements of point A of fig. 3a versus time in axial (u) and radial (w) direction

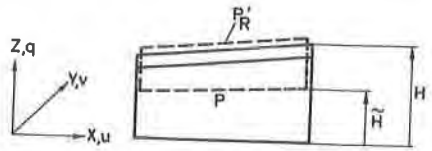


Fig. 4: Control volume for a 2 1/2 D STRUYA-mesh

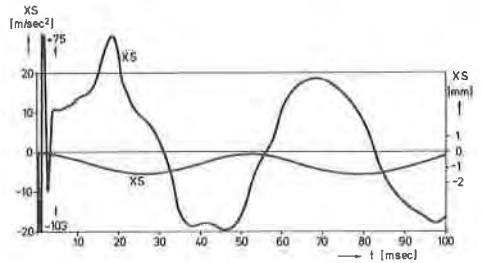


Fig. 5a: Acceleration and displacement of a "vessel" model

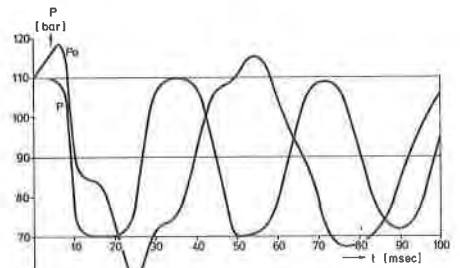


Fig. 5b: Pressure at the top of the downcomer for the case shown in fig. 5a with (P_0) and without coupling (P)

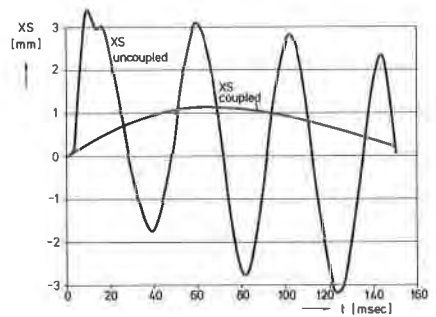


Fig. 6: Displacement of the lower core barrel end computed with STRUYA and an uncoupled and a coupled "beam" model.