FINITE MIXTURE MODEL APPROACH APPLIED IN THE ANALYSIS OF A TURBULENT BISTABLE FLOW ON TWO ROWS TUBE BANK

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ABSTRACT

This paper presents a study of a turbulent bistable flow on two rows of a triangular tube array where velocity time series obtained with the constant temperature hot wire anemometry technique in an aerodynamic channel are used as input data in a finite mixture model, to classify the observed data according to a family of probability density functions (PDF). In this context, the bistable phenomenon is considered as an incomplete data problem and PDF-estimation is performed using the observed data. The number of clusters was considered equal to two, and an expectation-maximization algorithm was applied to estimate the maximum log-likelihood function according to a known PDF with aid of a Monte Carlo method, to obtain the probability distribution function which is more likely to have produced the observed data. Wavelet transforms are applied in a multilevel decomposition of the signals in several bandwidth values, accordingly with a selected decomposition level, and to analyze their energy content. Flow visualizations in water channel are used in the identification of the bistable flow and qualitative analysis of this phenomenon.

INTRODUCTION

Banks of tubes or rods are found in several engineering applications, like in the nuclear and process industries being the most common geometry used in heat exchangers. The turbulent flow impinging on circular cylinders placed side-by-size presents a floppy and random phenomenon that changes the flow mode. This behavior is called in the literature as bistable flow and is characterized by a wide near-wake behind one of the cylinders and a narrow near-wake behind the other, which generates two dominant Strouhal numbers, each one associated with one of the two wakes formed: the wide wake is associated with a lower Strouhal number and the narrow wake with a higher one.

As flow induced vibration and structure-fluid interaction are very dependent of the arrangement or configuration of the cylinders, new studies are necessary to improve its understanding, since bistability can be an additional excitation mechanism on the tubes. Furthermore, tube arrangement is also equally important in the characterization of fluid flow and heat transfer of a tube bank.

Tube banks are the usual simplification for fluid flow and heat transfer in the study of shell-and-tube heat exchangers, where the coolant is forced to flow transversely to the tubes by the action of baffle plates. Geometric characterization of a tube bank is made by the P/D-ratio, being D the tube diameter and P, the pitch, which is the distance between the centerlines of adjacent tubes. Tube arrangement is also equally important in the characterization of fluid flow and heat transfer of a tube bank. According to Zdravkovich and Stonebanks [1], the leading feature of flow-induced vibration in tube banks is the randomness of dynamic responses of tubes, and even if the tubes are all of equal size, have the same dynamic characteristics, are arranged in regular equidistant rows and are subjected to an uniform steady flow, the dynamic response of tubes is non-uniform and random. The cross flow passing a tube in a bank is strongly influenced by the presence of the neighboring tubes. In the narrow gap between two tubes in a row, the strong pressure gradient will influence not only the flow in that region, but the flow distribution downstream of this point, in the narrow gap between two tubes in the next row, and so on.

The need for more efficient heat exchangers leads the operating conditions of these equipments to become critical, due to the reduction of the aspect ratio of the tube banks (pitch-to-diameter ratio) and the increasing of the flow velocity. As a consequence of the reduction of the flow area in the narrow gaps between the tubes, which causes velocity fluctuations, and the constant change of the flow direction, static and dynamic loads will be increased [2]. According to Blevins [3], the dynamic loads of the turbulent flow over small aspect ratio tube banks are characterized by broad band turbulence, without a defined shedding frequency. For large aspect ratio tube banks, the dynamic loads are basically associated with vortex shedding process.

THE BISTABLE EFFECT

The cross steady flow trough circular cylinder with same diameter placed side-by-side can present a wake with different modes [4], depending on the pitch-to-diameter ratio P/D. For intermediate pitch ratios (1.2<P/D<2.0), the flow is characterized by a wide near-wake behind one of the cylinders and a narrow near-wake behind the other. This phenomenon generates two dominants vortex-shedding frequencies, each one associated with a wake: the wide wake is associated with a lower frequency and the narrow wake with a higher one. The switching of the gap flow, which is biased toward the cylinder, from one side to other at irregular time intervals, is therefore known as a flip-flopping regime or bistable flow regime, as described by Bearman and Wadcock [5]. Previous studies show that this pattern is independent of Reynolds number, and it is not associated to cylinders misalignment or external influences, what suggest an intrinsically flow feature. The transition between the asymmetric states is completely random and it is not associated with a natural frequency. Through dimensional analysis, it was observed that the mean time between the transitions is on order 10^3 times longer than vortex shedding period, and the mean time intervals between the switches decreases with the increasing of Reynolds number. According to Kim and Durbin [6], there is no correlation between the bistable feature and the vortex shedding, due to the fact that Strouhal numbers are relatively independent from the Reynolds numbers. Due to the non-stationary characteristics of the bistability, wavelet transforms can be a useful tool for its analysis, as shown in the study of transient turbulent signals [7], in the switching phenomenon in two side-by-side cylinders [8] and more recently, in tube banks with square arrangement [9].

METHODOLOGY

Time series of axial and transversal velocity obtained with the constant temperature hot wire anemometry technique in an aerodynamic channel are used as input data in a finite mixture model, to classify the observed data according to a family of probability density functions (PDF). In this context, the bistable phenomenon is considered as an incomplete data problem and PDF-estimation is performed using the observed data (time series), where the number of clusters was considered equal to two. Wei and Tanner [10] use an expectation-maximization (EM) algorithm [11] together with a Monte Carlo (MC) method to perform maximum log-likelihood estimation (MLE) to know what the probability distribution function is more likely to have produced the observed data. Azzalini and Capitanio [12] developed PDF with skewness support. A skew Student's t density function can be defined through a stochastic representation of three random vectors, of which, two have standard normal distribution and the other follows a Chi-square function, where all vectors are sampled independently. According to Lin [13], finite mixture models have become more frequently used to provide a natural framework for unobserved heterogeneity in a population. A finite mixture model using the skew Student's t distribution has been recognized as a robust extension of normal mixtures [14]. The joint time-frequency domain analysis was made trough wavelet transform. The wavelet analysis can be applied to time varying signals, where the stationary hypothesis cannot be maintained, to allow the detection of non permanent flow structures. A discrete wavelet transform (DWT) is used to make a multilevel decomposition of a time signal in several bandwidth values, accordingly with the selected decomposition level. A continuous wavelet transform (CWT) is used to analyze the energy content of a signal through the so called spectrogram. In this work, Daubechies "db20" functions were used as bases of discrete wavelet transforms.

FINITE MIXTURE MODELS

The finite mixture model simulations were performed using as input data the experimental time series. A skew Student's t PDF was used in the estimation of the maximum log-likelihood function, which is considered as an inference problem. In this context, a Q function, or the expected complete data log-likelihood function, is defined. An EM algorithm is an iterative method for the computation of the maximizer of the posterior density, which has two steps: the E-step, which is the current estimative of the Q function; and the M-step, which is the maximizer of the Q function. Due to difficulties in evaluating the E-step of skew Student's t PDF, caused by the complexity of the target distribution, which does not admit a close-form solution to the Q function, the iterations may be executed by a MC method, performing independent draws of the missing values from the conditional distribution and then approximating the Q function. In the M-step, the Q function is maximized over the parameters vector and a new estimative is calculated. As MC error is introduced at the E-step, the monotonicity property is lost. To assess the convergence of the algorithm, the MC sample size was increased with the number of iterations and its stability was monitored with a tolerance of 10^{-3}.

EXPERIMENTAL TECHNIQUE

Aerodynamic channel

The velocity of the flow and its fluctuations are measured by means of a DANTEC *StreamLine* constant hot-wire anemometry system. One double straight/slant hot wire probe (type DANTEC 55P71 Special) and two single hot wire probes (type DANTEC 55P11) were used in the experiments, where their positions are show in Figs. 1c and 1d, respectively. The double probe has a straight wire perpendicular to the main flow, and a slant wire 45° with the probe axis. The single probes have each one wire perpendicular to the main flow. The aerodynamic channel used in the experiments is made of acrylic, with a rectangular test section of 0.146 m height, width of 0.193 m and 1.02 m of length (Fig. 2a). The air is impelled by a centrifugal blower of 0.64 kW, and passes through two honeycombs and two screens, which reduce the turbulence intensity to about 1% in the test section. Upstream the test section, placed in one of the side walls, a Pitot tube measures the reference velocity of the non-perturbed flow. Data acquisition is performed by a 16-bit A/D-board (NATIONAL INSTRUMENTS 9215-A) with USB interface. The acquisition frequency of time series was of 1 kHz, and a low-pass filter of 300 Hz was used to avoid aliasing. The circular cylinders, with external diameter of 25.1 mm, are made of Polyvinyl chloride (PVC), are rigidly attached to the top wall of test section and their extremities are closed. The probe support is positioned with 3D transverse system placed 200 mm downstream the outlets (Fig. 2b). The measurements were performed aligning the probes along the tangent to the external generatrices of the tubes (Fig. 2c). The mean error of the flow velocity determination with a hot wire was about +/- 3%. The Reynolds number of the experiment is 21,000, computed with the tube diameter and the reference velocity of 12.9 m/s.

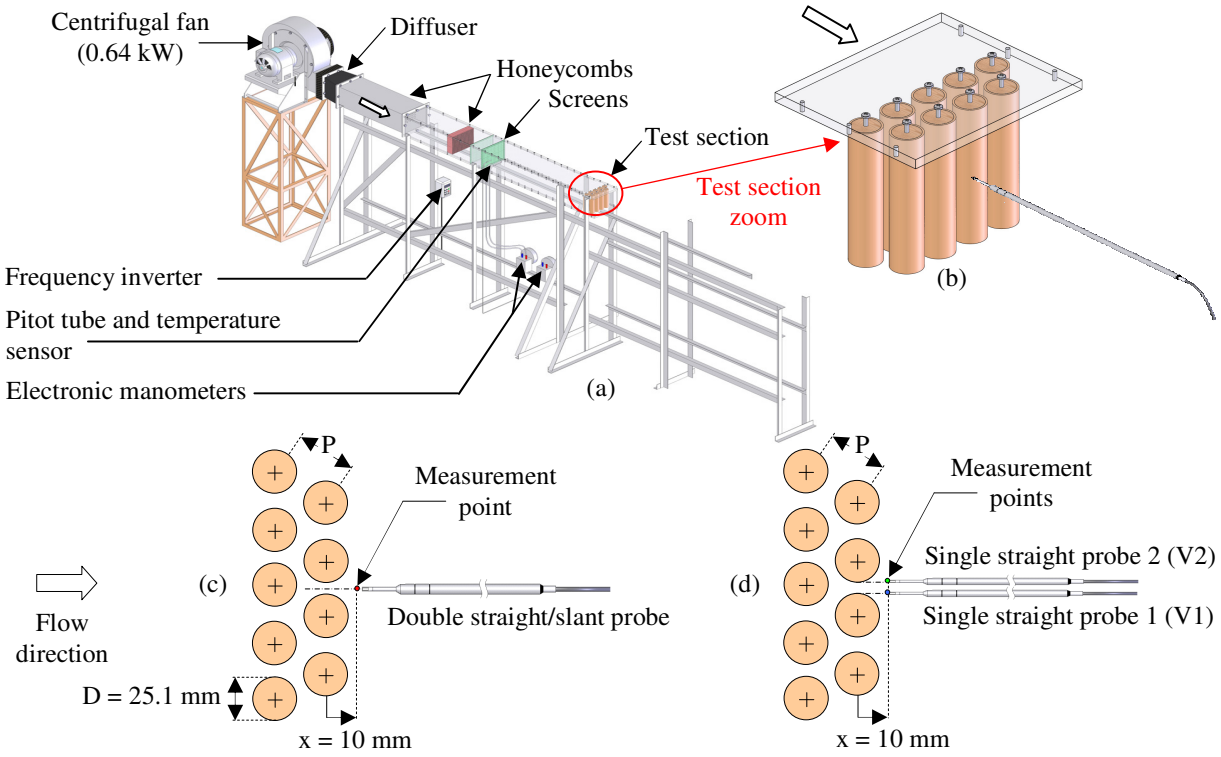

Fig. 1: Schematic views: (a) aerodynamic channel, (b) test section. Positions of the probes: (c) double straight/slant hot wire probe; (d) single straight hot wire probes.

Water channel

Flow visualizations were performed in a water channel which has a settling chamber with a honeycomb that acts as a flow straightener, a nozzle, a 30 m long open channel (10 m upstream, 20 m downstream the test section) with 0.5×0.6 m rectangular cross section, a vertical gage to control the water level, and a discharge tank with the return pipe to close the circuit. The experiments were conducted inside a test section placed in the visualization section of the water channel. The tube bank was completely submerged, so that the upper plate was at 0.08 m below the water surface to avoid the effect of gravitational waves. The cylinders are rigidly attached to the

base plate, and are built with commercial PVC tubes, with diameter of 0.06 m and 0.3 m height, covered by a thin white PVC film. A digital camera, placed above the bank, was used for taking digital movies at 30 frames per second in VGA resolution. Flow visualizations were performed with the injection of potassium permanganate directly in the free stream.

RESULTS

Measurements performed after two rows of tubes in a triangular arrangement with a pitch-to-diameter ratio of P/D=1.26 with a double hot wire probe placed in the center gap between and downstream the tubes are shown in Fig. 2, according to the position presented by Fig. 1c. Data acquisition is started before the blower is turned on, as in [7], with a transient flow from 5 to 15 seconds. Thereafter, the flow mode 1 (wide near-wake) is established, until 40 seconds, where the gap flow changes its direction to the mode 2 (narrow near-wake) lasting for the rest of the time acquisition. The axial and vertical signals (Figs. 2a and 2b) present similar behaviors, where an increase of velocity is observed in both signals, during the switching of the gap flow. The deviation angle of the flow tends to have smaller fluctuations when the flow direction changes from the wide near-wake to the narrow near-wake (Fig. 2c). A joint analysis with CWT and DWT of the time series is presented in Figs. 2d, 2e and 2f, where a reconstruction is performed accordingly with a selected decomposition level (n = 9, with frequency content from 0 to 0.976 Hz), together with the analysis of their energy content, or spectrogram. The frequency intervals of the spectrograms vary from 5 to 500 Hz, with a bandwidth of 5 Hz. Results show that the energy content of the signals increases when the direction of the gap flow changes and the mean values present higher standard deviation (Figs. 2d, 2e and 2f). The higher energy content of velocity signals is followed by a relative spreading of frequencies until approximately 300 Hz. For the wide near-wake mode, there is a decreasing in the energy content followed by a relative concentration of the frequencies. A similar characteristic can be observed from the spectrogram of the angle of deviation of the flow (Fig. 2f), where the higher energy content spreads until 500 Hz (wide near-wake mode) and the concentration of the frequencies of the narrow near-wake mode occurs up to 300Hz.

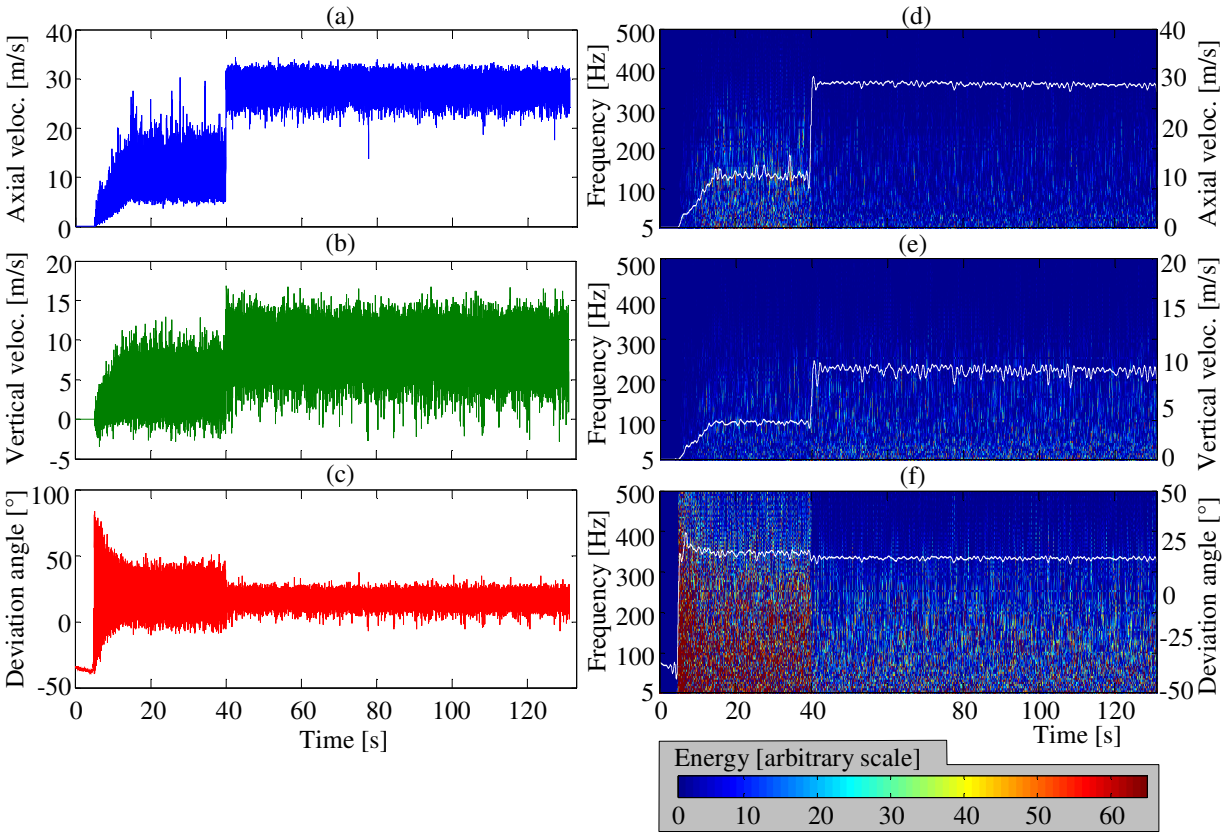

Fig. 2: Time series of (a) axial velocity, (b) vertical velocity and (c) deviation angle of the flow, with its joint CWT/DWT: (d), (e) and (f), respectively.

Figures 3a, 3b and 3c show in details the time series of Figs. 2a, 2b and 2c, respectively, from 20 to 60 seconds. The velocities PDF present the predominance of two major states of energy, with different shapes (Fig. 3d and Fig. 3e). The PDF of the angle of deviation of the flow presents a concentration around a single value (Fig. 3f). By separating two intervals of the Figs. 3a, 3b and 3c with length of 16768 data points, the power spectral density functions (PSD) of these dyadic series are presented in Figs. 3g and 3h, corresponding to flow modes 1 and 2, respectively, and show no peaks in frequency spectra. It is also observed a shift of energy levels of time signals.

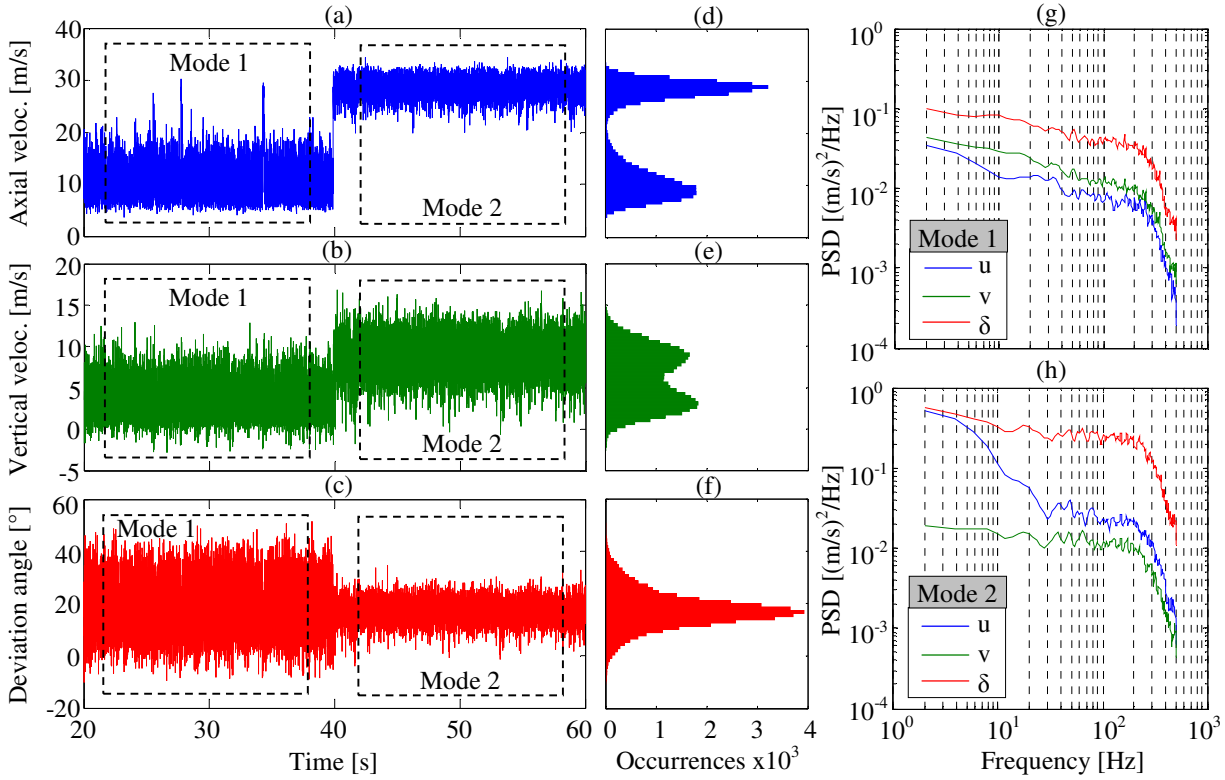

Fig. 3: Details of time series of Figs. 2, from 20 to 60 seconds: (a) axial velocity, (b) vertical velocity and (c) deviation angle of the flow, with its respective PDF (d, e and f). (g, h) Power Spectral Density functions (PSD) of the dyadic series emphasized in (a, b and c) for modes 1 and 2, respectively.

A joint analysis of axial and vertical velocity components is presented in Fig. 4a, where the results of the ordered pair of data show the concentration in two distinct areas, referred to both flow modes. A DWT reconstruction of level n = 5 (frequencies from 0 to 15.6 Hz) of the bivariate case is presented on the same figure, and shows five changes of the gap flow direction. This feature can be correlated to the space phase concept, where a particular state of a dynamical system can be represented as a point in this phase space. As the time varies, the point moves along a path. By analyzing this feature together with Fig. 3c, which presents the details of the axial velocity signal from 26 to 36 seconds of mode 1, is possible to observe two attempts of changing the gap flow direction of short time. The other change of the gap flow direction refers to transition to mode 2. The bivariate velocity PDF shows the presence of two prominences, where the higher velocity mode seems to be present in larger quantity in this case (Fig. 4b).

Results of the measurements with two single hot wire probes aligned along the tangent to the external generatrices of the cylinders are shown in Fig. 5, according to the position presented by Fig. 1d. The velocity signals V1 and V2 (Figs. 5a and 5b, respectively) show the change in the level of mean gap flow velocity at t = 66 seconds. The PDF of both signals present two prominences, are bell shaped and superimposed (Figs. 5c and 5d). The narrow near-wake mode presents a sharp PDF, while the wide near-wake is more distributed. The joint CWT/DWT of the velocity series (Figs. 5e and 5f) show that their energy contents increase when the direction of the gap flow changes, and the mean velocity present higher standard deviation. No fast change of flow direction was identified from the DWT with the selected decomposition level (n = 9, with frequency content from 0 to 0.976 Hz). The higher energy content is followed by a concentration of the frequencies until approximately 300 Hz.

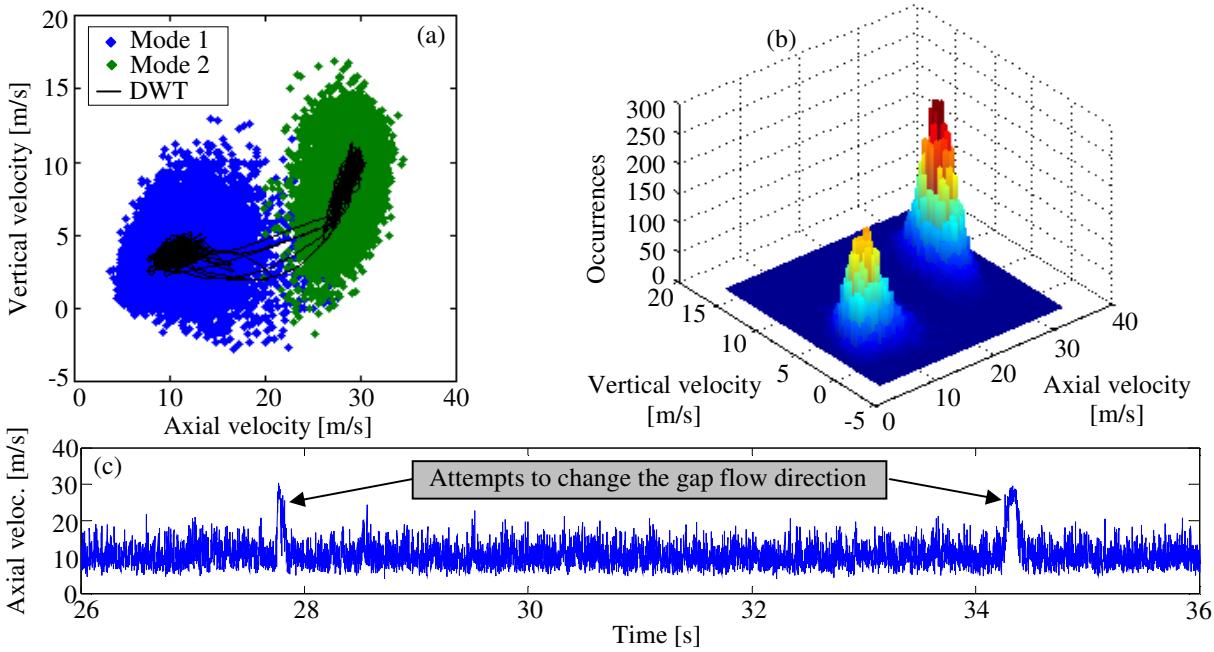

Fig. 4: (a) Ordered pair of data points of the flow velocity components. (b) Bivariate velocity PDF. (c) Details of the axial velocity signal from 26 to 36 seconds, showing two attempts to change the gap flow direction.

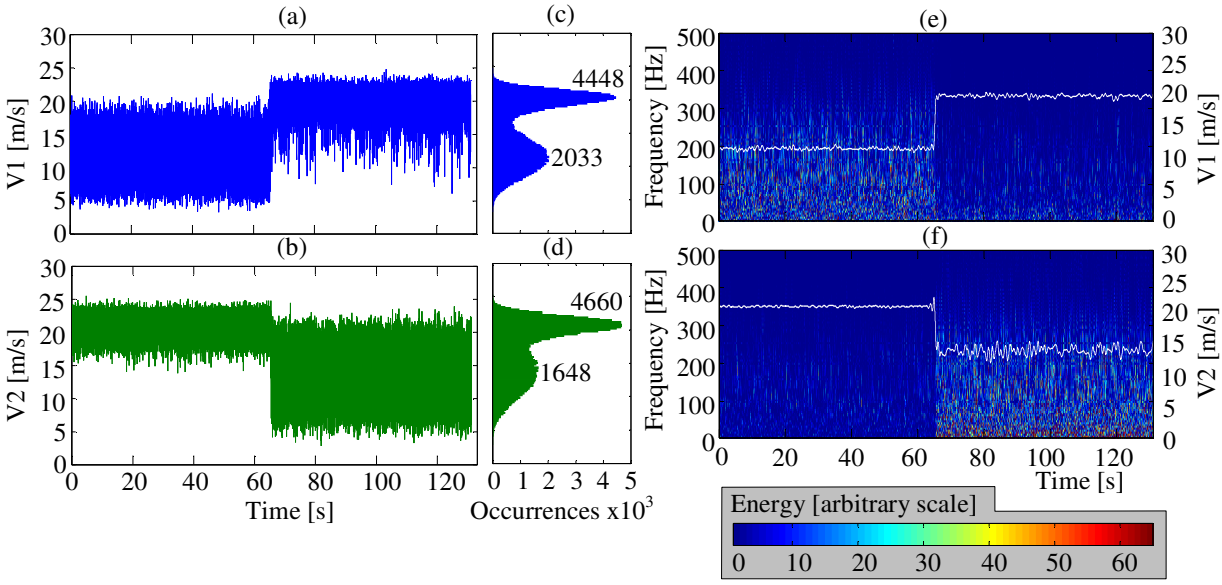

Fig. 5: (a, b) Velocity series. (c, d) PDF of the velocity series. (e, f) Joint CWT/DWT of the velocity series.

Finite mixture model simulations were performed to know what PDF is more likely to have produced the observed data. The results show a goodness of fit for both measurements (Fig. 6), with the numerical values of the parameters vector: mean value (μ), degrees of freedom (ν), maximum probability (p) and skewness (α). The PDF simulations with one double straight/slant hot wire probe for the axial velocity show no overlap, with high asymmetries (Fig. 6a). For the vertical velocity, the PDF are partially overlapped (Fig. 6b). The PDF of the deviation angle of the flow are completely overlapped and are almost symmetric (Fig. 6c). For the measurements with two single hot wire probes, the PDF for both velocity signals are partially overlapped (Figs. 6d and 6e), and its maximum difference is 18.9 % for the wide near-wake mode and 4.5 % for the narrow near-wake mode (Fig. 6f). Since each mode has distinct PDF, although similar, this can suggest that the flow modes are not symmetrical.

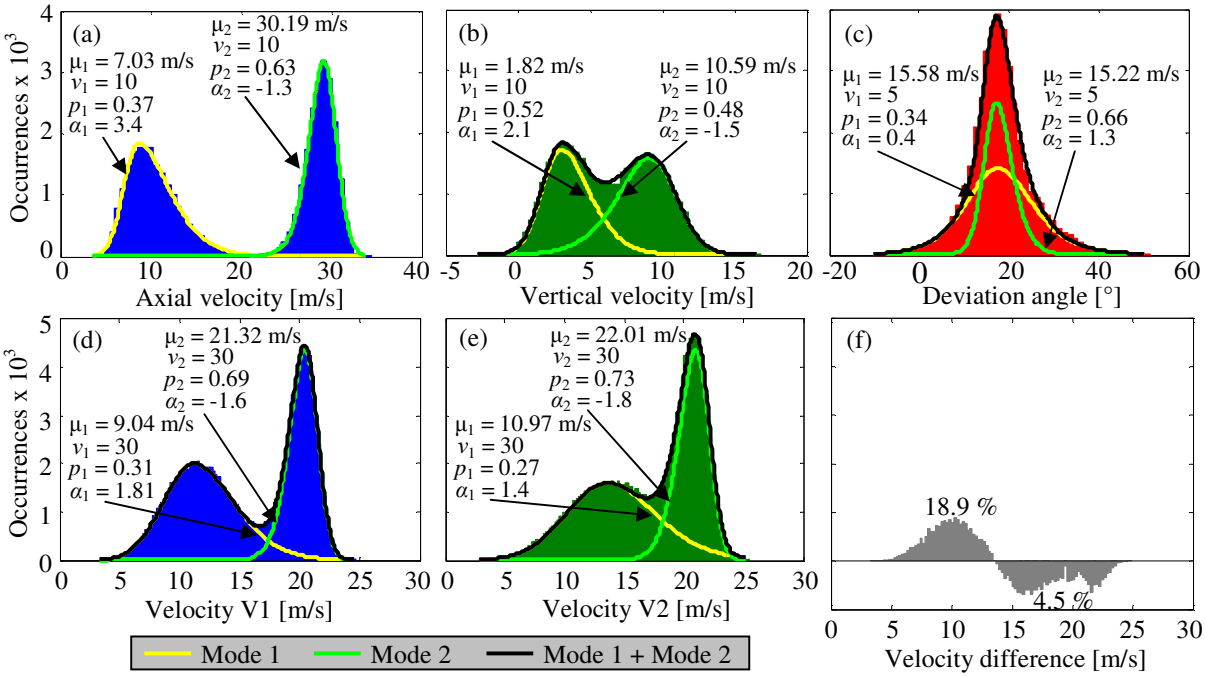

Fig. 6: MLE simulations via EM algorithm of PDF measurements with one double straight/slant hot wire probe: (a) axial velocity, (b) vertical velocity and (c) deviation angle of the flow. For measurements with two single hot wire probes the results are: (d) velocity V1 and (e) velocity V2. (f) Difference between the velocities PDF of (d, e).

Figure 7 shows the results of the flow visualization with injection of permanganate potassium direct in the free stream (Fig. 7a and Fig. 7b) with its respective flow patterns (Fig. 7c and Fig. 7d), where the bistable behavior is identified downstream the second row, in the 3rd tube, from left to right. These pictures are separated by a few minutes, long enough for the two flow modes are established. Similar results are observed for P/D=1.6 [15].

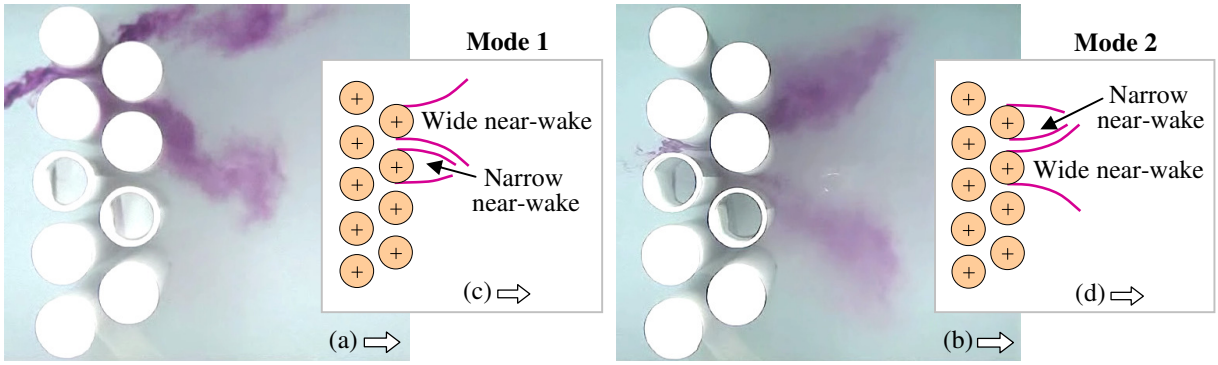

Fig. 7: (a, b) Results of flow visualization for modes 1 and 2, with their respective flow patterns (c, d).

CONCLUSIONS

This work presents a study about the bistable phenomenon on two rows tube bank. A finite mixture model tool is applied, where an expectation-maximization algorithm performs the maximum likelihood estimation according to a known PDF with aid of a Monte Carlo method, to know what the probability distribution function is more likely to have produced the observed data. This tool was valuable to determine the numerical values of the shape of the both PDF modes and the results show a goodness of fit. Results with one double straight/slant hot wire probe show that in the changes between the flow modes the increase in the axial velocity component is accompanied by increased of the transverse component, and their PDF present the predominance of two major states of energy, with different shapes. The angle of deviation of the flow tends to have smaller fluctuations when the flow direction changes from the wide near-wake to the narrow near-wake, and their PDF presents a concentration around a single

value. For the measurements with two single hot wire probes, the maximum difference of the PDF is 18.9 % for the wide near-wake mode and 4.5 % for the narrow near-wake mode. A joint analysis with discrete and continuous wavelet transforms of the time series is applied, where a multilevel decomposition in several bandwidth values, accordingly with a selected decomposition level, can be performed together with the analysis of their energy content (spectrogram). Results show that the energy content of the signals increases when the direction of the gap flow changes, and the mean velocity present higher standard deviation. The higher energy content is followed by a concentration of the frequencies until approximately 300 Hz. When the time series has higher variance, their energy content is higher. This analysis could be possible through the use of wavelet transforms, being valuable tools for the analysis of non-stationary turbulent signals. From the flow visualizations in water channel, the bistable behavior is clearly identified. Simulations of the bivariate fit of the phenomenon are in progress, as well as a search for individual degrees of freedom for each flow mode and increasing the numbers of clusters for the univariate case.

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REFERENCES

[1] M.M. Zdravkovich and K.L. Stonebanks, “Intrinsically non-uniform and metastable flow in a behind tube arrays”, *International symposium of flow-induced vibration and noise*, Vol. 1, 1988, pp. 61-73.

[2] L.A.M. Endres, C. Silva and S.V. Möller, “Experimental study of static and dynamic fluid flow loads in tube banks”, *Proceedings of the 13th International Conference on Structural Mechanics in Reactor Technology*, Porto Alegre, Brazil, 1995 August 13-18.

[3] R.D. Blevins, “Flow Induced Vibrations”, van Nostrand-Reinhold, New York, 1990.

[4] D. Sumner, S.S.T. Wong, S.J. Price and M.P. Païdoussis, “Fluid behavior of side-by-side circular cylinders in steady cross-flow”, *Journal of Fluids and Structures*, Vol. 13, 1999, pp. 309-338.

[5] P.W. Bearman and A.J. Wadcock, “The interaction between a pair of circular cylinders normal to a stream”, *Journal of Fluid Mechanics*, Vol. 61, Part 3, 1973, pp. 499-511.

[6] H.J. Kim and P.A. Durbin, “Investigation of the flow between a pair of circular cylinders in the flopping regime”, *Journal of Fluid Mechanics*, Vol. 196, 1988, pp. 431-448.

[7] M.L.S. Indrusiak, J.V. Goulart, C.R. Olinto and S.V. Möller, “Wavelet time–frequency analysis of accelerating and decelerating flows in a tube bank”, *Nuclear Engineering and Design* , Vol. 235, 2005, pp. 1875-1887.

[8] M.M. Alam, M. Moriya and H. Sakamoto, "Aerodynamic characteristics of two side-by-side circular cylinders and application of wavelet analysis on the switching phenomenon", *Journal of Fluids and Structures*, Vol. 18, 2003, pp. 325-346.

[9] C.R. Olinto, M.L.S. Indrusiak, L.A.M. Endres and S.V. Möller, "Experimental study of the characteristics of the flow in the first rows of tube banks", *Nuclear Engineering and Design*, Vol. 239, 2009, pp. 2022-2034.

[10] G.C.G. Wei and M. A. Tanner, “A Monte Carlo implementation of the EM algorithm and the Poor Man’s Data Augmentation Algorithms”, *Journal of the Statistical Association*, Vol. 85, No. 411, 1990, pp. 699-704.

[11] A.P. Dempster, N.M. Laird and D.B. Rubin, “Maximum likelihood from incomplete data via the EM algorithm”, *Journal of the Royal Statistical Society*, Series B, Vol. 39, No 1, 1977, pp. 1–38.

[12] A. Azzalini and A. Capitanio, “Distributions generated by perturbation of symmetry with emphasis on a multivariate skew t-distribution”, *Journal of the Royal Statistical Society*, Series B, Vol. 65, Part 2, 2003, pp. 367-389.

[13] T.I. Lin, “Robust mixture modeling using multivariate skew t distributions”, *Statistics and Computing*, Vol. 20, 2010, pp. 343-356.

[14] T.I. Lin, J.C. Lee and W.J. Hsieh, “Robust mixture modeling using the skew t distribution”, *Statistics and Computing*, Vol. 17, 2007, pp. 81-92.

[15] A.V. De Paula, L.A.M. Endres and S.V. Möller, “Some features of the turbulent flow in tube banks of triangular arrangement”, *Proceedings of the 20th International Conference on Structural Mechanics in Reactor Technology*, Espoo, Finland, 2009 August 9-14.