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Methods of Probabilistic Fracture Mechanics in the Analysis NPP Components

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ABSTRACT

The objective of this paper is to provide an overview over existing methods of stochastic fracture mechanics. It is noted that some of these methods are not yet applied in nuclear engineering but could significantly contribute to the solution of a number of pressing problems in nuclear engineering. In particular Markov chain models, solution by stochastic differential equation and central limit theorem models are discussed.

1. INTRODUCTION

There are at least two reasons to examine the problems of fracture mechanics - especially those of fatigue fracture - by the nondeterministic methods. The process of fatigue fracture is a mechanical process which strongly depends on the local properties of material. The materials used for NPP components - metals, alloys, etc. are not microscopically homogeneous. Some micro-defects, inclusions etc. are the origins of the crack initiation and the microheterogenity of material affects the crack propagation velocity even for the controlled constant deterministic cyclic loading [1,2]. Temporary accelerations and retardations of the crack growth velocity have been observed to cause a significant scatter of the life-times of specimen. This is a reason to consider some material parameters as random quantities and consequently to apply the probabilistic methods to solve problems of fatigue fracture. Another reason why probabilistic methods have to be used in the analysis of NPP components is the stochastic nature of the loading to which they are subjected. Deterministic models of load processes are not adequate to describe the real situations. Many types of loading, such as operational, - malfunctional (loss of offsite power, scram, loss of feedwater, etc.) as well as external hazard loading conditions (i.e. earthquakes, etc.) are random in terms of occurrence and / or intensity. Consequently they have to be modeled as stochastic functions or sequences of random events. The two origins of randomness of the fatigue process as discussed above are generally considered separately.

It is quite natural, that methods of probabilistic fracture mechanics have already been used to calculate failure probabilities of pressure vessels [3], containments [4], primary piping [5,6], and other components.

The purpose of this paper is to give a brief review of classes of existing proposals and approaches to the probabilistic analysis of fatigue fracture phenomena, some of them are not yet applied in nuclear engineering. It is the authors' belief, however, that these methods carry the potential to solve some of the pressing problems in this field and should therefore be utilized for this purpose.

The experiments on the one hand and some results of the various mechanical or physical based models on the other hand show that mathematical functions for the crack propagation rate should be described in general by following form

$$\frac{da}{dn} = F(a, S_{max}, S_{min}) \tag{1}$$

where a is the crack length, S_{max} and S_{min} are the maximum and minimum values of the far-field stress in the n-th load cycle respectively. In the literature one can find numerous proposals, (see for example [7]), for the function $F(\cdot)$ which involve one, two or even more material constants or functions. In most cases the proposal of *Paris* and *Erdogan* [8] is accepted to describe sufficiently well the crack propagation rate. The so called *Paris* law

$$\frac{da}{dn} = C(\Delta K)^m \tag{2}$$

is also used in most stochastic approaches to the fatigue problems. In eq. (2) ΔK denotes the stress intensity range factor

$$\Delta K = Y(a)\cdot(S_{max}-S_{min})\sqrt{\pi a} = Y(a)\cdot\Delta S\sqrt{\pi a} \tag{3}$$

where Y(a) is a factor depending on the geometry of the crack and specimen respectively, C and m are the material parameters.

It should be pointed out that all crack propagation laws were established for the constant amplitude loading, i.e. ΔS = const., and an extension to variable amplitude loading characteristics does not take into account the load sequence effects which may affect the crack propagation rate. Some existing proposals [9,10,11,12] which account for the retardation or acceleration of the crack rate following an overloading require in fact an explicite description of the applied loading. Their usefullness in the case of stochastic loading has not yet been examined and the possibility of their application in stochastic fatigue fracture will not be discussed here.

2. METHODS OF ANALYSIS

2.1 EFFECTIVE STRESS METHOD

The simplest method to take into account the randomness of a load process in the fatigue fracture analysis is to look for an equivalent or characteristic stress intensity range factor, ΔK_{eq}, that can be introduced into a deterministic equation, say Eq. (2). The value of ΔK_{eq}, most frequently found in the literature [13,14], is assumed to be equal to the root-mean-square (rms) value of ΔK, i.e.

$$\Delta K_{eq} = \Delta K_{rms} = Y(a)\ \Delta S_{rms}\ \sqrt{\pi a} \tag{4}$$

There are no theoretical arguments, however, for this assumption. Though some experiments show very good agreement with the predicted results [13] the single parameter, ΔS_{rms}, which is assumed to describe the whole influence of the randomness of the load on the crack growth cannot yield sufficient information about a stochastic process. It is obvious that identical values of ΔS_{rms} can be found for quite different characteristics of stochastic processes. It can be hardly expected that a structure yields the same time to failure if it is subjected either to a narrow - or wide band process - even if both processes possess equal ΔS_{rms} values. The application of this simple approach has to be carefully verified by experiments utilizing the real load spectrum. An appropiate choice of ΔK_{eq} for some given types of loading and for some typical structural components can however yield satisfactory assessments of the mean life-time.

2.2 SOLUTION BY STOCHASTIC DIFFERENTIAL EQUATIONS

The differential forms of the crack propagation laws lead many authors to apply the methods of stochastic differential equations to the analysis of fatigue fracture. In fact, if Eq. (1) is restated, i.e. expressed in the form

$$\frac{da}{dt} = F[a(t),\ \Delta S(t)]; \qquad a(t_o) = A_o \tag{5}$$

The crack length $a(t)$ could be considered as a Markov process and the derivation concerning some approximate methods of Markov processes [15] could be directly applied, where $\Delta S(t)$ is the stationary stochastic process with a finite correlation time τ_{cor}. The formal change of the argument n into t is made under the assumption that the number of load cycles within any time interval can be defined as their mean number so that

$$\frac{da}{dn} = \frac{1}{\nu+(t)}\ \frac{da}{dt} \tag{6}$$

where $\nu+(t)$ is the mean rate of maxima and $\nu+(t) = \text{const}$ for a stationary process $S(t)$. A direct application of the theory of Markov processes to Eq. (5) would require the amplitudes, $\Delta S(t)$, as a stochastic process with respect to time. There are no general theoretical results that would derive the amplitude stochastic process from the original process $S(t)$. Some results concerning a single amplitude ΔS_i [16,17,18], are not sufficient for the Markovian analysis of Eq. (5). For narrow-band stochastic processes the amplitude process is defined as the envelope process. There is automatically assumed that the neighboring extremes are symmetric with respect to the mean value of the process, $S_{i\ max} - \bar{S} = \bar{S} - S_{i\ min}$. It can be approximately true for very narrow-band processes only. Even then the application of the Markov approximation can be questionable because of relative long correlation time, $\tau_{cor} \ll [\frac{\partial F}{\partial a}]^{-1} = \tau_o$. As τ_{cor} increases for the decreasing band width of the process this strong inequality is not satisfied anymore for narrow-band processes. To overcome all these difficulties and to maintain the Markovian approximation to be valid some authors [19,20] propose to multiply the right hand side of a deterministic crack propagation law, Eq. (5), by a stochastic process X(t). The stochastic load amplitude process ΔS(t) is then replaced by some statistical parameters, say ΔS_{rms} [20], or the whole influence is put into the process X(t) directly, [19]. If the process X(t) is assumed to be a white noise [20], or a stationary random pulse sequence [19] the Markovian approximation can be successfully applied and yields some analytical solutions for the statistical moments of the random time when the crack reaches any given size. The only problem is the determination of the statistical parameters of the process X(t). Since all random effects have been put into the process X(t) one would like to have a general approach which allows to establish the parameters of X(t) from those of S(t) or of other sources of randomness. As it was shown in [19], this approach describes very well the experimental results if the necessary parameters are deduced from *these* results. The applicability of this approach depends on the possibility of life-time prediction by using some simple experiments where results would yield sufficient information to establish some necessary parameters to solve more complicated problems.

2.3 MARKOV CHAIN MODEL

Based on the assessment of experimental results [1,2] and theoretical considerations some authors proposed a Markov model of the fatigue crack propagation under constant amplitude loading. In a series of papers [21,22,23] the discrete Markov chain with the only non zero elements of the transition matrix on diagonal, p_{ii}, and the neighboring ones on their right hand side, $p_{i,i+1} = q_i$, was proposed to describe the observed scatter of crack length. In such a model $p_{ii} + q_i = 1$ and the time T_i that the process remains in any state i has the geometric distribution with parameter p_{ii}. Thus, the mean curve from the set of experimental curves, a = a(t), yields the mean of T_i, $E[T_i] = 1 + \frac{p_{ii}}{q_i}$, and the whole transition matrix of the Markov chain can be eventually determined.

Despite the very simplifying assumption about the transition matrix, i.e. that it contains only two non zero elements in every line (except the last one if the ultimate stage is absorbing) the authors show very good agreement with the results simulated according to their model with experiments. It should be pointed out that this model describes the experimental results well if the parameters which the model requires have been determined from these results as well. It is however worthy to notice that the model requires only a mean curve a(t) which can be obtained from a relative small number of samples.

The Markov chain model may also be applied to random pulse loading [24]. This pulse may be defined as duty cycle or some other load event such as an earthquake occurrence, etc.. The transition probabilities of a crack growing from one "stage", i.e. crack length to another during a particular load event may be defined by

$$p_{i,k} = P\{(k-i-\tfrac{1}{2})\Delta a < Z_i < (k-i+\tfrac{1}{2})\Delta a\} \tag{7}$$

where Z_i is the crack increment due to a load event starting from stage i and Δa a unit of crack growth. As due to randomness in both load events and material properties the crack increment Z_i is also random. Therefore eq. (7) may be expressed as

$$p_{i,k} = \int_{(k-i-1/2)\Delta a}^{(k-i+1/2)\Delta a} f_Z(z)dz \tag{8}$$

where Z is determined by using a crack growth law such as *Paris-Erdogan*, *Forman*, etc. based on random parameters.

2.4 CENTRAL LIMIT THEOREM

Another way of solving the fatigue fracture problems is to utilize the central limit theorem of probability. In most crack propagation laws the argumet "a" and those containing the load process and possibly random material parameters are separable so that eq. (1) can be written as

$$\frac{da}{F_1(a)} = F_2(S_{max}, S_{min})dn \tag{9}$$

If $\psi(a_o,a_c)$ denotes the integral of the left hand side of eq. (9) over the crack length interval (a_o,a_c) where a_o and a_c are the initial (at t=0) and ultimate, (at t=T after N(T) load cycles) crack lengths respectively the solution of the eq. (9) takes the form

$$\psi(a_o,a_c) = \sum_{i=1}^{N(T)} F_2(S_{i,max}, S_{i,min}) \tag{1o}$$

which for the laws of the type of eq. (2) can be expressed as follows

$$\psi(a_o,a_c) = \sum_{i=1}^{N(T)} (C_1+C_{2i})h(S_{i,m})\Delta S_i^m \tag{11}$$

$$N(T) = C_1W_1 + W_2 \tag{11a}$$

$$W_1 = \sum_{i=1}^{N(T)} h(S_{i,m})\Delta S_i^m; \quad W_2 = \sum_{i=1}^{N(T)} C_{2i}h(S_{i,m})\Delta S_i^m$$

where $h(S_{i,m})$ is a monotonic function that accounts for the effect of the load cycle mean, $S_{i,m} = \frac{1}{2}(S_{i\ max} + S_{i\ min})$, C_1 is a random variable describing random variations between mean values of the parameter C(eq. (2)) in different specimen and C_{2i} describes random nonhomogenity of material along the crack path within each specimen.

For a large number of cycles N(T) and for a general class of load processes the distribution of (W_1, W_2) is approximately binormal. This approach was extensively investigated in [25,26]. There it is assumed that C_{2i} are realizations of a white noise (independent random variables), ($\ln C_1$, m) have joint normal distribution and S(t) to be either a stationary narrow band Gaussian process or stationary Poisson pulse process.

It should be pointed out that the solution for the stationary narrow band Gaussian process S(t) is based on the assumption that the amplitude process $\Delta S(t)$ is given by the envelope R(t) of the process S(t), so that $\Delta S(t) = 2R(t) - \bar{S}$. This assumption introduces a priori the Rayleigh probability distribution of the amplitudes and a symmetry of extremes of the process with respect to its mean value causing the neighboring extremes to be fully correlated. Moreover, the number of load cycles is assumed to be equal to the mean number of upcrossings of the mean level $N(T) = \nu_o T$. In fact the stochastic load process S(t) is substituted for a harmonic process with constant period $\tau_p = \frac{1}{\nu_o}$ and random amplitude, $2R(t)-\bar{S}$. This model allows to determine the conditional parameters (mean and variance) of the normal random variable $\psi(a_o,a_c)$ and, eventually, the probability of failure can be calculated by use of the first order second moment method of the structural reliability. The restrictive assumption of the narrow-band properties of the load process is dropped in [27] where an arbitrary continuous stationary normal stochastic process S(t) is considered. The central limit theorem of probability is also assumed to hold for the random variable $\psi(a_o,a_c)$ eq. (11) given a_o, C_1,m are kept constant and the sample of the number of cycles n(T) is a large number. The mean and variance of $\psi(a_o,a_c)$ are determined from the Taylor expansion of the right hand side of eq. (11) where the randomness of the cycle number N(T) is retained and correlations between subsequent amplitudes are determined and taken into account. Some stochastic fluctuations of the exponent m due to the nonhomogenity of the material are also allowed. It is shown that these fluctuations and randomness of the load process affect most of all the life-time for the low as well as for the high cycle fatigue. They cause however the scatter of the life-time estimation rather for low cycle fatigue than for the high cycle case. The stochastic fluctuation of the parameter C yield a negligible effect on the solution.

3. DISCUSSION

The presented methods do not exhaust the subject of the stochastic approaches to fatigue fracture. They are thought to indicate the main trends which can be observed in the literature concerning randomness of loading and/or material properties in fatigue problems. There are three directions in which the developments tend to move:

- equivalent or characteristic loading to substitude for stochastic amplitudes
- Markov model which describes the crack propagation under stochastic loading and/or in presence of the randomness of material parameters
- central limit theorem of the probability which makes use from the fact that the crack length after n load cycles is a sum of random events.

The first approach and the Markov chain model utilizing constant amplitude loading provide a very good description of the experimental results if the necessary parameters have been determined from these results. The ability of these approaches to predict an unknown fatigue behaviour on the basis of experiments that were made under other conditions has not been investigated by the authors. The Markov chain model may also be used to predict crack growth between random pulse loading events. For this purpose the parameter estimates as used in the crack growth law are based on generic experimental data and introduced as random variables in the analysis.

In the central limit theorem approach the material parameters are determined from the deterministic experiments and those of the loading from statistical analysis of real load samples but there is still insufficient experimental verification between the predicted and real crack propagation.

All models presented here do not consider the load sequence effect explicitly. The first two may contain it implicitly - due to an appropriate choice of their parameters to describe the results already in hand. The third model as presented here does not provide this possibility. However, for future developments an explicit incorporation of the load sequence effect into a stochastic fatigue model seems to be desirable. Only then the experimental verification of the model and eventual calibration of its parameters can be accomplished with an agreement with the features occuring in the fatigue crack propagation under variable loading.

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REFERENCES

/ 1 / VIRKLER, D.A., HILLBERY, B.M., GOEL, P.K., The statistical nature of fatigue crack propagation, AFFDL-TR-78-43, Apr. 1978.

/ 2 / CLARK, W.G., Jr., HUDAK, S.J., Jr., Variability in fatigue crack growth rate testing, Journ. of Testing and Evaluation, Vol.3, No. 6, Nov. 1975, pp. 545 - 576.

/ 3 / MARSHALL, W., An assessment of the Integrity of PWR Pressure Vessels, UKAEA Report, Oct. 1976, H.M. Stationery Office, London, U.K.

/ 4 / WELLEIN, R., Beurteilung der Zuverlässigkeit der Stahlhülle mit Hilfe der Probabilistischen Bruchmechanik, Appendix III of BMFT Rep. No. 150 201 C, Techn. Univ. Munich, Sept. 1981, 79p.

/ 5 / HARRIS, D.O., LIM, E.Y., DEDHIA, D.D., WOO, H.H. and CHOU, C.K., Fracture mechanics models developed for piping reliability assessment in light water reactors - piping reliability project, NUREG/CR-2301, UCRL-15490, Wash. D.C., 1982.

/ 6 / SCHÄFER, R., SCHUËLLER, G.I. and KAFKA, P., Probabilistische Untersuchung des Rißfortschrittverhaltens von Reaktorkomponenten, IABG-Report TF-1605, Ottobrunn, F.R.Germany, March, 1984.

/ 7 / HOEPPNER, D.W., KRUPP, W.E., Prediction of component life by application of fatigue crack growth knowledge, Eng. Fract. Mechs., Vol. 6, 1974, pp. 47 - 70.

/ 8 / PARIS, P.C., ERDOGAN, F., A critical analysis of crack propagation laws, Journ. Basic Eng., Ser. D, Vol. 89, No. 4, 1963, pp. 528 - 534.

/ 9 / WHEELER, O.E., Spectrum loading and crack growth, Trans., ASME, J. Basic Eng., Vol. 94, 1972, pp. 181 - 186.

/ 10 / WILLENBORG, R.M., ENGLE, R.M., WOOD, H.A., A crack growth retardation model using an effective stress concept, WPAFB, TM-71-1-FBR, 1971.

/ 11 / ELBER, W., The significance of fatigue crack closure, Damage Tolerance in Aircraft Structures, ASTM STP 486, 1971, pp. 230 - 242.

/ 12 / NAYEB-HASHEMI, H., McCLINTOCK, F.A., Influence of overloads and block loading sequences on mode III fatigue crack propagation in A469 rotor steel, Eng. Fract. Mechs., Vol. 18, No. 4, 1983, pp. 763 - 783.

/ 13 / BARSOM, J.M., Fatigue crack growth under variable amplitude loading in ASTM A514-B steel, Progress in Flaw Growth and Fracture Toughness Testing, ASTM STP 536, 1973, pp. 147 - 167.

/ 14 / PARIS, P.C., The growth of cracks due to variations in load, Ph.D. thesis, Leigh Univ., 1962.

/ 15 / STRATANOVICH, R.L., Topics in the Theory of Random Noise, Vol. I, transl. by R.A. Silvermann, Gordon and Breach, New York, 1967.

/ 16 / RICE, J.R., BEER, F.P., On the distribution of rised and falls in a continuousrandom pulses, Trans. ASME, J. Basic Eng., June 1965, pp. 398 - 404.

/ 17 / YANG, J.-N., Statistics of random loading relevant to fatigue, Proc. ASCE, EM3, June 1974, pp. 469 - 475.

/ 18 / LINDGREN, G., Wave length and amplitude in Gaussian noise, Adv. Appl. Prob., Vol. 4, 1972, pp. 81 - 108.

/ 19 / LIND, Y.K., YANG, J.-N., On statistical moments of fatigue crack propagation, Eng. Fract. Mechs., Vol. 18, No. 2, 1983, pp. 243 - 256.

/ 20 / SOBCZYK, K., On the Markovian models for fatigue accumulation, J. de Méc. théorique et appliquée, Numéro special, 1982, pp. 147 - 160.

/ 21 / BOGDANOFF, J.L., KOZIN, F., A new cumulative damage model, Part 4, J. Appl. Mech., Vol. 47, No. 1, 1980, pp. 40 - 44.

/ 22 / BOGDANOFF, J.L., KOZIN, F., A critical analysis of some probabilistic models of fatigue crack growth, Eng. Fract. Mech., Vol. 14, 1981, pp. 59 - 89.

/ 23 / BOGDANOFF, J.L., A new cumulative damage model, Part 3, J. Appl. Mech., Vol. 45, No. 4, 1978, pp. 733 - 739.

/ 24 / OSWALD, G.F. and SCHUËLLER, G.I., Reliability of Deteriorating Structures, J. Eng. Fract. Mechs., 1984.

/ 25 / MADSEN, H.O., Probabilistic and deterministic models for predicting damage accumulation due to time varying loading, DIALOG 5-82, Danish Engineering Academy, Lyngby, Danmark, 1983.

/ 26 / MADSEN, H.O., Stochastic modeling of fatigue crack growth, Lecture Notes for Advanced Seminar on Structural Reliability, ISPRA Course, Ispra, Italy, June 1984.

/ 27 / DOLINSKI, K., On a stochastic model of fatigue crack growth, to be presented at ICOSSAR'85, Kobe, Japan, May 1985.