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EVALUATION OF SECTIONAL PROPERTIES OF A COMPLEX STRUCTURE FOR STICK MODEL

K.V. Subramanian[1], R.K. Ingle[1], H.S. Kushawaha[2], K.K. Vaze[2], G.R. Reddy[2] and G.V. Karandikar[3]

[1]Tata Consulting Engineers, Prabhadevi, Bombay, [2]Reactor Engineering Division, Bhabha Atomic Research Centre, Trombay, Bombay, [3]Nuclear Power Corporation, Bombay (India)

1.0 ABSTRACT

Seismic analysis of a large complex structure like a Reactor Building is usually carried out by modelling the structure as a stick model. This stick model is derived from the complex structure by evaluating for the equivalent model, sectional properties like flexural centre, area centre, area, shear area and moment of inertia along both the horizontal directions and torsional inertia. The Internal Structure of a Reactor Building consists of massive walls with complex geometrical configuration viz. discontinuity of shear walls, overhanging sections, large cutouts in floors etc. This paper outlines the method for evaluation of sectional properties for such a complex structural system for representation as an equivalent stick model.

2.0 INTRODUCTION

A Reactor Building consists of an Internal Structure (IS) and Calandria Vault (CV) contained in coaxial Inner and Outer Containment walls (ICW & OCW) cast monolithically with a circular Raft (Fig. 1). The OCW consists of cylindrical shell supporting a torispherical dome made up of reinforced concrete. The ICW also consists of cylindrical shell supporting a torispherical dome but made up of prestressed concrete. The IS is a complex three dimensional structure consisting of shear walls, columns and beams, supporting massive floors. The geometry of IS and CV is symmetric about the EW axis but not symmetric about NS axis (Fig 2 - 4). The IS is closed hollow box from EL 85000 to EL 100000 and an open section from EL 100000 to EL 115500. Above EL 115500, there are two boxes, housing the steam generators. The IS and ICW are connected by an annular steel plate at EL 130000 as the IS is flexible above EL 115500. This link permits transfer of lateral force but allows the two structures to rotate independently of each other.

3.0 EVALUATION OF SELECTIONAL PROPERTIES

The basic data required for analysis is the cross sectional properties, lumped masses and mass moments of inertia. The cross sectional properties consist of cross sectional area, shear area, moments of inertia, centre of rigidity and torsional inertia. Standard formulae are available for finding the sectional properties if the cross section of the member is simple and regular. (ASCE-4 86, Young) The sectional properties are dependent upon the constraints like rigidity of slab, length of member etc. These are normally ignored in conventional analysis. Restraint to warping can change the stress distribution thus affecting the centre of rigidity and torsional inertia.

Hence, the sectional properties of IS and CV are calculated by using Finite Element Model(FEM). However, standard formulae are used only for calculation of sectional properties of OCW, ICW and Raft. Computation of the sectional properties of the IS is described in the sections that follow. The IS is modelled by FEM using 4 noded flat shell elements.

3.1 Centre of Rigidity and Torsional Inertia

The centre of rigidity of a section is obtained by evaluating the torsional centre, as in the linear domain both are the same point on the crosssection. For different sections of the internal structure, torsional moments are applied, and the point on each section where horizontal displacement is zero is determined. This is the torsional centre. For determination of torsional inertia for the elements of the beam representing the IS, torsional loads are applied on the full FEM at all floor levels by application of a couple on the cross walls at each floor level. The torsional rotation at each floor level is calculated. If J is the torsional inertia of the crosssection, then

$$\frac{GJ}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} \theta_i \\ \theta_j \end{bmatrix} = \begin{bmatrix} T_{mi} \\ T_{mj} \end{bmatrix} \qquad \ldots(1)$$

where: T = Torsional moment G = modulus of rigidity J = Torsional Inertia
L = length of member θ = torsional rotation

With this element stiffness matrix , the stiffness matrix for the stick model for torsion is obtained. With the torsional rotations and applied torsional moments, the torsional inertia J for the various sections is evaluated.

3.2 Crosssectional Area, Shear Area and Moment of Inertia

On the FEM of the IS, lateral loads are applied in each direction such that the resultant passes through the centre of rigidity computed above. Thus for example, for direction X, loads are applied as distributed loads on the shear walls in this direction such that the resultant load passes through the flexural centre of the crosssection. Using the horizontal and vertical displacements for each load case, the equivalent displacement and rotation of the section is calculated. The element stiffness matrix given by equation (2) is used to assemble the stiffness equation of the equivalent beam system which has shear area and moment of inertia as unknowns for the horizontal load cases. On the same lines as above for the lateral loads, vertical loads are applied on the crosssection as a distributed load on all the shear walls such that the resultant passes through the centroid. The equivalent beam vertical deflection is evaluated as the average of corresponding deflections of the nodes of the shear walls at the section. The element stiffness matrix for axial stiffness is given in equation (3). Using the assembled axial stiffness equation, vertical load and vertical deflections obtained from application of this load, the cross sectional area is evaluated.

$$\frac{EI}{L^3(1+\emptyset)}\begin{bmatrix} 12 & 6L & -12 & 6L \\ & (4+\emptyset)L^2 & -6L & (2-\emptyset)L^2 \\ & & 12 & -6L \\ & & & (4+\emptyset)L^2 \end{bmatrix}\begin{bmatrix} \delta i \\ \theta i \\ \delta j \\ \theta j \end{bmatrix} = \begin{bmatrix} pi \\ Mi \\ Pj \\ Mj \end{bmatrix} \ldots(2) \qquad \begin{bmatrix} \frac{AE}{L} & \frac{-AE}{L} \\ & \frac{AE}{L} \end{bmatrix}\begin{bmatrix} \delta i \\ \delta j \end{bmatrix} = \begin{bmatrix} Pi \\ Pj \end{bmatrix} \ldots(3)$$

where ϕ	=	$12\ El\ /GA_S\ L^2$	θ	=	Rotation
A_S	=	Shear area	P	=	Applied shear forces
EI	=	Flexural rigidity	M	=	Applied moments
AE	=	Axial rigidity	δ	=	Displacement

3.2.1 Application of Lateral Loads

Transverse loads are applied on the FEM on the shear walls in the direction of the load for calculating sectional properties. As reflected in ASCE 4- 86, the effective shear area may be computed from the sum of the component shear areas of the individual walls parallel to the direction of the applied shear forces. From the point of influence of aspect ratio and effect of presence of the stiffness elements on the section, studies were carried out for two types of load application viz. loads are applied through the walls parallel to the direction of analysis (i.e. web) and loads applied through the webs and walls perpendicular to the direction of analysis (flanges). When loads are applied also on the flanges, flexural deflections are introduced which is local and should be neglected in global modelling. The sectional properties, for typical cross-sections of IS were determined for both the load applications. The computed properties for two sections (sections considered being between two floor levels) are given in Table 1.

3.3 Comparison of the Properties

Table 2 compares the properties of the elements representing the IS computed using standard formulae and the method described above. It may be observed the two are widely different.

4.0 DEVELOPMENT OF STICK MODEL

A 3-D stick model is developed using the sectional properties computed above. In this model, the stick is represented along the centre of rigidity and masses are lumped at the mass centre. (Subramanian, et. al.) The mass nodes are connected to the stick nodes by rigid links.(Fig.5) The OCW is represented by 23 elements, ICW is represented by 21 elements. The IS is represented by 16 elements and the CV by 10 elements. The raft is represented by 2 elements and the transverse, vertical, rotational and torsional springs are applied at the raft level to cater to the effect of soil-structure interaction. (ASCE 4-86) All the substructures are represented at the centre of the building.The connection between the IS and ICW at EL 130000 is modelled using springs representing the stiffness of the connecting link at the respective nodes.

5.0 STRAIN ENERGY EQUIVALENCE

To establish the equivalence of the 3-D stick model as representing the IS, the total strain energy in the two models (FE and 3-D stick of substructure IS) for an arbitrary loading system was compared. This was necessary as the sectional properties were determined by assuming decoupling of flexural and torsional effects for the applied loading system through the flexural centres of the cross-section. However, as the flexural centres of any two levels are not on the same line, torsional effects are introduced which are neglected.

The strain energy U_T is equal to the work done in distorting the system.

$$U = \tfrac{1}{2}\ \delta^T K\ \delta \qquad \ldots (4)$$

But, $P = K\delta$ or $P^T = \delta^T K$; Hence, $U = \tfrac{1}{2} P^T \delta$ ... (5)

where, δ=displacement vector, K=Global stiffness matrix, P=Total Load vector

An eccentric load applied on the structure, causes all the displacement components, depending upon the eccentricity between the flexural centre and the centre of applied loads. The strain energy viz.internal work done as per equation (4) is equal to the external work done computed using equation (5).

An arbritrary system of loads is applied at all levels in all the three directions (two horizontal and one vertical) and the corresponding deflections and strain energies in the 3-D stick model and the FEM of the IS are compared. As another load case, a torsional load was also applied on the system as a couple and the torsional rotation and strain energy in the two models compared. Finally, another load system with loads in all the three directions and a torsional load was applied on the two models and strain energies were compared. These are reported in Tables 3 & 4.

6.0 CONCLUSIONS

Comparison of deflections, torsional rotation and strain energy in the tables 3 & 4, show a very close match in between the two models. It can also be seen that the sectional properties are different from those calculated using standard formulae. Hence the influence of the complexities in the geometrical configuration can only be realistically established from the FEM. Such a close match establishes the equivalence of the 3-D stick model as representing the complex structural system and provides a confidence in the method adopted to represent such a complex structure for dynamic analysis.

7.0 ACKNOWLEDGEMENTS

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8.0 REFERENCES

Subramanian,K.V.,Ingle,R.K, Kushawaha,H.S., Vaze,K.K., Reddy,G.R., Conversion of a 3-D Structure into a 3-D Stick model, Ninth Symposium on Earthquake Engineering, Roorkee.Dec 1990.

ASCE 4 -1986, Seismic Analysis of Safety Related Nuclear Structures and Commentary, ASCE, 345, East 47th. Street, New York- 10017-2398

Young,W.C., 1989, Roark's Formulae for Stress and Strain, McGraw Hill Book Co.- Singapore.

TABLE - 1 Comparison of Properties - Internal Structure Sections

	Load applied(N-S)			Load applied (E-W)		
	On Webs & Flanges	On Webs only	% Diff	On Webs & Flanges	On Webs only	% Diff
EL 85.0 to 93.0						
Shear area (m^2)	205.91	203.35	1.24	260.58	260.24	0.13
Moment of inertia (m^4)	29632.43	29470.18	0.55	20563.84	20775.42	1.03
EL 93.0 to 100.0						
Shear area (m^2)	115.65	115.34	0.27	97.48	95.20	2.34
Moment of inertia (m^4)	12565.63	12784.40	1.74	7292.28	7359.32	0.92

TABLE -2 -Comparison of Sectional Properties for Internal Structure

SECTION	SHEAR AREA NS(m^2)	M. OF INERTIA @EW(m^4)	SHEAR AREA @ EW(m^2)	M. OF INERTIA @NS(m^4)	SECTION AREA (m^2)	TORSIONAL INERTIA NS(m^4)	RIGIDITY CENTTRE EW(m)	NS(m)
EL 85000 - FORM.	273	21065	265	46031	430	1010		
93000 - FEM.	200	27650	240	18350	420	39743	0.00	-0.85
EL 93000 - FORM.	156	11640	114	40354	234	122		
100000 - FEM.	90	3620	40	15675	150	17030	0.00	-2.25
EL100000 - FORM.	124	5515	97	9132	192	120		
104500 - FEM.	90	3260	40	10700	155	12553	0.00	-2.20
EL104500 - FORM.	121	5164	29	3378	151	92		
109000 - FEM.	90	3360	10	10600	120	9911	0.00	0.14
EL109000 - FORM.	129	5304	29	3378	153	108		
115500 - FEM.	90	4360	10	5750	65	10975	0.00	0.64
EL115500 - FORM.	42	551	56	3712	90	1261		
130000 - FEM.	20	320	40	5190	75	2660	0.00	0.48

TABLE-3 Comparison of δ, and St.Energy for Simultaneous applied Loads

Model	Level	Load (T) X	Y	Z	Displacements(m xe-3) X	Y	Z	Strain Energy (t-m) X	Y	Z
STICK	130	10000	10000	20000	45.390	73.804	6.795	226.950	369.020	67.955
FEM	130	10000	10000	20000	44.833	72.702	6.912	224.262	364.174	68.379
STICK	115	10000	20000	30000	35.781	34.582	5.725	178.905	345.820	85.871
FEM	115	10000	20000	30000	34.541	32.837	5.609	175.518	327.227	83.969
STICK	109	0	10000	10000	23.496	24.607	3.983	0.000	123.040	19.915
FEM	109	0	10000	10000	22.387	23.804	3.769	0.000	118.859	19.740
STICK	104	10000	10000	20000	13.565	16.500	3.628	67.825	82.506	36.280
FEM	104	10000	10000	20000	13.086	15.958	3.274	65.951	80.517	33.613
STICK	100	10000	10000	20000	9.350	9.767	2.740	46.751	48.830	27.404
FEM	100	10000	10000	20000	8.440	9.565	2.303	45.368	47.926	25.070
STICK	93	25520	10000	35520	2.346	2.753	0.849	29.930	13.766	15.080
FEM	93	25520	10000	35520	2.337	2.749	0.873	29.342	13.839	14.566

Total Stick : 1785.86 FEM : 1738.32 % age diff : 2.73%

TABLE-4 Comparison of Strain Energy due to Torsion and Total Strain Energy

Model	Level	Torque (t-m)	Rot θ(Rad) (x 10^{-3})	Strain En. (t-m)	%Diff	Load (T) X	Y	Z	Torque (t-m)	Strain En. (t-m)	%Diff
STICK	130	82969	0.837	34.70	0.23	10000	10000	20000	165939	815.140	2.20
FEM	130	82969	0.838	34.78		10000	10000	20000	165939	797.570	
STICK	115	62440	0.449	14.01	0.78	10000	20000	30000	124880	675.970	8.56
FEM	115	62440	0.452	14.12		10000	20000	30000	124880	622.650	
STICK	109	62440	0.383	11.95	0.67	0	10000	10000	124880	199.650	11.51
FEM	109	62440	0.385	12.03		0	10000	10000	124880	179.050	
STICK	104	113960	0.296	16.84	0.49	10000	10000	20000	227920	258.963	3.71
FEM	104	113960	0.297	16.91		10000	10000	20000	227920	249.707	
STICK	100	51520	0.194	4.99	0.42	10000	10000	20000	103040	143.350	3.58
FEM	100	51520	0.194	5.01		10000	10000	20000	103040	138.398	
STICK	93	66440	0.069	2.31	0.39	25520	10000	35520	132800	68.001	2.46
FEM	93	66440	0.070	2.32		25520	10000	35520	132800	66.365	

Total Stick:84.80 FEM: 85.18 Diff :0.45% Total Stick:2161.1 FEM:2053.7 Diff: 5.20%

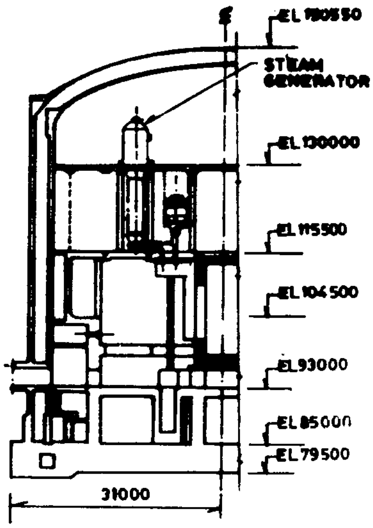

FIG-1

HALF SECTION OF REACTOR BUILDING

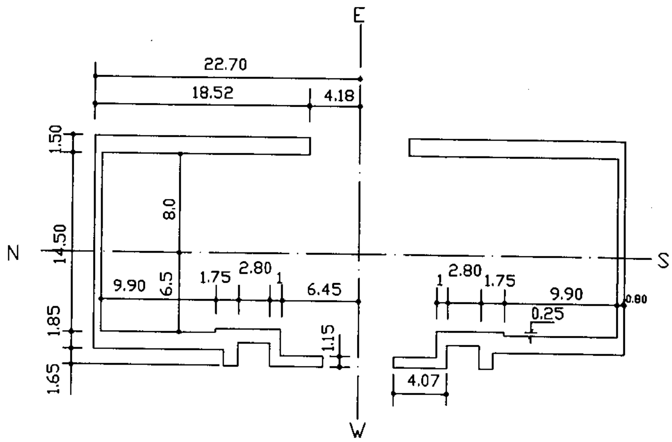

FIG-4 PLAN AT ELEVATION 104.5

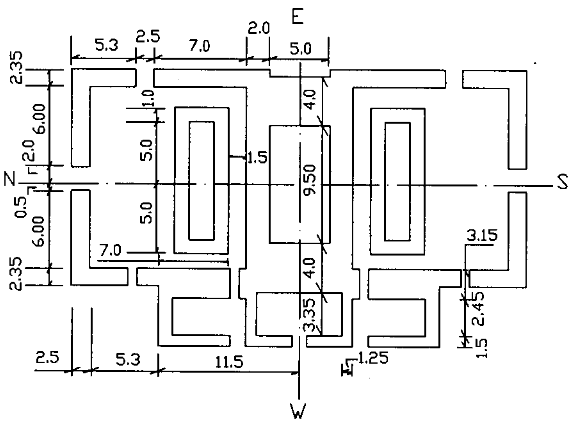

FIG.-2. PLAN AT ELEVATION 85.0

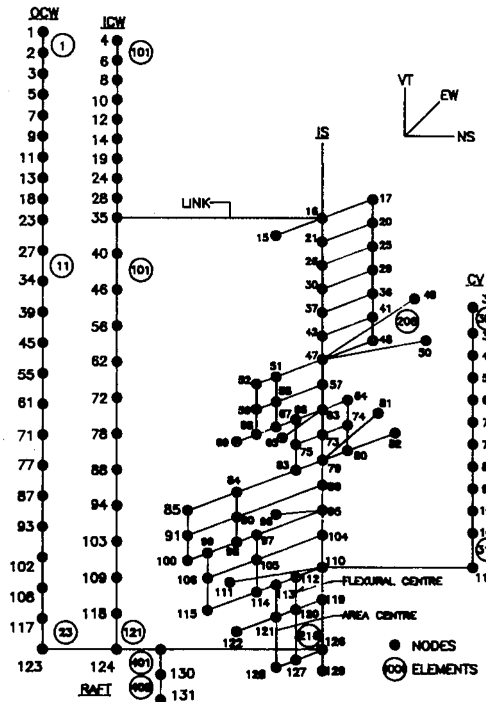

FIG – 5 3-D STICK MODEL FOR REACTOR BUILDING

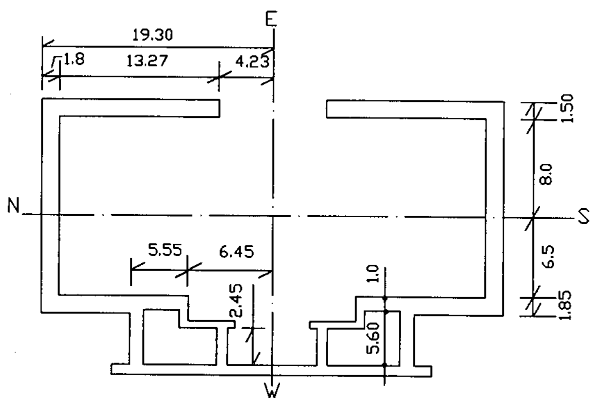

FIG-3 PLAN AT ELEVATION 100.00