CONCEPTUAL DESIGN OF PIPE WHIP RESTRAINTS USING INTERACTIVE COMPUTER ANALYSIS

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SUMMARY

Protection against pipe break effects necessitates a complex interaction between failure mode analysis, piping layout, and structural design. Many iterations are required to finalize structural designs and equipment arrangements. The magnitude of the pipe break loads transmitted by the pipe whip restraints to structural embedments precludes the application of conservative design margins.

A simplified analytical formulation of the nonlinear dynamic problems associated with pipe whip has been developed and applied using interactive computer analysis techniques. In the dynamic analysis, the restraint and the associated portion of the piping system are modeled using the finite element lumped mass approach to properly reflect the dynamic characteristics of the piping/restraint system. The analysis is performed as a series of piecewise linear increments. Each of these linear increments is terminated by either formation of plastic conditions or closing/opening of gaps. The stiffness matrix is modified to reflect the changed stiffness characteristics of the system and re-started using the previous boundary conditions. The formation of yield hinges are related to the plastic moment of the section and unloading paths are automatically considered.

The conceptual design of the piping/restraint system is performed using interactive computer analysis. Input parameters include system geometry, stiffness and material characteristics, gap size, and time history of applied forces. The output data file consisting of time history of displacements, velocities, accelerations, member loads, and yield hinge locations are scanned for selected parameters and reviewed at the interactive terminal. If the results of the analysis are acceptable the complete output data file is printed at the interactive terminal or at an off-line high speed printer.

The validity of the analytical formulation has been established using a general purpose finite element analysis program wherein the effects of strain-rate, strain-hardening, and gaps have been included. Results of the comparison indicate close agreement between the restraint design loads obtained from the large-order finite element analysis and the analytical formulation proposed in this paper. The application of the simplified analytical approach with interactive computer analysis results in an order of magnitude reduction in engineering time and computer cost.

1. Introduction

Protection against postulated pipe break effects is accomplished by separation of safety related systems and components, intermediate barriers, and pipe whip restraints. The evolution of a plant design especially in the preliminary engineering phase requires many iterations to determine not only acceptable structural designs and equipment arrangements but also to minimize the pipe break loads which are transmitted by the pipe whip restraints to the structural embedments.

Simplified analytical formulations based either on the energy approach or a consideration of one or two degree-of-freedom dynamic models provide rapid solutions to the many iterations encountered by the plant designer. Unless empirical scaling factors are applied to the simplified analytical formulations, the conservative load estimates, especially for larger diameter high energy systems, result in overdesigned structural embedments and subsequent load carrying structural elements.

A general purpose finite element program which considers material and geometric nonlinearities can establish realistic design load estimates for postulated pipe break effects. Schedule considerations, engineering manpower requirements, and computer costs frequently preclude the consideration of this approach especially during the preliminary engineering phase.

This paper presents an analytical formulation which is combined with interactive computer capability to meet the primary objectives of realistic pipe break design loads, low analytical costs, and minimum impact on engineering and design schedules.

2. Computer Program Formulation

The analytical formulation of the nonlinear dynamics problem was simplified as much as possible to be consistent with the preliminary nature of the input data and the extent of information required during the conceptual design phase of the restraint. The simplified analytical approach was incorporated into an interactive computer program (SIMPLE) with special emphasis on input/output data management by the pipe whip restraint designer.

Piecewise nonlinear elastic-plastic analysis of piping systems or other simple beam structures subjected to dynamic loading can be represented by the following equation for each elastic step:

$$\left[M\right]\left\{\ddot{x}\right\} + \left[K\right]\left\{x\right\} + \left[C\right]\left\{\dot{x}\right\} = \left\{F(t)\right\} \qquad (1)$$

where,

$\left[M\right]$ = Diagonal Mass Matrix

$\left[K\right]$ = Symmetric Stiffness Matrix

$\left[C\right]$ = Damping Matrix

$\left\{F(t)\right\}$ = Forcing function vector

$\left\{x\right\}$, $\left\{\dot{x}\right\}$, $\left\{\ddot{x}\right\}$ = Displacement, velocity and acceleration vector.

After an elastic step has been terminated by either the formation of an elastic-plastic transition region or a closure/opening of a gap, equation (1) is rewritten as follows:

$$\left[M\right]\left\{\ddot{\delta}\right\} + \left[K_{NEW}\right]\left\{\delta\right\} + \left[C\right]\left\{\dot{\delta}\right\} = \left\{F^{*}(t)\right\} \qquad (2)$$

where,

$$\left\{F^{*}(t)\right\} = \left\{F(t)\right\} - \left[K_{OLD}\right]\left\{x_E\right\}$$

$\left\{\delta\right\} = \left\{x - x_E\right\}$ = new reference displacement vector

$\left\{x_E\right\}$ = displacement vector at the end of previous elastic step

$\left[K_{NEW}\right]$ = new stiffness matrix (new geometry, hinges and/or new members included)

$\left[K_{OLD}\right]$ = stiffness matrix of the previous elastic step.

and the total displacement vector is given by:

$$\left\{x\right\} = \left\{\delta\right\} + \left\{x_E\right\} \tag{3}$$

The linear acceleration method is used to transform eqs. (1) and (2) to a set of "n" simultaneous linear algebraic equations where the "n" unknowns are the displacements defined at $t_o + \Delta t$. For each degree of freedom, the acceleration between time t_o and $t_o + \Delta t$ is approximated by

$$\ddot{x}(t) = \ddot{x}(t_o) + \frac{\ddot{x}(t_o + t) - \ddot{x}(t_o)}{\Delta t}(t - t_o) \tag{4}$$

The velocity at any time within the same interval is:

$$\dot{x}(t) = \dot{x}(t_o) + \int_{t_o}^{t} \ddot{x}(t)\,dt$$

$$= \dot{x}(t_o) + (t - t_o)\,\ddot{x}(t_o) + \frac{\ddot{x}(t_o + t) - \ddot{x}(t_o)}{2\Delta t}(t - t_o)^2 \tag{5}$$

At time $t_o + \Delta t$ the velocity and displacement are:

$$\dot{x}(t_o + \Delta t) = \dot{x}(t_o) + \frac{\Delta t}{2}\left\{x(t_o + \Delta t) + x(t_o)\right\} \tag{6}$$

$$x(t_o + \Delta t) = x(t_o) + \dot{x}(t_o)\,\Delta t + \left(\frac{\Delta t}{6}\right)^2\left\{2\ddot{x}(t_o) + \ddot{x}(t_o + \Delta t)\right\} \tag{7}$$

Applying eqs. (6) and (7) to the system of equations (1) and/or (2), they become

$$\left[A\right]\left\{x(t_o + \Delta t)\right\}\left\{\bar{B}\right\} \tag{8}$$

where,

$\left[A\right]$ is the symmetric matrix whose elements are given by

$$A_{ij} = M_{ij}\frac{6}{\Delta t^2} + K_{ij} + 3\frac{C_{ij}}{\Delta t} \quad (M_{ij} = 0, \text{ when } i \neq j)$$

$\left\{\bar{B}\right\}$ is a column vector given by

$$\left[B\right]\left\{x(t_o)\right\} + \left[D\right]\left\{\dot{x}(t_o)\right\} + \left[G\right]\left\{\ddot{x}(t_o)\right\} + \left\{F(t_o + \Delta t)\right\}$$

and,

$$B_{ij} = M_{ij}\frac{6}{\Delta t^2} + 3\frac{C_{ij}}{\Delta t}$$

$$D_{ij} = M_{ij}\frac{6}{\Delta t} + 2C_{ij} \quad (M_{ij}\ 0, \text{ when } i \neq j)$$

$$G_{ij} = 2M_{ij} + C_{ij}\frac{\Delta t}{2}$$

At a particular stage of the computation $x(t_o)$, $\dot{x}(t_o)$, $\ddot{x}(t_o)$, and $F(t_o+\Delta t)$ are known, and therefore, eq. (8) represents a set of "n" simultaneous linear algebraic equations. The "Cholesky" square root method, an efficient form of the decomposition method, is used to solve eq. (8). Solution convergence with minimal computation errors are obtained for Δt values equal to 1/10 of the shortest natural period of the system.

The flow chart for interactive computer program SIMPLE is presented in Figure 1. The mass of the piping system is entered at discrete nodal points as part of the input data. Member stiffness matrices which are used to assemble the global stiffness matrix are determined in accordance with the classical formulation for straight beams where bending, shear, and axial stiffnesses are included. Closing and opening of gaps are accounted for by considering members between the joints where the gaps are defined. These members offer zero stiffness as long as the gap is open while positive resistance when the gap is closed.

At each increment of time the existence of plastic conditions is checked in each member. When a plastic condition is reached, the plastic behavior which follows is accounted for by reducing the value of the elastic modulus of elasticity. The transition from a plastic to an elastic condition is accomplished by reassigning to the member the original modulus of elasticity.

Each of the piecewise linear increments is terminated by either a closing/opening of a gap or a formation of a plastic condition and then reinitiated with the conditions left by the previous linear increment. Because of the incremental nature of the computations, displacements and member forces or moments calculated for a given time increment may exceed allowable values. If an overshoot condition occurs an iterative process based on a reduction of calculation increments maintains the amount of overshoot within specified bounds.

3. Program Verification

The analytical basis of the computer program SIMPLE incorporates simplifying assumptions consistent with the accuracy of the input data available during the preliminary engineering phase of a nuclear power plant. The results generated using the SIMPLE program were compared with the results obtained from general purpose programs such as MARC-CDC. Gap size, pipe diameter and wall thickness, shape of forcing function and restraint stiffness characteristics were changed for a series of mathematical models to verify program adequacy.

Figure 2 represents one of the piping/restraint configurations which was investigated. Six nodal mass points are used for the SIMPLE model as compared to fourteen nodal mass points for the MARC-CDC model. Additional model data are presented in Figure 3.

Calculated force-time histories in the pipe whip restraint and displacement-time histories of the pipe are plotted in Figure 4. A comparison of the maximum restraint load to the pipe thrust load at t=0 indicates a dynamic amplification factor of 1.5. The MARC-CDC force-time history curve rises more gradually to the maximum value and then decreases more rapidly than the corresponding SIMPLE curve. The maximum restraint load predicted by SIMPLE for this case is within 1 percent of the load calculated with MARC-CDC. Maximum restraint loads determined from the program SIMPLE for all comparisons were found to be less than 6 percent higher than those obtained from general purpose programs with nonlinear dynamic analysis capability.

4. Interactive Computer Analysis

The conceptual design of pipe whip restraints is an interactive process. The designer using past experience, available nomographs, or simple hand calculations makes an initial estimate of the pipe whip restraint load and sizes primary load carrying members accordingly. Interactive computer analysis enables the designer to implement a series of design refinements with an immediate retrieval of results.

Figure 5 presents the basic interactive analysis flow chart. The input data base consisting of geometry, member structural properties, nodal masses, and applied forcing functions are entered at the time sharing terminal by the designer. The input data base is altered during subsequent iterations by changing only the affected parameters. Upon completion of problem execution an output data file is created containing time histories of displacements, velocities, accelerations, member and restraint loads, and identification of elastic to plastic and plastic to elastic transitions. The query facility enables the designer to access selected output data for immediate display on the time sharing terminal. Upon review of the selected output data the designer has the option of either modifying the input data base for a subsequent design iteration or printing the total output data by an off-line high speed printer or by the printer at the time sharing terminal.

Table I indicates a comparison of productivity and computer cost ratios for a general purpose program to program SIMPLE. The priority levels selected for batch processing of the general purpose program were based on comparable batch and time sharing unit costs. Computer run turnaround ranged from one to three runs per day. Time sharing terminal to central processor connect charges represented a significant percentage of total time sharing costs up to completion of the first iteration. Activity duration ratios include consideration of waiting periods associated with batch run processing. The average duration of a problem which required three iterations was approximately two days using the program SIMPLE and the interactive analysis approach and approximately twenty days using a general purpose finite program and batch processing.

5. Conclusions

A simplified analytical formulation of the nonlinear dynamics problem associated with the design and analysis of pipe whip restraints can be used to generate realistic design loads. Maximum restraint load magnitudes calculated by the simplified approach are in close agreement with results obtained using more sophisticated general purpose finite element programs. The application of interactive computer analysis techniques for conceptual design of pipe whip restraints minimizes analytical and component hardware costs while reducing the potential for schedule delays.

Table I

A COMPARISON OF PRODUCTIVITY AND COMPUTER COSTS

	Performance Ratio of General Purpose Program to Program SIMPLE		
Work Unit	Manhours	Computer Cost	Activity Duration
Problem Start to Completion of First Iteration	4.8 - 8.1	3.7 - 5.4	3.9 - 6.4
Completion of Each Subsequent Iteration	2.9 - 5.2	6.6 - 8.5	11.6 - 18.0

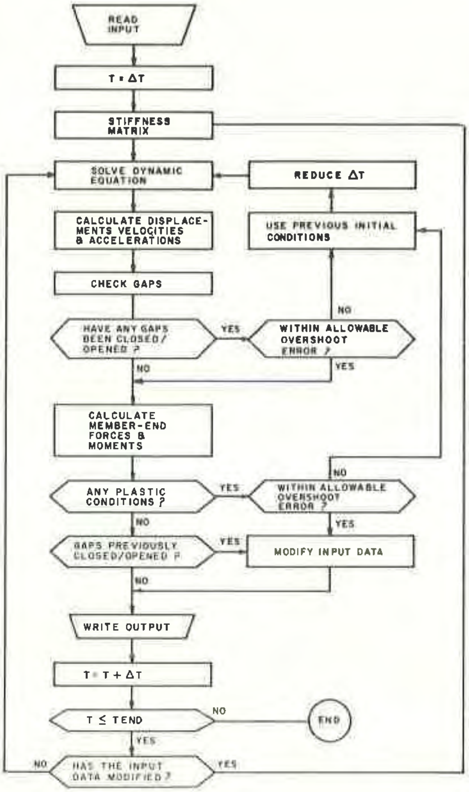


Figure 1 - Program SIMPLE Flow Chart

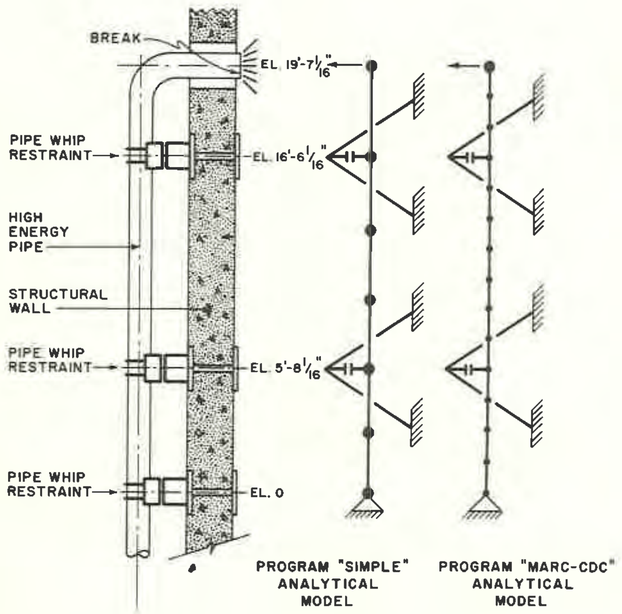
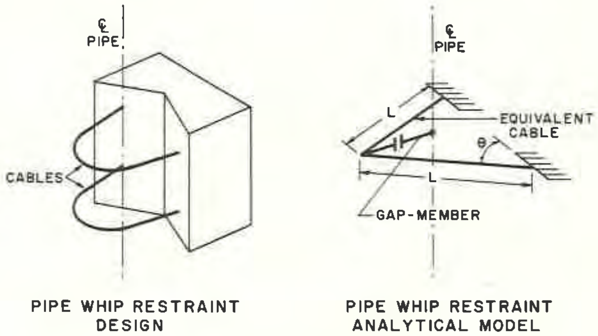


Figure 2 - Analtyical Models Used for Program Verification



ANALYTICAL MODEL DATA

PROPERTIES	PIPE	GAP MEMBER	CABLE	FLUID
MATERIAL	A106-GR B		WIRE ROPE IMPROVED PLOWED STEEL	WATER
SIZE	12" SCH 100	GAP = .1126	O.D. = 1"	
DYNAMIC YIELD	37,000 PSI		147,000 PSI	
LENGTH L			17"	
ANGLE θ			35°	
PRESSURE				1050 PSI
TEMPERATURE	550° F		100° F	550° F
EQUIV. STIFFNESS		8.18 X 10^5 LB/IN		

Figure 3 - Analytical Model Data

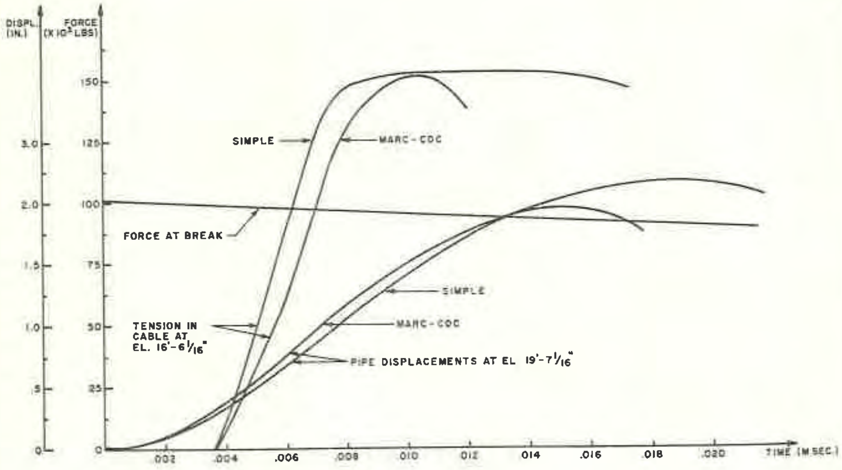


Figure 4 - A comparison of Pipe Whip Restraint Force and Pipe Displacement Time Histories

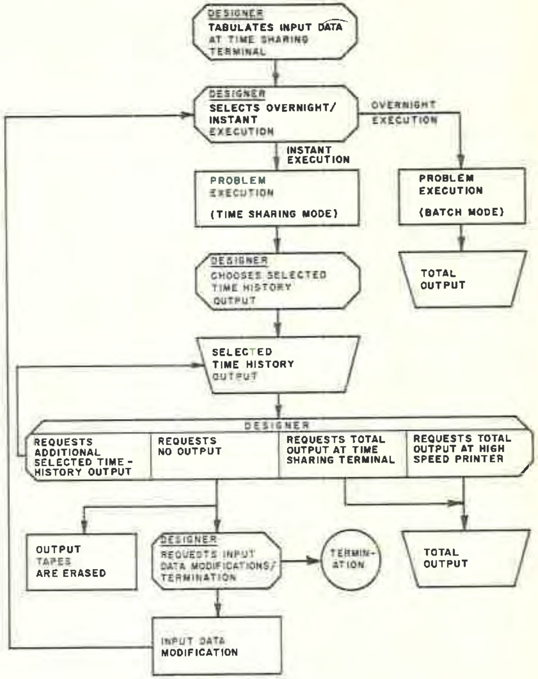


Figure 5 - Interactive Analysis Flow Chart