COUPLED FLUID STRUCTURAL ANALYSIS FOR A SPHERICAL BWR CONTAINMENT WITH PRESSURE SUPPRESSION SYSTEM

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Summary

The condensation of steam, blown into the water pool of the pressure suppression system of a boiling water reactor, causes pressure oscillations in the pool and, as a consequence, corresponding vibrations of the surrounding walls. However, as a feed back, also the structural deformations of the walls have a considerable influence on the pressure fields in the water pool. Therefore, a theoretical investigation of the dynamics of the pressure suppression system cannot be subdivided in a separate analysis of the fluid behaviour, followed by calculations of the structural response. Rather an analysis taking into account the fluid structural coupling must be carried through.

Often this is achieved by a step-by-step technique, where in the simplest case for small time steps either the pressures or the accelerations at the fluid-structural interface are extrapolated, separate codes for fluid and structural dynamics check whether the extrapolated values satisfy the interface conditions and an iterative improvement is made if necessary. Although in this method standard fluid and structural dynamics codes can be used as moduls and non-linearities can be treated easily, an essential drawback is that often a very large number of time steps is required in order to obtain numerical stability.

Therefore, in this paper a so-called simultaneous coupling technique is used (computer code SING-S), where the unknown structural loadings at the fluid-structural interfaces are eliminated by direct substitution of relations describing the fluid dynamics. Neglecting the fluid compressibility, equations of motion for the coupled problem are obtained which have the same form as the equations of motion for the structural dynamics without coupling. Only the masses are changed. They include now the added mass effect from the fluid. Consequently, for the further treatment of the coupled problem similar methods may be used as in pure structural dynamics. In this work the eigenfrequencies and the corresponding mode shapes (eigenvectors) are determined first. Then the problem is solved by modal superposition.

In the applications to the dynamics of the pressure suppression system it is assumed that the excitation phenomena due to steam condensation, i.e. the radius of a collapsing steam bubble is given as a function of time. The stiffness of the part of the spherical containment which forms the outer wall of the condensation chamber has been obtained with a semi-analytical approach described in paper B8/3. The flexibilities of the other walls of the condensation chamber are neglected. Appropriate relations describing the threedimensional fluid dynamics of the water pool have been obtained by a boundary integral method described in paper B8/2.

As results eigenfrequencies and vibration modes as well as radial displacements of the spherical containment shell versus time will be presented. It can be seen that the added mass effect of the water pool reduces the eigenfrequencies and influences the vibration modes considerably. Most critical are bending effects of the spherical containment shell in the bottom region of the water pool.

1. Description of the Structural Dynamics by Means of Stiffness and Mass Matrices

The flexible structure is approximately represented by a finite number of points, each of them with one or more degrees of freedom (a maximum of three translational and three rotational degrees of freedom per point). In the wetted region one translational degree of freedom must be normal while the others must be tangential to the fluid structure interface. Furthermore it is assumed that the displacements must be small in comparison to characteristic dimensions and that the structural material must be linear elastic.

For the degrees of freedom in the wetted region and normal to the fluid structure interface

- the displacements are described by the components of the vector $\bar{s}^1$
- and the corresponding loads per unit area of the wetted structural parts are described by the components of the vector $\bar{q}^1$.

For all other degrees of freedom

- the displacements are described by the components of the vector $\bar{s}^2$
- and the corresponding loads by the components of the vector $\bar{q}^2$.

With $\ddot{\bar{s}}^1$ and $\ddot{\bar{s}}^2$ as the second time derivatives of $\bar{s}^1$ and $\bar{s}^2$ the structural dynamics is governed by the equation of motion:

$$\begin{aligned} \bar{\bar{A}}^{11}\bar{s}^1 + \bar{\bar{A}}^{12}\bar{s}^2 + \bar{\bar{B}}^{11}\ddot{\bar{s}}^1 + \bar{\bar{B}}^{12}\ddot{\bar{s}}^2 &= \bar{q}^1 \\ \bar{\bar{A}}^{21}\bar{s}^1 + \bar{\bar{A}}^{22}\bar{s}^2 + \bar{\bar{B}}^{21}\ddot{\bar{s}}^1 + \bar{\bar{B}}^{22}\ddot{\bar{s}}^2 &= \bar{q}^2 \end{aligned} \tag{1.1}$$

$\bar{\bar{A}}^{11},\ldots,\bar{\bar{B}}^{22}$ are sub-matrices of the stiffness matrix $\bar{\bar{A}}$ and the mass matrix $\bar{\bar{B}}$:

$$\bar{\bar{A}} = \begin{bmatrix} \bar{\bar{A}}^{11} & \bar{\bar{A}}^{12} \\ \bar{\bar{A}}^{21} & \bar{\bar{A}}^{22} \end{bmatrix} \qquad \bar{\bar{B}} = \begin{bmatrix} \bar{\bar{B}}^{11} & \bar{\bar{B}}^{12} \\ \bar{\bar{B}}^{21} & \bar{\bar{B}}^{22} \end{bmatrix}$$

Both, $\bar{\bar{A}}$ and $\bar{\bar{B}}$, may be determined by standard methods of structural mechanics, such as finite element methods.

In the application to the spherical containment of a boiling water reactor with pressure suppression system, however, the description by standard finite elements was found to be inadequate. Therefore the semi-analytical method SPHERE described in paper B8/3 was used in order to calculate the stiffness matrix $\bar{\bar{A}}$.

With appropriate initial conditions, eqs. 1.1 would provide unique solutions for the structural displacements $\bar{s}^1$ and $\bar{s}^2$, if the loads $\bar{q}^1$ and $\bar{q}^2$ were known. In coupled fluid structural dynamics problems, however, the load $\bar{q}^1$ in the wetted region is not explicitely given. Rather it stems from the fluid dynamics which depends on the motion of the fluid boundary being identical with the structural displacements $\bar{s}^1$ in the wetted region. With other words, due to the fluid dynamics, there is a feedback from the displacements $\bar{s}^1$ on the loads $\bar{q}^1$. Therefore relations from the fluid dynamics will be used in order to eliminate the unknown loads $\bar{q}^1$. The loads $\bar{q}^2$ in the unwetted region are assumed to be given.

2. Description of the Fluid Dynamics by a Boundary Integral Method with Dipole Panels

During elimination of the unknown fluid dynamic loads $\bar{q}^1$ it is important that, besides the displacements $\bar{s}^1$ of the wetted structure, no additional unknowns have to be introduced. This means, that the three-dimensional fluid dynamics problem must be reduced to a boundary problem, where at the interface with the structure the normal displacements of the boundary agree with the structural displacements $\bar{s}^1$ and at the other boundaries either the pressures or the displacements are given. For this purpose the boundary integral equation method described in paper B8/2 will be used. Therefore, the fluid boundary is approximately represented by a finite number of points which usually are the centers of the dipole panels forming the fluid boundary.

For points belonging to boundary regions wetting the flexible structure

- the normal accelerations are described by the components of the vector $\bar{a}^1$
- and the corresponding pressures by the components of the vector $\bar{p}^1$.

Furthermore, location and sequence of these boundary points must be the same as location and sequence of the points representing the wetted structure.

For points representing boundary regions with prescribed normal accelerations

- these accelerations are described by the components of the vector $\bar{a}^2$.

A special case are rigid boundaries (rigid walls) with vanishing normal accelerations.

For points representing boundary regions with prescribed pressures

- these pressures are described by the components of the vector $\bar{p}^3$.

Here, a special case are free fluid surfaces with constant pressures.

If the boundary points introduced above are interpreted as the points j in paper B8/2, the boundary integral equation method yields the following relations:

$$\bar{\bar{C}}^1\bar{X} = \bar{a}^1 \qquad \bar{\bar{C}}^2\bar{X} = \bar{a}^2$$
$$\bar{\bar{D}}^1\bar{X} = \bar{p}^1 \qquad \bar{\bar{D}}^3\bar{X} = \bar{p}^3 \tag{2.1}$$

Eqs. 2.1 correspond with eqs. 3.4 in paper B8/2. The elements of the matrixes $\bar{\bar{C}}^1$ and $\bar{\bar{C}}^2$ correspond with C^{ξ}_{jk}, the elements of the matrices $\bar{\bar{D}}^1$ and $\bar{\bar{D}}^3$ correspond with C^{p}_{jk}. The components of the vector $\bar{X}$ are the unknowns X_k.

According to paper B8/2 the number of components of the vector $\bar{X}$ must agree with the number of boundary points j. Consequently the matrix

$$\bar{\bar{F}} = \begin{bmatrix} \bar{\bar{D}}^1 \\ \bar{\bar{C}}^2 \\ \bar{\bar{D}}^3 \end{bmatrix}$$

is quadratic. It will be inverted and then splitted

$$\begin{bmatrix} \bar{\bar{D}}^{*1} & \bar{\bar{C}}^{*2} & \bar{\bar{D}}^{*3} \end{bmatrix} = (\bar{\bar{F}})^{-1}$$

in such a way that the number of columns of $\bar{\bar{D}}^{*1}$, $\bar{\bar{C}}^{*2}$ and $\bar{\bar{D}}^{*3}$ agree with the number of rows of $\bar{\bar{D}}^1$, $\bar{\bar{C}}^2$ and $\bar{\bar{D}}^3$, respectively. Now the following equation for the unknown vector $\bar{X}$ can be obtained from the second, third and fourth equation of 2.1:

$$\bar{X} = \bar{\bar{D}}^{*1}\,\bar{p}^1 + \bar{\bar{C}}^{*2}\,\bar{a}^2 + \bar{\bar{D}}^{*3}\,\bar{p}^3 \tag{2.2}$$

With this result, vector $\bar{X}$ can be eliminated in the first equation of 2.1:

$$\bar{a}^1 - (\bar{\bar{C}}^1\bar{\bar{D}}^{*1})\bar{p}^1 = \bar{\bar{C}}^1(\bar{\bar{C}}^{*2}\bar{a}^2 + \bar{\bar{D}}^{*3}\bar{p}^3) \tag{2.3}$$

Eq. 2.3 is the required relationship from the fluid dynamics. The unknown vectors $\bar{a}^1$ and $\bar{p}^1$ describe the normal accelerations and pressures at those parts of the fluid boundary which wet the flexible structure. All other quantities are known. Accelerations and pressures inside the fluid region do not occur, i.e., the three-dimensional fluid dynamics problem has been reduced to a boundary problem.

3. Coupling between Fluid Dynamics and Structural Dynamics

Provided that the positive normal accelerations of the fluid boundary are directed inside the fluid region, the equilibrium and compatibility conditions at the fluid structural interface read:

$$\bar{p}^1 = -\bar{q}^1 \qquad \text{and} \qquad \bar{a}^1 = \ddot{\bar{s}}^1 \tag{3.1}$$

Here, analogous to free fluid surfaces in paper B8/2, it has been assumed that the dynamic pressure at the fluid-structure interface can be neglected in comparison with characteristic pressures.

Now, in fluid dynamics equation 2.3 vectors $\bar{a}^1$ and $\bar{p}^1$ can be replaced by the structural accelerations $\ddot{\bar{s}}$ and the loads $\bar{q}^1$. Then, with the resulting relation, the unknown loads $\bar{q}^1$ in the structural dynamics equations 1.1 can be eliminated. Finally, the following equations of motion for the coupled problem are obtained:

$$\begin{aligned} (\bar{\bar{C}}^1\bar{\bar{D}}^{*1}\bar{\bar{A}}^{11})\bar{s}^1 + (\bar{\bar{C}}^1\bar{\bar{D}}^{*1}\bar{\bar{A}}^{12})\bar{s}^2 + (\bar{\bar{C}}^1\bar{\bar{D}}^{*1}\bar{\bar{B}}^{11}+\bar{\bar{E}})\ddot{\bar{s}}^1 + (\bar{\bar{C}}^1\bar{\bar{D}}^{*1}\bar{\bar{B}}^{12})\ddot{\bar{s}}^2 &= \bar{\bar{C}}^1(\bar{\bar{C}}^{*2}\bar{a}^2+\bar{\bar{D}}^{*3}\bar{p}^3) \\ \bar{\bar{A}}^{21}\bar{s}^1 + \bar{\bar{A}}^{22}\bar{s}^2 + \bar{\bar{B}}^{21}\ddot{\bar{s}}^1 + \bar{\bar{B}}^{22}\ddot{\bar{s}}^2 &= \bar{q}^2 \end{aligned} \tag{3.2}$$

$\bar{\bar{E}}$ is the identity matrix.

Eqs. 3.2 are of the same type as the structural dynamics equations and the matrices governing the coupled problem have the same dimensions as the stiffness and mass matrices for the structural dynamics. Only the symmetry of the matrices disappears for coupled problems. Together with appropriate initial conditions eq. 3.2 yields unique solutions for the structural displacements $\bar{s}^1$ and $\bar{s}^2$. Now the other unknown quantities describing the coupled problem may easily be obtained. Using solutions $\bar{s}^1$ and $\bar{s}^2$, the first of eqs. 1.1 yields the load $\bar{q}^1$ of the wetted structural parts. Then, with eqs. 3.1 and 2.2, the vector $\bar{X}$ can be calculated, which allows for the determination of pressure and accelerations at any fluid point by means of eqs. 3.3 in paper B8/2.

Based on eqs. 3.2 the so-called added mass matrix can be found. Left-hand multiplication with $(\bar{\bar{C}}^1\bar{\bar{D}}^{*1})^{-1}$ of the first equation in 3.2 yields

$$\bar{\bar{A}}^{11}\bar{s}^1 + \bar{\bar{A}}^{12}\bar{s}^2 + \left[\bar{\bar{B}}^{11} + (\bar{\bar{C}}^1\bar{\bar{D}}^{*1})^{-1}\right]\ddot{\bar{s}}^1 + \bar{\bar{B}}^{12}\ddot{\bar{s}}^2 = \ldots$$

Comparison with the first equation in 1.1. shows that coupling with fluid dynamics can be described, if the structural mass matrix $\bar{\bar{B}}^{11}$ is increased by the added mass matrix

$$(\bar{\bar{C}}^1\bar{\bar{D}}^{*1})^{-1}.$$

While $\bar{\bar{B}}^{11}$ is basicly a diagonal matrix, the added mass matrix is fully populated. The transfer of prescribed forces or accelerations from the fluid to the structure as described by the right-hand sides of eq. 3.2 is not included in the added mass concept.

In the above procedures, both fluid and structural dynamic conditions are satisfied simultaneously. Therefore this method is sometimes referred to as "simultaneous coupling". Similar methods, however, with applications to the completely different field of space vehicle design, were used by Guyan, Ujihara and Welch [1], Khabbaz [2], and Chung and Rush [3]. For the fluid dynamics part, also Khabbaz takes advantage of a singularity method which is similar to that used in this paper. Basic differences concern the types of singularities (sources and dipoles, respectively) and their arrangement (panels). A similar way of solution for fluid-structural coupling in a blowdown suppression system was proposed by Schweiger and Mayr [4]. To describe the inertia effect of the fluid, they also use a singularity method, but modeling of the geometry is not carried through in detail. A large number of contributions to the dynamics of pressure suppression systems was presented at Session B5 and J6 of the 4th Int. Conf. on Structural Mechanics in Reactor Technology held in San Francisco in August 1977. More recently, also Di Maggio, Bleich and McCormick have dealt with the fluid-structure coupling aspects of pressure suppression systems, however, under axisymmetric conditions [5]. Class tackles the same problem [6]. Finally, some preliminary results have been published by the author [7, 8]. A more detailed paper on the treatment of coupled problems in fluid-structural dynamics is under preparation [9].

More common than the "simultaneous coupling" is the "step-by-step-coupling", where equilibrium between the fluid dynamic pressure and the structural load or compatibility between fluid flow and structural deformations is restored approximately only after discrete time steps. However, for the pressure suppression system with an almost incompressible fluid contained in relatively flexible thin walled shells, step-by-step-coupling would require an extremely large number of time steps in order to avoid numerical instabilities.

4. Discussion of the Results

Now, eqs. 3.2 for the coupled problem can be solved in similar ways as for the structural dynamics alone. Here, the solution has been obtained by modal superposition. Details are described in [9]. For the numerical calculations the computer code SING-S has been written.

The eigenfrequencies of a 120°-section of the spherical containment shell coupled with the water pool were computed as 12.49, 14.90, 17.87, 20.45, 21.50, 22.30, 22.79, 23.62,...Hz. Some corresponding mode shapes are shown in fig. 1. At the lower three rows of panels the shell was assumed to be rigid. At the upper five rows, however, the shell is flexible, but the radial displacements vanish almost. This is due to the absence of fluid coupling in this region. The eigenfrequencies without fluid coupling are 51.97, 56.01, 58.82, 59.10, 59.50, 60.05, 60.23, 60.87,...Hz. The significant differences confirm the necessity for coupled analyses.

For the transient analysis three simultaneously collapsing steam bubbles at the circumferential angels of 0°, 120°, 240° were assumed as shown in fig. 2. The radius decrease of the bubbles versus time was prescribed according to Class [10] with a total collapse time of 0.05 sec. The initial bubble radius of 0.5 m represents just a reference value. Results for other bubble sizes can be easily obtained, taking into account that the system response is proportional to the bubble volume.

Some of the calculated radial shell displacements versus time are shown in fig. 3. Again, in the unwetted region the deformations are very small. The maximum displacements occur close to the bottom region of the pool. Consequently relatively high bending stresses are expected in the lower part of the shell. It is interesting to note that with respect to the circumferential distribution the largest displacements are observed between two collapsing steam bubbles. At these locations the first superposition of the disturbances from both bubbles occur.

With SING-S many calculations for different parameters have been performed, indicating, that the deformations grow with decreasing collapse time. The existence of additional, not collapsed bubbles reduces the deformations. Details are presented in [11].

References

/_1_7 GUYAN, R.J., UJIHARA, B.H., WELCH, P.W., "Hydroelastic of Axisymmetric Systems by a Finite Element Method", Proc. Conf. Matrix Methods in Structural Dynamics, Dec. 69

/_2_7 KHABBAZ, G.R., "Dynamic Behaviour of Liquids in Elastic Tanks", AIAA Journ., 9, Oct. 71

/_3_7 CHUNG, T.J., RUSH, R.H., "Dynamically Coupled Motion of Surface-Fluid-Shell Systems", Journ. Appl. Mech., Sept. 76

/_4_7 SCHWEIGER, W., MAYR, M., "On the Solution of a Certain Fluid-Structure-Coupling Problem using the Boundary-Integral Equation Method", Mech. Res. Comm. 3, 1976

/_5_7 DI MAGGIO, F.L., BLEICH, H.H., MC CORMICK, J.M., "Dynamic Response of a Containment Vessel to Fluid Pressure Pulses", Computers & Structures, 8, 1978

/_6_7 CLASS, G., "Untersuchung der bei der Dampfkondensation im Druckabbausystem von Siedewasserreaktoren entstehenden Wandbelastungen", Kernforschungsz. Karlsruhe, KFK 2549

/_7_7 GÖLLER, B., HAILFINGER, G., KRIEG, R., "Vibrations of the Pressure Suppression System of a Boiling Water Reactor", Int. Conf. Vibration in Nucl. Plant, May 78, Keswick, U.K.

/_8_7 KRIEG, R., Coupled Problems in Transient Fluid and Structural Dynamics in Nuclear Engineering, Part I: Safety Problems Solved by Flow Singularity Methods", Appl. Math. Modelling, 2, 1978

/_9_7 KRIEG, R., GÖLLER, B., HAILFINGER, G., "Transient, Three-dimensional Potential Flow Problems and Dynamic Response of the Surrounding Structures, Part II: Simultaneous Coupling between Fluid and Structural Dynamics", to be published in Journ. of Comp. Physics

/_10_7 CLASS, G., "Theoretische Untersuchung der Druckpulsentstehung bei der Dampfkondensation im Druckabbausystem von Siedewasserreaktoren-Rechenprogramm KONDAS". KFK 2487

/_11_7 HAILFINGER, G., KRIEG, R., "Schwingungen eines kugelförmigen Siedewasserreaktor-Containments bei vorgegebenen Dampfblasenkollapsen im Druckunterdrückungssystem. Gekoppelte fluid-strukturdynamische Rechnungen für einen 120°-Ausschnitt mit SING-S." Kernforschg. Karlsruhe, KFK 2779B, to be published

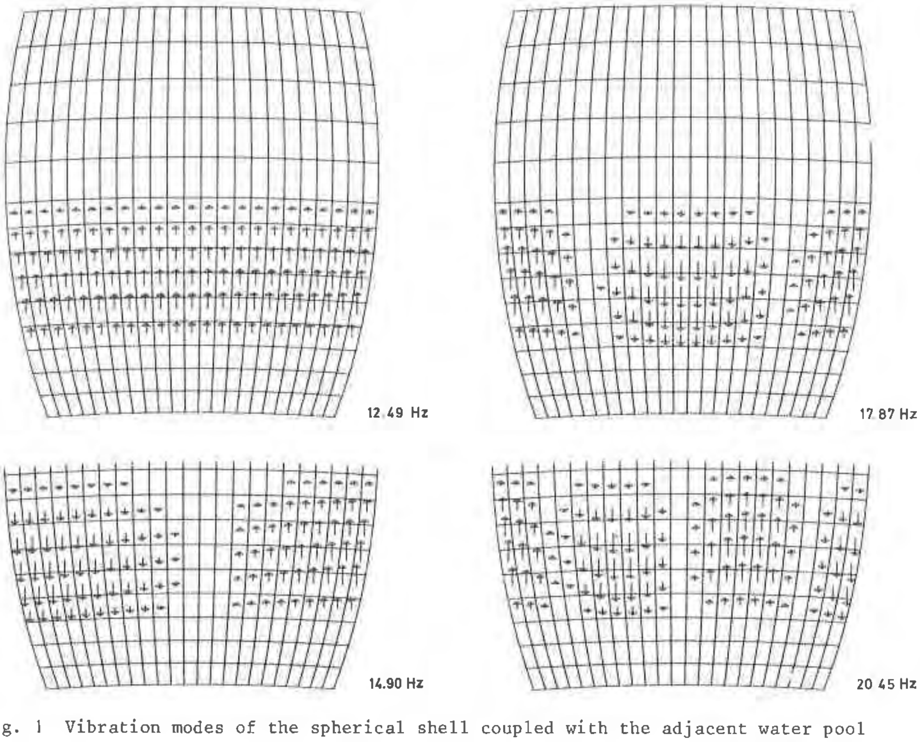

Fig. 1 Vibration modes of the spherical shell coupled with the adjacent water pool

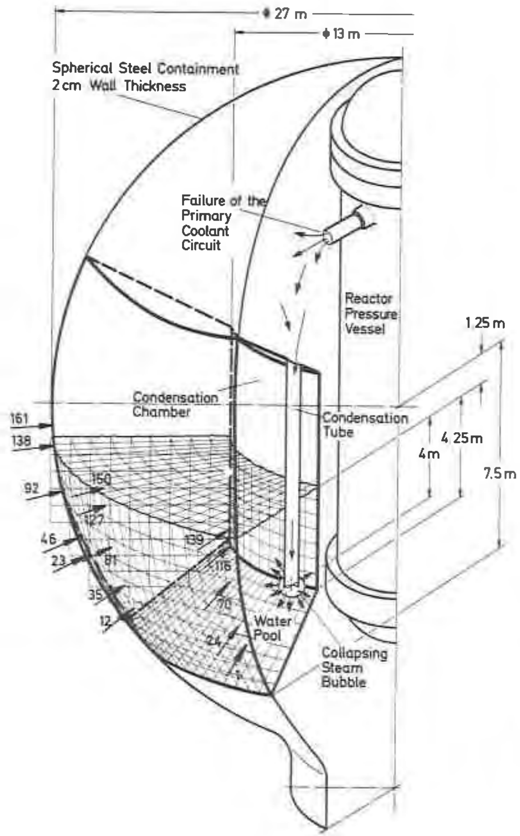

Fig. 2 60^{o}-section of a spherical containment with pressure suppression system

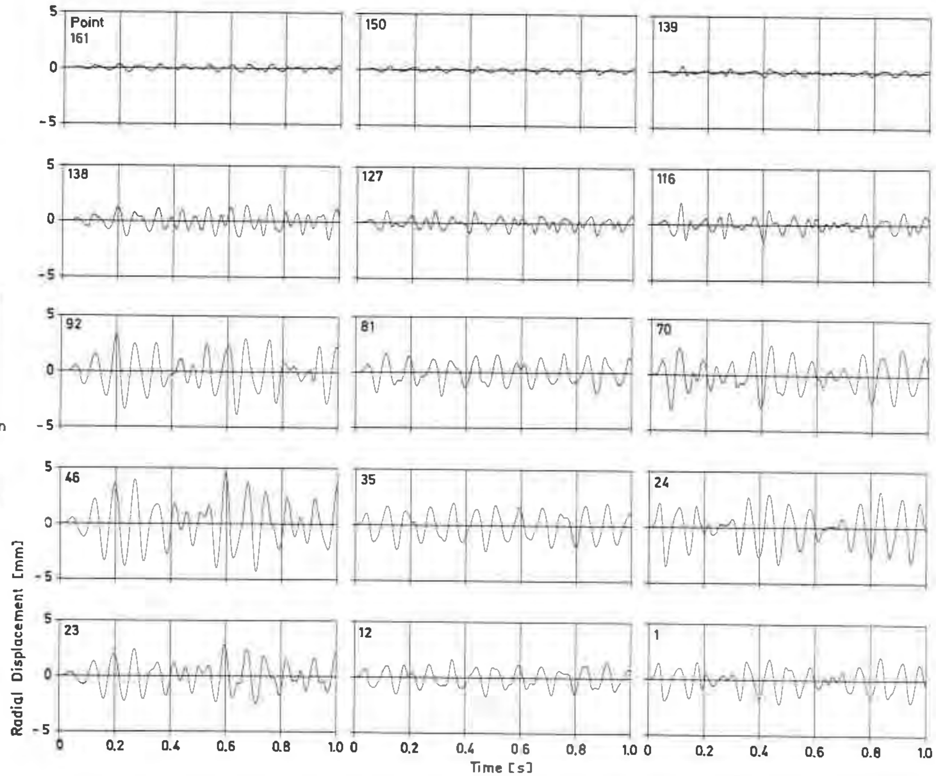

Fig. 3 Radial displacements versus time for some characteristic points of the spherical containment shell.
Numbers and locations of the characteristic points are shown in fig. 2