

Response Variability and Reliability of Systems with Uncertain Material Property

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1. INTRODUCTION

Most advanced and complex structural systems such as nuclear power plants require the well-sophisticated method to ensure structural integrity. Reliability analysis is one of the useful and essential tools and has been extended to even system stochasticity problems where system parameters such as mechanical properties, geometry and boundary conditions are uncertain.

Hisada and Nakagiri (1985), based on the discretization of spatial randomness of a mechanical property, combines the finite element scheme with the stochastic field theory to establish the stochastic finite element method (SFEM) and showed many practical applications by using this method. Bucher and Shinozuka (1988) idealize the spatially varying bending flexibility of beams as one-dimensional, Gaussian, statistically homogeneous stochastic field and derive the analytical solution without employing such discretization.

Once the stochastic characteristics of the system response are evaluated, the problems are then incorporated with the reliability concept in which a limit state describes state of excursion of the stochastic response. A reliability of the deterministic system subjected to random dynamic loading has been well developed as a first excursion theory. Although the reliability analyses of systems with uncertain mechanical properties have been reported, only a few locations are selected to be examined in most of the reliability analyses. Thus, the computed reliability is not the one as a structural member, but only the one at the selected locations. Recent study on the system stochasticity problems indicates that the response also constitute its own stochastic field particularly when the spatially varying mechanical property is assumed to be a stochastic field. So, needed is an advanced reliability analysis that can treat the response as a stochastic field.

The present paper focuses on the response variability and reliability of simple stochastic systems and introduces the concept of "a reliability as a structural member". A linear beam with spatially varying bending flexibility is selected as a simple structural system. The loading condition is considered deterministic and static. The resulting response becomes a stochastic field which is a one-dimensional, non-Gaussian and statistically inhomogeneous stochastic field. Then, "the reliability as a member" is computed by means of the first excursion theory which can treat the inhomogeneity of the response field. As numerical examples, two kinds of beams; a canti-lever beam and a both-end fixed beam, are analyzed and the results will be compared with those from the direct Monte Carlo simulation method.

2. RESPONSE OF STOCHASTIC BEAMS

2.1 Stochastic representation of flexural rigidity

A problem to be analyzed is to evaluate the response statistics of a beam whose bending flexibility is expressed in terms of the one-dimensional, statistically homogeneous, Gaussian stochastic field.

$$1/EI(x) = 1/EI_0 \{1 + f(x)\} \quad (1)$$

where $1/EI_0$ is a mean bending flexibility, and $f(x)$ is a zero mean one-dimensional stochastic field whose auto-correlation function $R_{ff}(\xi)$ is assumed to be given.

2.2 Stochastic evaluation of response

The beam is considered to be subjected to a deterministically distributed load $p(x)$. If the beam is statically indeterminate, the stress as well as the deflection responses become stochastic. The beam with the order of N static indeterminacy has N static indeterminate forces B_k . Therefore, the deflection and deflection angle responses $u(x)$ can be written in the following (Bucher and Shinozuka 1988).

$$u(x) = u_0(x) + \sum_{k=1}^N B_k u_k(x) \quad (2)$$

where $u_0(x)$ is a response vector of the associated statically determinate system subjected to $p(x)$, and $u_k(x)$ is the response vector of the associated statically determinate system subjected to k -th statically indeterminate unit force. Note that $u_i(x)$ ($i = 0$ to N) become the Gaussian stochastic fields, and can easily be evaluated from the following.

$$u_i(x) = \begin{Bmatrix} w_i(x) \\ \theta_i(x) \end{Bmatrix} = \int_0^l \frac{1}{EI} M_i(t) h(x, t) dt \quad (3)$$

where $M_i(t)$ is the moment response due to B_i , and $h(x, t)$ is a Green function vector of the associated statically determinate system. The statically indeterminate force must satisfy the following boundary conditions.

$$\begin{Bmatrix} \delta_{10} \\ \cdot \\ \delta_{N0} \end{Bmatrix} + \begin{bmatrix} \delta_{11} & \cdot & \delta_{1N} \\ \cdot & \cdot & \cdot \\ \delta_{N1} & \cdot & \delta_{NN} \end{bmatrix} \begin{Bmatrix} B_1 \\ \cdot \\ B_N \end{Bmatrix} = 0 \quad (4)$$

where δ_{ik} ($k \neq 0$) is the i -th boundary deflection component of the associated statically determinate system subjected to the unit B_k at x_k , and is expressed in terms of

$$\delta_{ik} = \int_0^l \frac{1}{EI} h_i(x_i, t) h_k(x_k, t) dt, \quad \delta_{i0} = \int_0^l \frac{1}{EI} M_0(t) h_i(x_i, t) dt \quad (5)$$

where $h_i(x_i, t)$ are the Green functions of the associated statically determinate system. In Eq. (4), δ_{i0} is the i -th deflection component of the associated statically determinate system subjected to $p(x)$, and $M_0(t)$ is the moment distribution of the associated statically determinate system subjected to $p(x)$.

Note that the response field describing Eq. (2) is not Gaussian since B_k are evaluated through the matrix inversion of Eq. (4). This inevitably leads to the approximation of the response.

2.3 First-order perturbation

As seen in Eq. (2), $\mathbf{u}(x)$ is expressed in terms of the nonlinear transformation of several Gaussian random variables and stochastic fields, e.g. $\mathbf{u}_0(x)$, $\mathbf{u}(x)$ and δ_{ik} . So, the first-order perturbation approximation is utilized as in the following.

$$\mathbf{u}(x) \approx \mathbf{u}(x)|_{X_i=\langle X_i \rangle} + \sum_{i=1}^m \frac{\partial \mathbf{u}(x)}{\partial X_i} \Big|_{X_i=\langle X_i \rangle} \{X_i(x) - \langle X_i(x) \rangle\} \quad (6)$$

where m is the total number of Gaussian random variables and stochastic fields in Eq. (2), and $X_i(x)$ are involved Gaussian random variables.

It then follows that the cross covariance function of the deflection vector $\mathbf{u}(x)$ can easily be evaluated.

$$C_{\mathbf{u}\mathbf{u}}(x, y) \approx \sum_{i=1}^m \sum_{j=1}^m \frac{\partial \mathbf{u}(x)}{\partial X_i} \Big|_{X_i=\langle X_i \rangle} \left\{ \frac{\partial \mathbf{u}(y)}{\partial X_j} \Big|_{X_j=\langle X_j \rangle} \right\}^t \langle \Delta X_i(x) \Delta X_j(y) \rangle \quad (7)$$

2.4 Numerical examples

Now let us demonstrate a simple example, a cantilever beam subjected to the concentrated force P at the free end, as is shown in Fig. 1. We now take the following auto-correlation function of $f(x)$.

$$R_{ff}(\xi) = \sigma_f^2 \left\{ 1 - 3 \left(\xi/b \right)^2 \right\} / \left\{ 1 + \left(\xi/b \right)^2 \right\}^3 \quad (8)$$

where σ_f is the standard deviation of $f(x)$ and is set equal to 0.2, b is a correlation length.

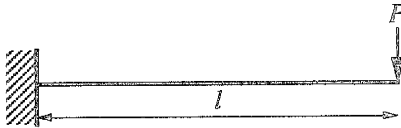


Fig. 1. A cantilever beam

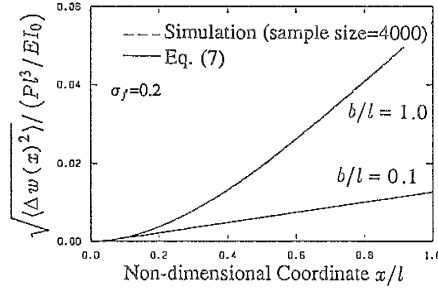


Fig. 2. Standard deviation of deflection

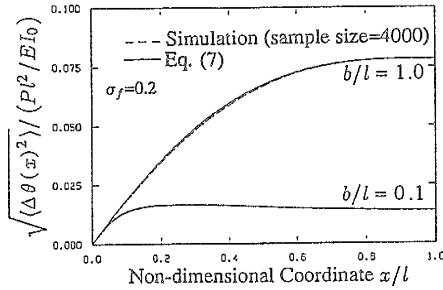


Fig. 3. Standard deviation of deflection angle

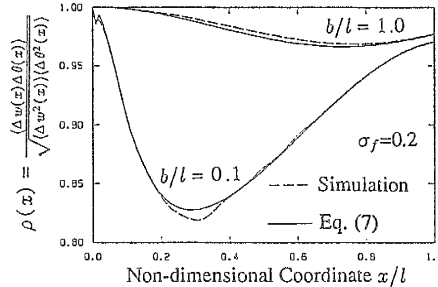


Fig. 4. Correlation coefficient between deflection and deflection angle

Figures 2 to 4 are the second-order statistics associated with the beam deflection and deflection angle. In these figures, the Monte Carlo simulation (Shinozuka, 1987) with 4000 sample functions is also carried out. These comparison shows fairly good agreements between the results from two different methods.

A both end-fixed beam with uniformly distributed load p is now analyzed. This beam system is statically indeterminate, so Eq. (6) is used. Figures 6 to 8 are the spatial plots of the second-order statistics of the responses. The solid lines are computed from Eq. (7), and the dashed lines are from the Monte Carlo simulation with 4000 sample functions based on Eqs. (2) and (4). It can be observed from these figures that the approximation of Eq. (6) is sufficiently valid.

3 RELIABILITY EVALUATION

3.1 First excursion probability

In classical reliability analyses, the stochastic responses at the certain locations of systems are selected as random variables not as stochastic fields, and the associated threshold is probabilistically compared with them to evaluate the reliability. However, if one wants to evaluate the reliability of stochastic systems, stochastic responses at a few selected locations may not be enough to estimate "reliability as a structural member" since the stochastic responses continuously and spatially fluctuate. This indicates the need of evaluating the probability that the response field stays in a safe region within an individual structural member. This is indeed well known as the first excursion problem. So, the following probability that the deflection of a single beam member exceeds a specified threshold level within the beam length is considered hereafter.

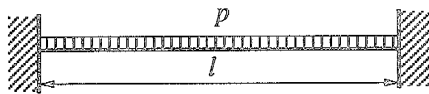


Fig. 5. A both-end fixed beam

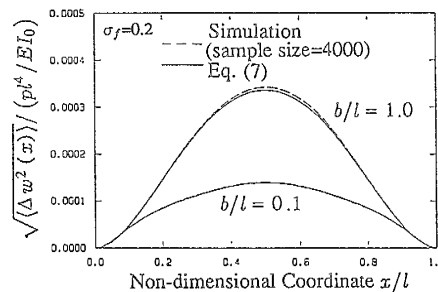


Fig. 6. Standard deviation of deflection

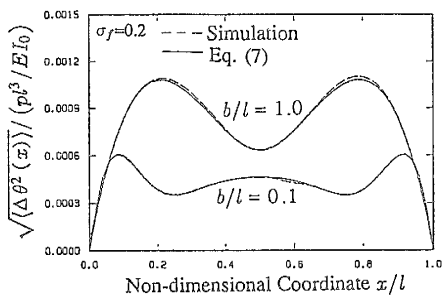


Fig. 7. Standard deviation of deflection angle

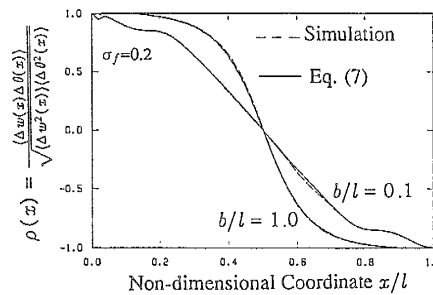


Fig. 8. Correlation coefficient between deflection and deflection angle

$$P_f(l, \alpha) = P \left\{ \max_{0 \leq x \leq l} \{w(x) - \langle w(x) \rangle\} > \alpha \right\} \quad (9)$$

where $w(x)$ implies the beam deflection and α is a positive threshold.

As seen in Eq. (2), the deflection constitutes a stochastic field which is neither Gaussian nor statistically homogeneous. Therefore, the classical first excursion theory cannot directly be applied. In this study, Eq. (6), which is the first-order approximation consisting of the linear combination of Gaussian stochastic fields, is used instead of the exact expression of Eq. (2).

For the evaluation of the first excursion probability of the Gaussian, nonstationary process, Shinozuka's upper and lower bounds (1964, 1968) are used since these bounds require only the second-order statistics of the response. The upper bound is now estimated as follows.

$$P_u(l, \alpha) = \frac{1}{2\pi} \int_0^l \frac{\sigma_\theta}{\sigma_w} \left[\sqrt{1 - \rho^2} \exp \left\{ -\frac{1}{2(1 - \rho^2)} \left(\frac{\alpha}{\sigma_w} \right)^2 \right\} + H(\rho) \sqrt{2\pi\rho} \frac{\alpha}{\sigma_w} \exp \left\{ -\frac{1}{2} \left(\frac{\alpha}{\sigma_w} \right)^2 \right\} \right] dx \quad (10)$$

where σ_w , σ_θ and ρ are respectively the standard deviation of the deflection, deflection angle and the correlation coefficient function between these two responses, and $H(\rho)$ is a Heaviside unit step function.

On the other hand, the lower bound is

$$P_l(l, \alpha) = 1 - \Phi(\alpha/\sigma_w^*) \quad (11)$$

where σ_w^* is the maximum standard deviation within the beam length, and Φ is a standard Gaussian probability distribution function.

The above two bounds imply that the upper bound takes into account the excursion of the response at every location of the beam, while the lower bound considers only one location where the standard deviation of the response takes the maximum value.

3.2 Numerical examples

Figures 9 and 10 are the probability of excursion versus the threshold level α for the canti-lever beam and for the both end-fixed beam, respectively. The small circles imply the results from the Monte Carlo simulation method, and the dashed lines represent the upper and lower bounds given in Eqs. (10) and (11). The canti-lever beam subjected to the concentrated force at the free end is statically determinate, so the deflection response is linear with respect to $f(x)$. This exactly means the response constitutes the Gaussian inhomogeneous stochastic field. From Fig. 9, the upper and lower bounds coincide each other. This means that when the reliability of the canti-lever beam subjected to the concentrated force at the free end is discussed, only the maximum response, which is equal to that of the free end, is enough. While the reliability of the both end-fixed beam subjected to the uniform load is considered, the lower bound gives the unconservative results, compared with the results from the Monte Carlo simulation method. Detail examination of Fig. 10 shows that the simulation result gets closer to the upper bound in the lower threshold level. This states that the reliability as a structural member is needed rather than that of a selected location.

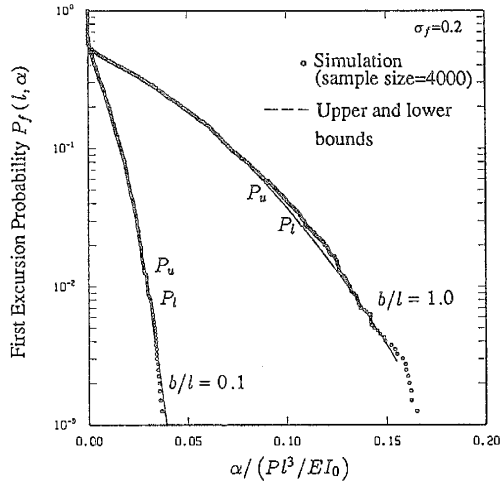


Fig. 9. First excursion probability of deflection (a canti-lever beam)

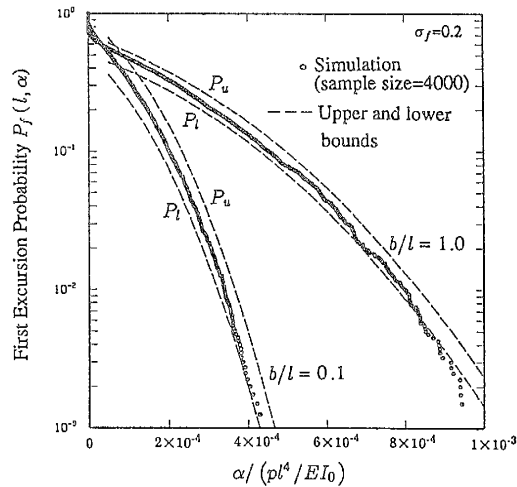


Fig. 10. First excursion probability of deflection (a both-end fixed beam)

4 CONCLUSION

This study clearly shows that when the spatially varying bending flexibility is assumed to be a stochastic field, the beam response also constitutes the inhomogeneous stochastic field, and "a reliability as a member" can exactly be evaluated by means of the classical first excursion theory. Although this study is very preliminary, it can help us to understand the response behavior of stochastic systems and also can give us a useful insight to estimating the reliability as a structural member.

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