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Displacement estimates of pipe elbows prior to plastic collapse loads

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ABSTRACT: The classic limit analysis gives only information about the limit load (or limit multiplier) and the plastic collapse mechanism. Based in the Zarka's method and using the finite element method and mathematical programming it is possible to obtain not only the limit load but also the estimated elastoplastic displacement. This circumstance is very useful because construction codes require limitation of the realistic elastoplastic displacements. The present work is dealing with such calculations and the structure of pipes and elbows are considered. The comparison with the results obtained by classical limit analysis is illustrated by some examples.

1. INTRODUCTION

The direct determination of structural carrying load capacity by mean of classical limit and shakedown formulation do not provide directly information about elastoplastic displacements associated to the limit load.

In view to satisfy construction codes, it's necessary to obtain an estimation of displacements that has to be bounded for the structure in-service. On the other hand Zarka and Casier [8] have introduced a simplified method with a non-vanishing kinematic hardening modulus that allows to compute plastic strains and displacements at the limit state (Buff [2]). This work is dealing with the estimation of the structure's displacements of elbows using finite element method and nonlinear mathematical programming technique. The plastic limit load corresponding to rigid perfectly plastic material is simulated by using a very small linear kinematic hardening coefficient.

One-dimensional finite elements of displacement type, including warping and ovalisation effects, are performed(Militello and Huespe [4]).

2. RESIDUAL AND BACK-STRESS RELATION

Let us consider a deformable solid of volume V constituted by a perfectly elastic material. Suppose that body undergo a initial deformation E^I. Defining $\bar{E}$ as the Hook's matrix, the purely elastic stress response of structure is given by the following equation:

(1) $$T = T^E = \bar{E}(E - E^I) \quad \text{with} \quad E = E^E + E^I$$

and where E^E are the purely elastic strains and E^I the initial strains respectively.

Suppose now that the body is constituted by an elastoplastic material with a linear kinematic hardening. The following expression is used in the case of infinitesimal deformations:

$$(2) \qquad E = E^E + E^R + E^I = E^e + E^p + E^I$$

where E^R is the residual strain composed by an elastic E^{Re} and plastic E^{Rp} parts, E^e the elastic strain component and E^p the plastic strain component.

The stress response in this case take the following form:

$$(3) \qquad T = T^E + T^R$$

whereT^R is the residual stress tensor.

The Von Mises convex, in the kinematic linear hardening law, take the following normalised form:

$$(4) \qquad F = f(S - X) - 1$$

where $X = \overline{\overline{H}}E^P$ is the hardening parameter and $\overline{\overline{H}}$ the hardening coefficient tensor.

Then, the residual deformation component can be written by:

$$(5) \qquad E^R = E^{Re} + E^P = \overline{\overline{E}}^{-1}T^R + \overline{\overline{H}}^{-1}X.$$

Defining the residual back stress by the relation below:

$$(6) \qquad Y = X - \mathrm{dev}T^R,$$

substituting it in expression (5) and using the modified elasticity matrix $\overline{\overline{E}}_M$ defined by the following equality:

$$(7) \qquad \overline{\overline{E}}_M^{-1}(.) = \overline{\overline{E}}^{-1}(.) + \overline{\overline{H}}^{-1}\mathrm{dev}(.)$$

we obtain the residual stress tensor of an elastoplastic structure in a explicit form:

$$(8) \qquad T^R = \overline{\overline{E}}_M\left(E^R - \overline{\overline{H}}^{-1}Y\right)$$

Comparing equation (1) and (8) we can obtain the residual stress response of an elastoplastic structure using the same method as the linear case when we take into account the equivalencies between u and u^R,T^E and T^R, $\overline{\overline{E}}$ and $\overline{\overline{E}}_M$, E and E^R, E^I and $\overline{\overline{H}}^{-1}Y$.

The residual displacements from the back stress tensor are obtained in the following:

1. discretizing the structure by means of finite element and using the fact that the residual stress tensor satisfy the internal homogeneous equilibrium we have:

(9) $$\sum_e \int_{V^e} B^T \sigma^R dV = 0.$$

2. Using the compatibility between residual strains and residual displacements gives the linear equation system:

(10) $$u^R = K_M^{-1} g \quad \text{with} \quad g = \sum_e \int_{V^e} B^T \overline{E}_M \overline{H}^{-1} \sigma^Y dV^e$$

and where $K_M = \sum_e \int_{V^e} B^T \overline{E}_M B dV^e$ is the stiffness matrix made up from the modified elasticity matrix.

3. PLASTIC CONVEX EVOLUTION CRITERIA IN THE BACK-STRESS SPACE

The next step shows how to obtain an approximation of the back-stress tensor by a minimisation process. Generalising the principle of the maximum work of plastic deformation for linear kinematics hardening material model we can write:

(11) $$\left[(T - X) - \left(T^* - X^*\right)\right] : \dot{E}^P \geq 0$$

where: $F\left(T^* - X^*\right) \leq 0$ and $F(T - X) = 0$.

If we choose the following stress field:

(12) $$T^* = T^E + \overline{T}^R \quad \text{and} \quad T = T^E + T^R$$

where $\overline{T}^R$ is the time independent residual stress field defined in the Melan's theorem, we obtain:

(13) $$\left(\overline{Y} - Y\right) : \dot{E}^p \geq 0, \qquad \overline{Y} = X^* - \text{dev}\overline{T}^R$$

The above expression means that plastic deformation direction $\dot{E}^P$ always points to the interior of plastic criteria convex in the back-stress space. Now, from equation (5) and (6) we can write:

(14) $$\dot{Y} = \dot{X} - \text{dev}\left(ZH^{-1}\dot{X}\right) = \left[\overline{H} - \text{dev}Z(.)\right]\dot{E}^p$$

where Z is an operator that connects residual stress to plastic strain tensors. From the fact that $\overline{H}$ is positive definite and devZ(.) is negative definite, $\dot{Y}$ takes also the direction of the interior of the plastic criteria convex but the normality law is not valid. The evolution of the plastic collapse convex C_0 in the back-stress space is defined by:

(15) $$C_0 = \{Y, F(-Y) \leq 0\} \quad \text{and} \quad C_t = \left\{Y, F\left(S^E(t) - Y\right) \leq 0\right\}$$

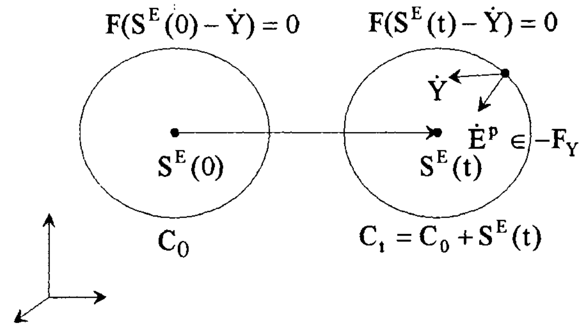

Figure 1: Evolution of plastic criteria convex with radial load in back-stress space.

We can see that the C_t convex can be obtained from C_0 by a rigid convex translation defined by the purely elastic deviatoric stress S^E, function of the applied load.

4. LIMIT LOADS

Here we are interested to obtain back-stress field of the stabilised problem in the case of radial loads. The variation of this loads are function of the same parameter $\bar{\alpha}$:

(16) $$(a,b)=\bar{\alpha}(a^0,b^0)$$

Two methods can be used; an incremental calculation to follow the evolution of back-stress or a direct calculation using an approximated method. The direct calculation supposes that the real back-stress path can be approximated by the projection $Y(t)$ of the initial back-stress state Y_0 over the convex state C_t as presented below:

(17) $$\min_{Y\in C_t}\left[(Y_0-Y):(Y_0-Y)\right]$$

where: $C_t=\left\{Y;\ \ F\left(\bar{\alpha}_1 T^{Eo}(x,t)-Y(x)\right)\le 0\right\}$

To calculate the displacement prior to collapse loads, it is necessary first to establish the level $\bar{\alpha}$ of limit load . This could be done or by a direct method using the Melan or Koiter's theorem or using the Zarka's method with a very little kinematic hardening coefficient to simulate the perfect elastoplastic material (Bohani [1.]).

5. NUMERICAL RESULTS

The following example illustrates the convergence of results obtained in one hand by classical solution of Koiter, using a regularisation of plastic dissipation [3.] and in other hand by Zarka's simplified analysis[5.]. Straight and elbow pipes with or without ovalisation effects are analysed. It appears that the discretization has some importance to obtain convergence in the simplified analysis.

The first example is based on a straight pipe, fixed at one end and subjected to an axial traction at the other end, with the following geometric and material characteristics a/L=0.25, h/a=0.2, $E = 24.100.000 \quad N/mm^2$, $\nu = 0.3$, $\sigma_y = 5. \quad N/mm^2$.

For the numerical solution only 1 element is used with a $3 \times 8 \times 1$ numerical integration order. At the figure 2a we compare the solution of the limit load using the classical Koiter's method with a plastic dissipation regularisation and the Zarka's simplified method. The differences between limit loads are about only 2.4%.

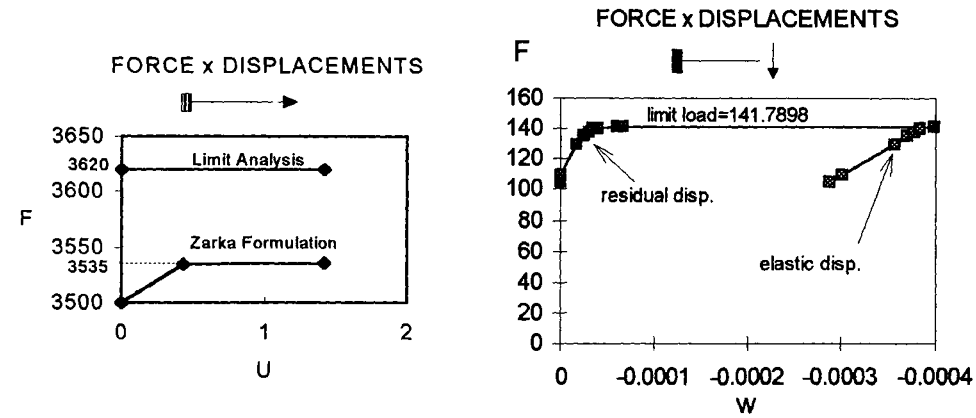

Figure 2: Displacement-load curve for a straight pipe subjected to a) an axial traction and b) a transversal load.

At the figure 2b the evolution of elastic and residual displacements of the same straight pipe but now subjected to a transversal load are presented with the increasing of loads. For the numerical analysis 4 elements and a $3 \times 8 \times 3$ numerical integration points are used. It appears that the structure reaches the plastic collapse load when the load level is about $F = 141.8 \quad N$.

The third example presented is an elbow pipe fixed at one end and subjected to a transversal load at the other one. The structure is characterised by the following geometric and material constants: a/R=0.15, L/a=0.666..., $E = 2.100.000 \quad N/mm^2$, $\nu = 0.3$, $\sigma_y = 500 \quad N/mm^2$.

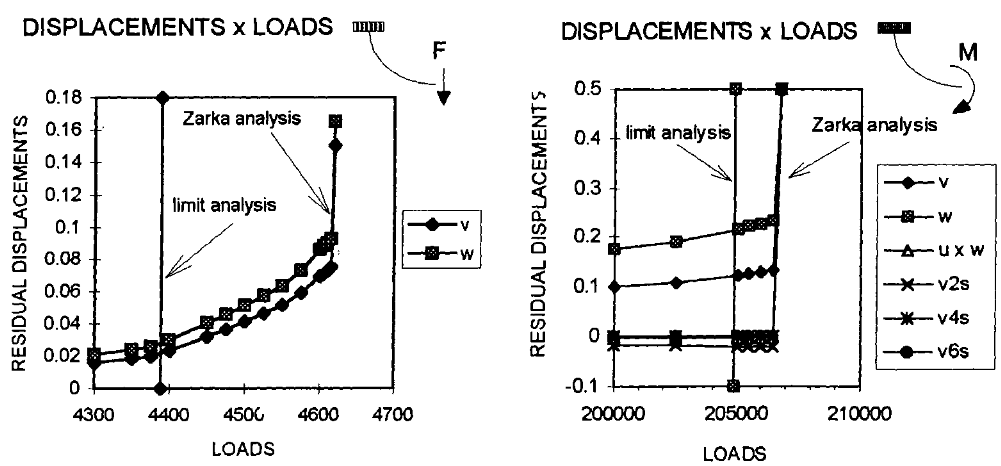

Figure 3: Displacement-load curve for an elbow pipe submitted to a) an in-plane

transversal load and b) in-plane moment with ovalisation effects included.

At the figure 3a both the solution of the limit load using the classical Koiter's method and the Zarka's simplified method are shown. To obtain convergence in numerical calculations 3 elements are used for $F_Z \leq 4575$, 4 elements for $F_Z > 4575$ with a $3\times12\times3$ numerical integration order. The differences between limit loads don't exceed 5,15%.

The fourth example is the same elbow presented above submitted to a in-plane moment and considering the following shell d.o.f.: v_2^s, v_4^s, v_6^s. In this case 5 elements and a $3\times24\times3$ numerical integration order are used. The differences between limit loads obtained from classical Koiter's analysis and Zarka's simplified analysis are presented in figure 3b and are about 5.15%.

6. SUMMARY AND CONCLUSIONS

Zarka's method seems to play a complementary role in the direct calculation of limit loads (radial and variable loads). The classical limit load formulation does not permit to obtain displacement estimation. This function is fulfilled by the simplified Zarka's method without a large amount of cpu time. At the numerical topic, limit loads obtained from simplified analysis are shown very close to ones from direct method. To obtain displacements prior to collapse, we propose to proceed first a classic limit load calculation and then to make a simplified Zarka's calculation.

7. REFERENCES

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