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ROBUSTNESS OF CERTAIN TESTS OF CENSORING OF EXTREME SAMPLE VALUES
II: SOME EXACT RESULTS FOR EXPONENTIAL POPULATIONS*

by

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1. Introduction

In [1] we have given general formulae for evaluating the effect of an incorrect choice of population distribution on certain tests of sample censoring. (For definitions, etc. see [2][3].)

These formulae apply when the form of population distribution is known, but certain parameters θ have to be estimated. The formulae apply when possibly incorrect values θ^* are used for θ .

When θ has to be estimated, there will usually be random variation in θ^* . The overall effect of this can be evaluated by taking expected values of the formulae in [1] with respect to suitable distributions for θ^* .

We shall suppose throughout the paper that there is sampling from a population in which the measured character has an absolutely continuous distribution.

2. General Discussion

As in [1] and earlier papers we suppose r values $X_1 \leq X_2 \leq \dots \leq X_r$ are available and wish to test whether they are a complete random sample, or come from a complete random sample of size $(s_0 + r + s_r)$, from which the s_0 smallest and s_r largest values have been censored. We denote this last hypothesis by H_{s_0, s_r} ; $H_{0,0}$ corresponds to the case of no censoring.

The critical regions of the tests described in [1][2] are all defined in terms of the probability integral transforms

$$(1) \quad Y_1 = \int_{-\infty}^{X_1} f(x|\varrho) dx ; Y_r = \int_{-\infty}^{X_r} f(x|\varrho) dx$$

of the minimum and maximum values X_1, X_r respectively. Here $f(x|\varrho)$ is the density function of X . If, in fact, an incorrect value of ϱ is used - ϱ^* , say - then instead of calculating Y_1, Y_r we actually calculate

$$(2) \quad Y_j^* = \int_{-\infty}^{X_j} f(x|\varrho^*) dx \quad (j = 1, r) .$$

Now provided $f(x|\varrho) > 0$ for all x and ϱ , (1) and (2) define Y_j^* as a unique function $h^*(Y_j)$ of Y_j . So when we use a critical region $w_{(k)}$ defined by

$$w_{(k)}(Y_1^*, Y_r^*) \geq C_\alpha^{(k)}$$

($C_\alpha^{(k)}$ is a constant which would give $100\alpha\%$ significance level if $H_{0,0}$ were valid *and* the correct ϱ were used) we are actually using a critical region $w_{(k)}^*$ defined by

$$(3) \quad w_{(k)}(h^*(Y_1), h^*(Y_r)) \geq C_\alpha^{(k)}$$

in terms of the correct Y_1 and Y_r . The latter have joint density function (under H_{s_0, s_r})

$$(4) \quad \frac{(r+s_0+s_r)!}{s_0!(r-2)!s_r!} y_1^{s_0} (y_r - y_1)^{r-2} y_r^{s_r} \quad (0 \leq y_1 \leq y_r \leq 1)$$

For a given ϱ^* , the power $\beta_{(k)}(s_0, s_r | \varrho^*)$ of the test with critical region $w_{(k)}^*$ is the integral of (4) over the region

$$(5) \quad w_{(k)}(h^*(y_1), h^*(y_r)) \geq C_\alpha^{(k)}.$$

The unconditional, overall power $\beta_{(k)}(s_0, s_r)$ is the expected value, with respect to variation in ϱ^* of $\beta_{(k)}(s_0, s_r | \varrho^*)$. In particular the actual significance level is

$$(6) \quad \beta_{(k)}(0, 0) = r(r-1)E \left[\int_{w_{(k)}^*} \int (y_r - y_1)^{r-2} dy_1 dy_r \right]$$

(ϑ^* , of course appears implicitly in $w_{(k)}^*$.) This will not in general be equal to α , if $C_\alpha^{(k)}$ is calculated in accordance with the formulae in [2]. It will, in particular cases, be possible to choose $C_\alpha^{(k)}$ so that the overall significance level is α , but the necessary formulae will depend on the density function of X , and on that of ϱ^* , while the values in () do not.

As a concrete example we consider the case of normal variation.

In [1] it was shown that if $\varrho = (\xi, \sigma)$ and

$$f(x | \xi, \sigma) = (\sigma\sqrt{2\pi})^{-1} \exp\left\{-\frac{(x - \xi)^2}{2\sigma^2}\right\}$$

then $\beta(s_0, s_r | \xi^*, \sigma^*)$ depends only on

$$\Delta^* = (\xi^* - \xi)/\sigma \quad \text{and} \quad \epsilon^* = \sigma^*/\sigma .$$

If ξ^* and σ^{*2} are calculated from an independent random sample of size N , as the sample mean and variance respectively then

(i) Δ^* and ϵ^* are independent

(ii) Δ^* is distributed $N(0, N^{-1})$ which gives useful practical indications

(iii) ϵ^* is distributed $(N - 1)^{-\frac{1}{2}} \chi_{N-1}$.

This information is sufficient to enable us to calculate

$$(7) \quad \beta(s_0, s_r) = E[\beta(s_0, s_r | \xi^*, \sigma^*)]$$

the expectation being with respect to variation of ξ^* and σ^* .

Although this may be evaluated explicitly using numerical cubature applied to $\beta(s_0, s_r | \xi^*, \sigma^*)$ which is itself quite a complicated function, the calculations are quite heavy even with the assistance of an electronic computer. (It is hoped to provide some numerical tables in the third report in this series.) It is rather easier to use a two-entry table of values of $\beta(s_0, s_r | \xi^*, \sigma^*)$ and (very roughly) average with respect to the distributions of ξ^* and σ^* .

A further possible complication is that it may be necessary to estimate θ from the r values X_1, \dots, X_r actually available, so that θ^* and (Y_1, Y_r) are no longer mutually independent. We do not consider this situation in the present paper, except for a brief discussion in the final

section.

3. Exponential Populations

The main aim of the present paper is to evaluate performance of the three tests described in [2] when applied to a population in which the (unorthodox) X 's have an exponential distribution with expected value θ . We will suppose θ to be estimated as the arithmetic mean of a random sample (independent of the r values to be analysis) of size N , so that:

$$(8) \quad \theta^*/\theta \quad \text{is distributed as} \quad (2N)^{-1} \chi_{2N}^2 .$$

Under these conditions it is possible to get explicit formulae for properties of the tests which are relatively simple (as compared, for example, with the normal case).

This discussion will, it is hoped, help to make clearer the general description in Section 2.

The population density function is $\theta^{-1} \exp(-x/\theta)$ ($\theta, x > 0$) corresponding to a $\frac{1}{2}\theta\chi_2^2$ distribution.

$$\text{We have } Y = \int_0^X \theta^{-1} \exp(-x/\theta) dx = 1 - e^{-X/\theta} \quad (X > 0) .$$

Hence

$$(9) \quad Y^* = 1 - e^{-X/\theta^*} = 1 - (1 - Y)^{\theta/\theta^*} .$$

The critical regions to be used in testing for extreme censoring (1) form below (2) symmetrically or (3) in general, are (see [2]) respectively

$$(10.1) \text{ (Below)} \quad Y_1 \geq C_\alpha^{(1)}$$

$$(10.2) \text{ (Symmetric)} \quad Y_1(1 - Y_r) \geq C_\alpha^{(2)}$$

$$(10.3) \text{ (General)} \quad Y_1 + (1 - Y_r) \geq C_\alpha^{(3)}$$

If an incorrect value, θ^* , is used for θ , the regions actually used are, respectively

$$(11.1) \quad 1 - (1 - Y_1)^{\theta/\theta^*} \geq C_\alpha^{(1)}$$

$$(11.2) \quad \{1 - (1 - Y_1)^{\theta/\theta^*}\} (1 - Y_r)^{\theta/\theta^*} \geq C_\alpha^{(2)}$$

$$(11.3) \quad 1 - (1 - Y_1)^{\theta/\theta^*} + (1 - Y_r)^{\theta/\theta^*} \geq C_\alpha^{(3)}$$

4. Censoring from Below

The inequality (11.1) can also be written

$$(11.1)' \quad Y_1 \geq 1 - (1 - C_\alpha^{(1)})^{\theta^*/\theta} = 1 - (1 - C_\alpha^{(1)})^T$$

where we put

$$(12) \quad T = \theta^*/\theta .$$

Since, given H_{s_0, s_r} the distribution of Y_1 is standard beta with parameters $s_0 + 1, r + s_r$ we have

$$(13) \quad \beta_{(1)}(s_0, s_r | \theta^*) = \Pr[Y_1 \geq 1 - (1 - C_\alpha^{(1)})^T | s_0, s_r] \\ = I_{(1-C_\alpha^{(1)})^T}(r + s_r, s_0 + 1)$$

where $I_p(\alpha, \gamma) = \{B(\alpha, \gamma)\}^{-1} \int_0^p t^{\alpha-1} (1-t)^{\gamma-1} dt$ is the incomplete beta function ratio.

In particular the conditional significance level is

$$(14) \quad \beta_{(1)}(0, 0 | \theta^*) = I_{(1-C_\alpha^{(1)})^T}(r, 1) = (1 - C_\alpha^{(1)})^{rT}.$$

The unconditional significance level is

$$(15) \quad E[\beta_{(1)}(0, 0 | \theta^*)] = E[(1 - C_\alpha^{(1)})^{rT}] \\ = E\left[(1 - C_\alpha^{(1)})^{\frac{1}{2} \chi_{2N}^2 / N} \right] \\ = \{1 - rN^{-1} \log(1 - C_\alpha^{(1)})\}^{-N}.$$

In order to make the significance level equal to α , we must choose $C_\alpha^{(1)}$ ($= C_\alpha^{(1)}(N)$, say) to satisfy

$$\{1 - rN^{-1} \log(1 - C_\alpha^{(1)})\}^{-N} = \alpha$$

i.e.

$$(16) \quad C_\alpha^{(1)}(N) = 1 - \exp\{r^{-1}N(1 - \alpha^{-1/N})\}.$$

If the value $C_{\alpha}^{(1)} = 1 - \alpha^{1/r}$, which is appropriate when θ is known, is used then the overall significance level is actually

$$(17) \quad (1 - N^{-1} \log \alpha)^{-N} .$$

As would be expected this tends to α as N tends to infinity.

The power with respect to $H_{s_0, 0}$ (i.e. actual censoring from below) is

$$(18) \quad \begin{aligned} \beta_{(1)}(s_0, 0) &= E \left[I_{(1-C_{\alpha}^{(1)})^T (r, s_0 + 1)} \right] \\ &= \frac{(r+s_0)!}{(r-1)!s_0!} \sum_{j=0}^{s_0} (-1)^j \binom{s_0}{j} (r+j)^{-1} E \left[(1-C_{\alpha}^{(1)})^{\frac{1}{2}(r+j) \chi_{2N}^2 / i!} \right] \\ &= \frac{(r+s_0)!}{(r-1)!s_0!} \sum_{j=0}^{s_0} (-1)^j \binom{s_0}{j} (r+j)^{-1} \{1 - (r+j)N^{-1} \log(1-C_{\alpha}^{(1)})\}^{-N} . \end{aligned}$$

Inserting the value for $C_{\alpha}^{(1)}$ from (16) (which gives overall significance level equal to α) we obtain

$$(19) \quad \beta_{(1)}(s_0, 0) = \frac{(r+s_0)!}{(r-1)!s_0!} \sum_{j=0}^{s_0} (-1)^j \binom{s_0}{j} (r+j)^{-1} \{1 + (1+jr^{-1})(\alpha^{-1/N} - 1)\}^{-N} .$$

Table 1 gives values of $\beta_{(1)}(s_0, 0)$ for $r = 5(5)25$, $N = 5, 10, 25, 50$ and $s_0 = 2(2)8$ with $\alpha = 0.05$. These are powers when the actual significance level is 0.05. Table 2 gives the actual significance levels and powers obtained if the value $C_{\alpha}^{(1)} = 1 - \alpha^{1/r}$ is used. In this case

$$(20) \quad \beta_{(1)}(s_0, 0) = \frac{(r+s_0)!}{(r-1)!s_0!} \sum_{j=0}^{s_0} (-1)^j \binom{s_0}{j} (r+j)^{-1} \{1 - (1+jr^{-1})N^{-1} \log \alpha\}^{-1}.$$

(Some values of $\beta_{(1)}(s_0, s_r)$ with $s_r \neq 0$ are also given.)

5. Symmetrical and General Censoring

Similar calculations for the tests with critical regions (11.2) and (11.3) are complicated by the fact that both Y_1 and Y_r appear in the inequalities. In these cases we use the fact that the joint density functions of $Z_1 = 1 - Y_1$ and $Z_r = 1 - Y_r$ under H_{s_0, s_r} is

$$(21) \quad \frac{(r+s_0+s_r)!}{s_0!(r-2)!s_r!} (1 - z_1)^{s_0} (z_1 - z_r)^{r-2} z_r^{s_r} \quad (0 \leq z_r \leq z_1 \leq 1)$$

and so the conditional power $\beta_{(k)}(s_0, s_r | \theta^*)$ is obtained by integrating (21) over the regions

$$(22.2) \quad \left[1 - z_1^{\theta/\theta^*} \right] z_r^{\theta/\theta^*} \geq C_\alpha^{(2)} \quad \text{for } k = 2 \quad (\text{symmetrical censoring})$$

or

$$(22.3) \quad 1 - z_1^{\theta/\theta^*} + z_r^{\theta/\theta^*} \geq C_\alpha^{(3)} \quad \text{for } k = 3 \quad (\text{general censoring})$$

The resultant value has in each case, to be averaged over the distribution θ^* (or equivalently of $T = \theta^*/\theta$).

Making the transformations $v_j = z_j^{\theta/\theta^*}$ ($j = 1, 2$) we have

$$(23) \quad \beta_{(k)}(s_0, s_r | \theta^*) = \frac{(r+s_0+s_r)!}{s_0!(r-2)!s_r!} \iint T^2 (1-v_1^T)^{s_0} (v_1^T - v_r^T)^{r-2} v_1^{T-1} v_r^{(s_r+1)T-1} dv_1 dv_r$$

The regions of integration are:

$$(24.2) \quad \text{For } k = 2: (1 - v_1)v_r \geq C_\alpha^{(2)} ; 0 \leq v_r \leq v_1 \leq 1$$

$$(24.3) \quad \text{For } k = 3: 1 - v_1 + v_r \geq C_\alpha^{(3)} ; 0 \leq v_r \leq v_1 \leq 1 .$$

The overall power is obtained by taking the expected value with respect to the distribution of T , which is $(2N)^{-1} \chi_{2N}^2$.

By performing the integration first with respect to T , and then with respect to v_r , we can reduce the triple integral to a finite sum of single integrals.

The integrand in (23) is

$$\begin{aligned} & T^2 \left\{ \sum_{i=0}^{s_0} (-1)^i \binom{s_0}{i} v_1^{iT} \right\} \left\{ \sum_{j=0}^{r-2} (-1)^j \binom{r-2}{j} (v_1^{r-2-j} v_2^j)^T \right\} v_1^T v_r^{(s_r+1)T} (v_1 v_r)^{-1} \\ &= (v_1 v_r)^{-1} T^2 \sum_{i=0}^{s_0} \sum_{j=0}^{r-2} (-1)^{i+j} \binom{s_0}{i} \binom{r-2}{j} v_1^{(r+i-j-1)T} v_r^{(s_r+j+1)T} . \end{aligned}$$

The expected value of this with respect to variation in T is

$$\begin{aligned} & (v_1 v_r)^{-1} \sum_{i=0}^{s_0} \sum_{j=0}^{r-2} (-1)^{i+j} \binom{s_0}{i} \binom{r-2}{j} (2N)^{-2} \times \\ & \times E \left[\chi_{2N}^4 \exp\{ (2N)^{-1} [(r+i-j-1) \log v_1 + (s_r+j+1) \log v_r] \chi_{2N}^2 \} \right] \\ (25) \quad &= (v_1 v_r)^{-1} N^{-1} (N+1) \sum_{i=0}^{s_0} \sum_{j=0}^{r-2} (-1)^{i+j} \binom{s_0}{i} \binom{r-2}{j} \times \\ & \times [1 - N^{-1} \{ (r+i-j-1) \log v_1 + (s_r+j+1) \log v_r \}]^{- (N+2)} . \end{aligned}$$

(Using the relation $E[\chi_v^4 \exp(A\chi^2)] = v(v+2)(1-2A)^{-\frac{1}{2}v-2}$ for $A < \frac{1}{2}$.)

The indefinite integral of (25) with respect to v_r is

$$v_1^{-1} \sum_{i=0}^{s_0} \sum_{j=0}^{r-2} (-1)^{i+j} \binom{s_0}{j} \binom{r-2}{j} (s_r + j + 1)^{-1} \times$$

$$[1 - N^{-1} \{(r + i - j - 1) \log v_1 + (s_r + j + 1) \log v_r\}]^{-(N+1)} .$$

For $k = 2$, the range of integration for v_r is $(1 - v_1)^{-1} C_\alpha^{(2)} \leq v_r \leq v_1$ and for v_1 , $v_1(1 - v_1) \geq C_\alpha^{(2)}$ i.e. $V_1^- \leq v_1 \leq V_1^+$ where

$$(26) \quad V_1^\pm = \frac{1}{2} \left[1 \pm \sqrt{1 - 4C_\alpha^{(2)}} \right] .$$

Hence

$$(27.2) \quad \beta_{(2)}(s_0, s_r) = \frac{(r+s_0+s_r)!}{s_0!(r-2)!s_r!} \sum_{i=0}^{s_0} \sum_{j=0}^{r-2} (-1)^{i+j} \binom{s_0}{i} \binom{r-2}{j} (s_r + j + 1)^{-1} \times$$

$$\times \left[(r + s_r + i)^{-1} \left\{ [1 - N^{-1}(r + s_r + i) \log V_1^+]^{-N} \right. \right.$$

$$\left. - [1 - N^{-1}(r + s_r + i) \log V_1^-]^{-N} \right\}$$

$$\left. - \int_{V_1^-}^{V_1^+} v_1^{-1} \{1 - N^{-1}[(s_r + j + 1) \log \frac{C_\alpha^{(2)}}{1 - v_1} + (r + i - j - 1) \log v_1]\}^{-(N+1)} dv_1 \right] .$$

When $k = 3$, the range of integration for v_r is $v_1 - (1 - C_\alpha^{(3)}) \leq v_r \leq v_1$, and the range of integration for v_1 is $1 - C_\alpha^{(3)} \leq v_1 \leq 1$.

Hence

$$(27.3) \beta_{(3)}(s_0, s_r) = \frac{(r+s_0+s_r)!}{s_0!(r-2)!s_r!} \sum_{i=0}^{s_0} \sum_{j=0}^{r-2} (-1)^{i+j} \binom{s_0}{j} \binom{r-2}{j} (s_r + j + 1)^{-1} \times$$

$$\times \left[(r + s_r + i)^{-1} \left\{ 1 - [1 - N^{-1}(r + s_r + i) \log (1 - C_\alpha^{(3)})]^{-N} \right\} \right.$$

$$\left. - \int_{1-C_\alpha^{(3)}}^1 v_1^{-1} \left\{ 1 - N^{-1} [(r+i-j-1) \log v_1 + (s_r+j+1) \log (v_1 - 1 + C_\alpha^{(3)})] \right\}^{-N} dv_1 \right].$$

6. Comments on Tables

Note that in all cases, for $s_0 + s_r$ constant, the power decreases as s_0 decreases. This is to be expected, since this test is designed to detect alternative hypotheses of the type $H_{s_0,0}$. Table 1 shows that the use of $C_\alpha^{(1)} = 1 - \alpha^{1/r}$ as critical limit increases the significance level, and also the power. As might be expected, this effect decreases as N increases.

If $C_\alpha^{(1)}(N)$ is used so that the actual significance level is α , Table 2 shows that the power increases with N . This is not always the case in Table 1. Only for $s_0 + s_r$ sufficiently big, and s_0 not too small, does the power increase for N in Table 1.

As $N \rightarrow \infty$ the values in both Tables tend to the values for the case when the population distribution is known, (which do not depend on the actual distribution, so long as it is known and continuous).

I would like to express my gratitude to Mr. Kerry L. Lee for programming the computation of extensive tables of which these are a part.

7. No Prior Estimate of Parameter Available

In this case we may (assuming the null hypothesis $H_{0,0}$ to be valid) estimate θ by θ^* , the arithmetic mean of the r available X 's. If $H_{0,0}$ is valid then θ^* is distributed as $(2r)^{-1}\theta\chi_{2r}^2$, but X_1, X_2, \dots, X_r are not independent of θ^* . In fact

$$\Pr\{X_1(r\theta^*)^{-1} < K | H_{0,0}\} = 1 - r(1 - K)^{r-1} + \binom{r}{2}(1 - 2K)^{r-1} \dots$$

(the series terminates at the m -th term when $1 - mk < 0 \leq 1 - (m - 1)K$).

In this case, the actual significance level for $k = 1$ (i.e. when testing for censoring from below) is

$$\begin{aligned} & \Pr\{Y_1^* > C_\alpha^{(1)} | H_{0,0}\} \\ (28) \quad & = \Pr\{X_1(r\theta^*) > -r^{-1} \log(1 - C_\alpha^{(1)})\} \\ & = r\{1+r^{-1}\log(1-C_\alpha^{(1)})\}^{r-1} - \binom{r}{2}\{1+2r^{-1}\log(1-C_\alpha^{(1)})\}^{r-1} + \dots \end{aligned}$$

It is possible to choose $C_\alpha^{(1)}$ to make the actual significance level equal to α but we cannot give an explicit formula, as in (16).

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TABLE 1
Power w.r.t. H_{s_0, s_r} of (11.1) with $C_\alpha^{(1)} = 1 - \alpha^{\frac{1}{r}}$ ($\alpha = 0.05$)

r= N=		5			
s_0	s_r	5	10	25	50
0	0	.0956	.0728	.0590	.0545
2	0	.3896	.3572	.3334	.3245
1	1	.1810	.1438	.1192	.1106
4	0	.6287	.6237	.6211	.6204
3	1	.4350	.4035	.3793	.3699
6	0	.7789	.7955	.8109	.8174
5	1	.6348	.6316	.6309	.6309
4	2	.4681	.4385	.4150	.4056
8	0	.8673	.8919	.9124	.9205
7	1	.7679	.7858	.8032	.8109
6	2	.6398	.6383	.6394	.6403

r= N=		10				15			
s_0	s_r	5	10	25	50	5	10	25	50
0	0	See r=5 values				See r=5 values			
2	0	.4285	.4003	.3796	.3719	.4433	.4167	.3973	.3901
1	1	.2218	.1853	.1605	.1517	.2379	.2019	.1774	.1686
4	0	.7005	.7050	.7101	.7124	.7271	.7345	.7418	.7449
3	1	.5321	.5140	.5007	.4957	.5695	.5564	.5472	.5439
6	0	.8546	.8754	.8925	.8993	.8800	.9006	.9169	.9231
5	1	.7516	.7638	.7755	.7805	.7924	.8082	.8221	.8278
4	2	.6121	.6050	.6008	.5994	.6672	.6677	.6697	.6708
8	0	.9311	.9514	.9657	.9707	.9497	.9668	.9779	.9816
7	1	.8748	.8979	.9165	.9238	.9073	.9287	.9448	.9507
6	2	.7900	.8082	.8248	.8318	.8399	.8606	.8782	.8852

(cont.)

TABLE 1 (cont.)

s ₀	s _r	r=20				r=25			
		N=5	10	25	50	5	10	25	50
0	0	See r=5 values				See r=5 values			
2	0	.4511	.4253	.4067	.3997	.4559	.4307	.4124	.4056
1	1	.2465	.2109	.1865	.1778	.2519	.2165	.1922	.1836
4	0	.7409	.7496	.7579	.7614	.7493	.7588	.7676	.7713
3	1	.5891	.5786	.5716	.5690	.6013	.5923	.5865	.5845
6	0	.8925	.9127	.9282	.9340	.8999	.9197	.9347	.9402
5	1	.8128	.8298	.8444	.8503	.8251	.8426	.8574	.8633
4	2	.6957	.6998	.7045	.7067	.7132	.7192	.7254	.7282
8	0	.9582	.9734	.9829	.9860	.9630	.9770	.9856	.9883
7	1	.9224	.9423	.9567	.9618	.9310	.9498	.9630	.9677
6	2	.8639	.8849	.9019	.9085	.8778	.8986	.9150	.9213

TABLE 2

Power w.r.t. H_{s_0, s_r} of (11.1) with $C_{\alpha}^{(1)} = 1 - \exp\{-r^{-1} N(\alpha^{-1/N} - 1)\}$ ($\alpha = 0.05$)

s ₀	s _r	r=5			
		N=5	10	25	50
2	0	.2408	.2736	.2973	.3059
1	1	.0964	.0993	.1008	.1013
4	0	.4396	.5188	.5764	.5975
3	1	.2678	.3075	.3370	.3480
6	0	.5965	.7027	.7746	.7996
5	1	.4368	.5200	.5826	.6062
4	2	.2874	.3329	.3677	.3810
8	0	.7099	.8216	.8885	.9096
7	1	.5751	.6855	.7634	.7913
6	2	.4346	.5210	.5881	.6139

(cont.)

TABLE 2 (cont.)

r=	N=	10				15			
		5	10	25	50	5	10	25	50
s ₀	s _r								
2	0	.2731	.3132	.3420	.3526	.2858	.3286	.3594	.3706
1	1	.1241	.1327	.1385	.1405	.1355	.1465	.1541	.1568
4	0	.5174	.6077	.6701	.6923	.5484	.6414	.7042	.7261
3	1	.3537	.4133	.4371	.4733	.3895	.4562	.5043	.5219
6	0	.7008	.8054	.8678	.8877	.7402	.8402	.8965	.9137
5	1	.5682	.6689	.7376	.7617	.6204	.7229	.7893	.8118
4	2	.4212	.4981	.5548	.5759	.4794	.5656	.6270	.6492
8	0	.8196	.9107	.9542	.9659	.8575	.9352	.9700	.9784
7	1	.7253	.8325	.8946	.9137	.7798	.8776	.9289	.9437
6	2	.6090	.7173	.7898	.8147	.6782	.7851	.8509	.8723

r=	N=	20			25				
		5	10	25	50	5	10	25	50
s ₀	s _r								
2	0	.2926	.3369	.3686	.3802	.2969	.3420	.3743	.3861
1	1	.1417	.1541	.1626	.1656	.1456	.1588	.1679	.1712
4	0	.5650	.6591	.7217	.7434	.5754	.6699	.7323	.7538
3	1	.4091	.4791	.5292	.5474	.4214	.4934	.5446	.5631
6	0	.7608	.8574	.9100	.9257	.7733	.8676	.9178	.9326
5	1	.6481	.7503	.8145	.8358	.6652	.7667	.8292	.8497
4	2	.5115	.6015	.6641	.6864	.5317	.6235	.6865	.7087
8	0	.8763	.9477	.9766	.9834	.8874	.9541	.9801	.9861
7	1	.8073	.8985	.9436	.9561	.8237	.9103	.9516	.9628
6	2	.7144	.8180	.8787	.8977	.7364	.8372	.8942	.9117